

Securing progression in mathematics

Following the training in the generic unit *Securing progression*, it is important to now consider how the key messages of the training apply to mathematics. As part of the whole-school focus on assessment for learning, this subject development material is intended to help you consider the key messages of the training unit and identify any areas requiring development in your department.

This unit is aimed at schools that have established a secure foundation of assessment for learning practice across departments and have undertaken developmental work in curricular target setting (Unit 6.1). It will enable teachers to consolidate, crystallise and deepen their understanding of the assessment for learning practice covered in Units 3–6 and help schools focus and sustain further development.

The unit focuses particularly on two key characteristics of assessment for learning:

- sharing learning goals with pupils;
- helping pupils to know and recognise the standards they are aiming for.

Objectives

- To understand the need to plan for different types of learning outcomes and curricular targets.
- To understand the need for different intervention strategies to enable pupils to make progress towards achieving curricular targets.
- To identify strategies which enable pupils to take responsibility for their own progress and become more independent learners.

Key messages

- Teachers need to be able to identify different types of learning outcomes and curricular targets if they are to plan the most appropriate intervention strategies to enable pupils to make progress.
- This unit focuses on curricular targets that are important across the curriculum and within individual subjects. These are the recurring processes, skills and the understanding of concepts that can be performed at a range of levels.
- Progression towards achieving curricular targets must be at the forefront of teachers' short- and medium-term planning and should be explicitly and assertively taught and assessed. Assessment for learning techniques can support this process and make a significant impact on standards of attainment.
- There are many intervention strategies that can help individual pupils to progress over the short and medium term. They require teachers and pupils to understand detailed progression in the processes, skills or concepts being taught. Consequently, pupils' rates of learning can increase dramatically, as can their independence and motivation.
- For pupils to become more independent learners, they need to take increasing responsibility for identifying where they are in their learning, where they are trying to get to and what steps they need to take to get there. They need to be taught how to do this.

Reviewing existing practice in securing progression

The following progression table provides a tool for a department to self-review current practice and to help identify an appropriate starting point.

As a department, agree and highlight the statements that best reflect the practice of the whole department. At the bottom of the column is a reference to the tasks that will support your current practice and provide the appropriate material to develop from this point.

Having completed this review you should read 'Making effective use of the subject development material' on page 4.

	Focusing	Developing	Establishing	Enhancing
Teachers	<p>The subject leader has identified that:</p> <ul style="list-style-type: none"> teachers have a limited understanding of progression in the key concepts and skills in their subject progression in key concepts and skills is not clearly mapped out in schemes of work pupils have subject-specific curricular targets but teachers do not explicitly teach to them or monitor pupils' progress towards them there is no clear link between learning outcomes in lessons and curricular targets. 	<p>Teachers work collaboratively to develop their understanding of progression in the key concepts and skills for their subject.</p> <p>Schemes of work address progression in key concepts and skills in each subject.</p> <p>Teachers plan lessons to objectives and outcomes linked to curricular targets.</p> <p>Teachers communicate curricular targets to pupils and plan opportunities to review their progress towards them.</p> <p>Teachers are beginning to discuss success criteria with pupils so that pupils understand what is good about their work and how to improve it.</p>	<p>Teachers regularly review progression in key concepts and skills for both subject-specific and cross-curricular targets to crystallise their understanding and refine success criteria.</p> <p>Teachers' understanding of progression in key concepts and skills routinely informs medium- and short-term planning.</p> <p>Teachers use a range of strategies to explore success criteria with pupils and use them to inform peer and self assessment activities.</p> <p>Teachers regularly review progress towards curricular targets with pupils.</p> <p>Teachers regularly discuss links between the learning objectives and outcomes and curricular targets with pupils.</p>	<p>There is a coherent whole-school system for planning for progression towards subject and cross-curricular targets.</p> <p>School effectiveness is measured by the progress pupils make towards their curricular targets.</p> <p>All teachers understand progression in the skills and concepts to be taught. Progression towards these is at the forefront of collaborative short- and medium-term planning.</p> <p>Teachers work with pupils to identify success criteria which enable them to independently make progress towards curricular targets.</p>
Pupils	<p>The subject leader has identified that:</p> <ul style="list-style-type: none"> pupils are often unable to recognise when they are making progress or to explain what success would look like most pupils know the learning outcomes in lessons but few can relate them to longer-term curricular targets pupils do not recognise that the specific concepts or skills they are learning in one lesson can be applied across many contexts within the subject pupils tend to be reliant on teachers for assessing and directing their next steps in learning. 	<p>Typically, pupils can recognise the progress they are making and can identify what makes a successful learning outcome in a lesson.</p> <p>When prompted, most pupils can explain how the learning outcomes in a lesson will help them make progress towards their personal targets.</p> <p>Pupils are beginning to recognise how they can apply their learning in key concepts and skills across a subject.</p> <p>Pupils are beginning to use success criteria to identify what is good about their work and how they could improve it.</p>	<p>Most pupils understand the progression in key concepts and skills and can provide examples.</p> <p>Typically pupils can explain how the learning outcomes in a lesson will help them make progress towards their personal targets.</p> <p>In lessons, pupils can discuss the strengths of their work and the areas they need to improve.</p> <p>Pupils can use success criteria to identify for themselves precisely what they need to learn and what steps they need to take.</p> <p>Pupils can articulate clearly what their subject and cross-curricular targets are and know how they are progressing towards these.</p>	<p>All pupils understand progression in the skills and concepts they are learning.</p> <p>All pupils can explain their curricular targets and can evaluate the progress they are making towards them.</p> <p>All pupils can use success criteria to engage in extended and focused dialogue about their learning.</p> <p>All pupils can relate learning outcomes in a lesson to progression towards their curricular targets.</p> <p>Pupils independently identify and take their next steps in learning.</p>
	Begin with task 6.2A	Begin with task 6.2B	Begin with task 6.2C	Begin with task 6.2C

Making effective use of the subject development material

The tasks you have been referred to are intended to focus the development or extension of securing progression in mathematics and provide guidance on how to embed this into the regular practice in mathematics lessons.

The results of the self-review will have suggested the appropriate task(s) to support your department's development needs.

To make best use of the supporting material the following sequence will be helpful:

1. Read the task and the supporting exemplification.

This describes how a department has approached the task and worked through each of its stages. It is given as an example of how the task might be addressed. It is not intended that you follow this approach, which is given as a guide to the process that will support improvements in your subject.

2. Identify what the department did and the impact it had on pupils.

Discuss as a team the example provided and establish the key areas that helped to develop this practice and the impact it had on pupils. It will be helpful to identify the changes in teachers' practice and how this impacted on pupils' learning.

3. Agree and plan the actions that will develop your practice.

As a department, agree how you intend to approach this task. Clarify what you are focusing on and why. The example given will act as a guide, but be specific about which classes, which lessons and which aspects of the curriculum will be your points of focus.

4. Identify when and how you will evaluate its impact on pupils.

The purpose of focusing on this is to improve pupils' achievement and attainment in mathematics. You will need to be clear on what has helped pupils to learn more effectively in your subject. Part of this will be how your practice has adapted to allow this. You should jointly identify what has worked well and which areas require further attention.

5. Having evaluated these strategies consider what steps are required to embed this practice.

You will need to undertake an honest evaluation of what you have tried and the impact it has had on your teaching and on pupils' learning. One outcome might be that you need to spend longer on improving this area or you may be in a position to consider the next task.

Other departments in the school will have been focusing on this area and you should find out about the progress they have made.

You may find that some teachers in the department will require further time to develop and consolidate new practice, while others will be ready to progress further through the tasks in this area (while continuing to support their colleagues). Practice across a department will need to be consolidated before focusing on a new area of assessment for learning.

The subject development tasks

Handouts and slides referenced in the text are found in Unit 6.2, 'Securing progression' of the *Assessment for learning whole school training materials* folder (DfES 0043-2004 G), and on the *Assessment for learning whole school training materials* CD-ROM, 2nd edition (DfES 1240-2005 G CD).

Task 6.2A

As a department, select a sequence of between six and ten lessons from a medium-term plan or study unit. Identify the learning outcomes you would expect from each lesson and from the block of lessons as a whole.

Place these in their appropriate positions on **handout 6.2.2** to identify whether they are short-term learning outcomes or longer-term curricular targets, and where they sit on the study unit to cross-curricular (vertical) continuum.

Agree which of the curricular targets (i.e. those to the right of the vertical line) your pupils experience most difficulty in progressing towards. Identify how far the short-term learning outcomes help pupils to progress towards achieving these longer-term curricular targets.

As a department, plan how to go about addressing the issues this activity has raised and agree actions.

Task 6.2B

Select one curricular target your pupils experience difficulty in progressing towards. As a department use the following steps to help you produce 'staged success criteria' for this curricular target (see **handout 6.2.6**).

1. For one year group or class, select between five and ten pieces of work which focus on the identified curricular target. Select pupils from across the ability range. Video or audio recordings can be used for performance-based outcomes.
2. Rank the pupil outcomes then analyse them to identify two or three important factors to explain why each is better than the one below it.
3. Produce staged success criteria for the curricular target in question along the lines of those in handout 6.2.6. These should describe progress towards the curricular target in question. Make them as 'pupil friendly' as possible.
4. Use the criteria in a lesson to inform peer and self assessment which helps pupils to improve their work. You may find it helpful to watch the geography lesson sequence which supports Unit 6.2 again or to use **handout 6.2.7** for ideas.

continued

Task 6.2C

Select a curricular target your pupils experience difficulty in progressing towards. If you have already carried out **task 6.2B** consider choosing a different curricular target. Ensure that all teachers in the department are secure in their understanding of progression towards this curricular target. If they are not, follow steps 1 to 3, as described in **task 6.2B**, so you agree the progression necessary to achieve the curricular target.

Select two or three strategies from **handout 6.2.9** to trial in the classroom and use these over the course of several lessons to support pupils' progress towards the curricular target.

As a department, evaluate the successes and issues for the strategies trialled. Consider:

- which enabled most pupils to make most progress;
- which built the greatest pupil independence;
- which generated most pupil motivation.

The following pages provide examples of each task.

Task 6.2A

As a department, select a sequence of between six and ten lessons from a medium-term plan or study unit. Identify the learning outcomes you would expect from each lesson and from the block of lessons as a whole.

Place these in their appropriate positions on handout 6.2.2 to identify whether they are short-term learning outcomes or longer-term curricular targets, and where they sit on the study unit to cross-curricular (vertical) continuum.

Agree which of the curricular targets (i.e. those to the right of the vertical line) your pupils experience most difficulty in progressing towards. Identify how far the short-term learning outcomes help pupils to progress towards achieving these longer-term curricular targets.

As a department, plan how to go about addressing the issues this activity has raised and agree actions.

Context

Following the whole-school focus on securing progression, the mathematics department decided to strengthen their work on achieving curricular targets. In the previous term, Year 8 teachers had analysed pupils' responses to questions on the Year 8 optional tests. They found that pupils targeted to achieve level 5 or higher at the end of Key Stage 3 had not performed well on the algebra questions. Closer analysis of the targeted group revealed a particular weakness with solving equations and simplifying expressions. Teachers were aware that the test questions assessed a limited part of this strand of algebra and they were keen to look at this in greater depth.

Process

In order to understand the issues with this strand of mathematics the department agreed to focus on two of its algebra units: algebra 2 and 4. The learning objectives for these units are given in **appendix 6.2A.1**.

The department worked to identify the learning outcomes for both units of work and positioned these on **handout 6.2.2** (see **appendix 6.2A.2**) to identify whether they are short-term learning outcomes or longer-term curricular targets, and where they sit on the subject to cross-curricular continuum.

The department identified that the curricular targets pupils had most difficulty in progressing towards were:

- you will be able to simplify linear expressions by collecting like terms;
- you will be able to solve linear equations, including those with unknowns on either or both sides, without and with brackets.

Further discussion revealed that the most important short-term learning outcomes for these curricular targets were not explicitly taught in Year 8, so the department looked back at the medium-term planning for Year 7 and added two further learning outcomes (shaded on **appendix 6.2A.2**).

Evaluation

Teachers recognised the importance of these curricular targets for progression across Key Stages 3 and 4, so decided to check their planning and collaboratively review the progression in their teaching at the next department meeting.

Algebra learning objectives

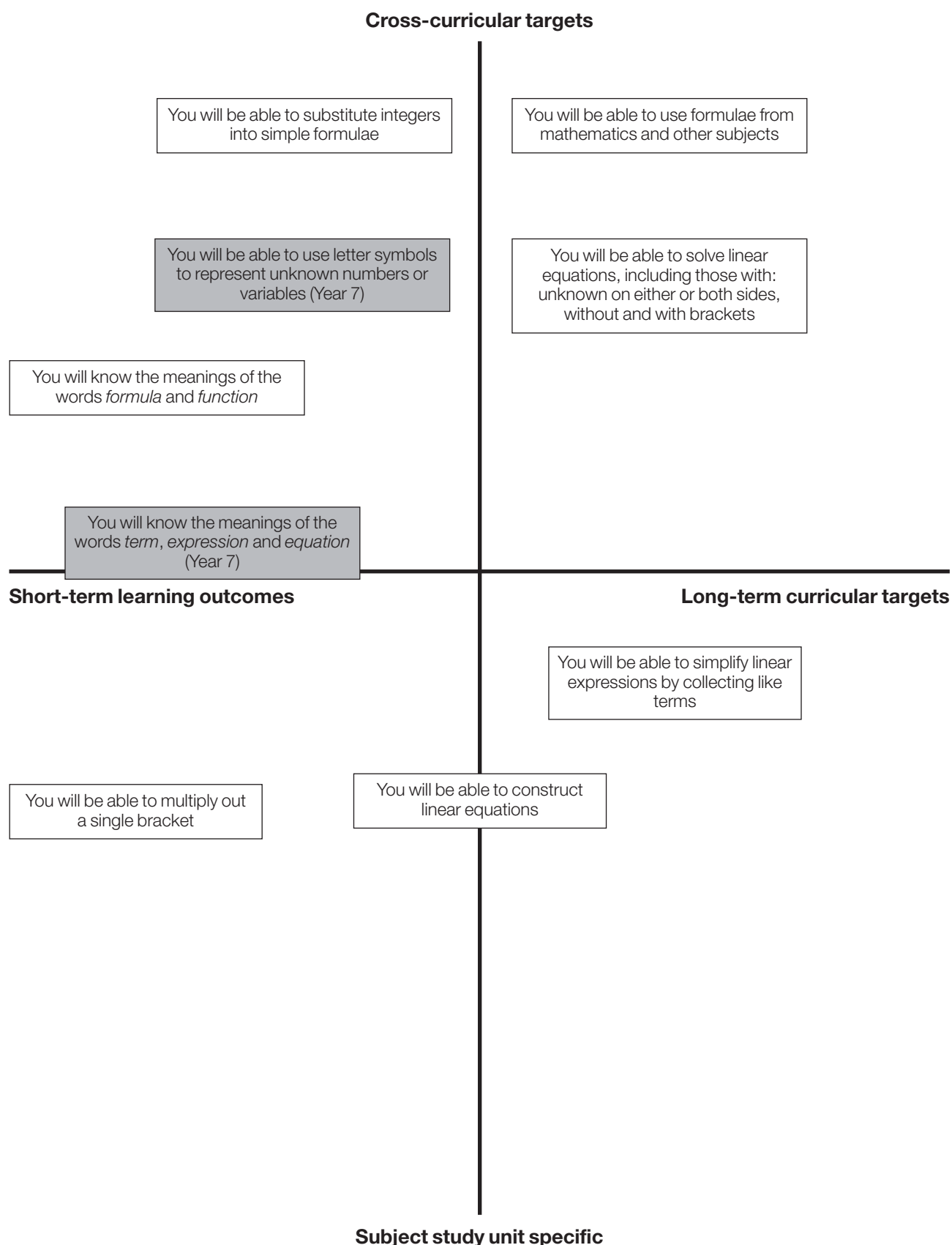
Algebra 2

- Begin to distinguish the different roles played by letter symbols in equations, formulae and functions; know the meanings of the words *formula* and *function*.
- Know that algebraic operations follow the same conventions and order as arithmetic operations; use index notation for small positive integer powers.
- **Simplify or transform linear expressions by collecting like terms; multiply a single term over a bracket.**
- Use formulae from mathematics and other subjects; **substitute integers into simple formulae**, and positive integers into expressions involving small powers (e.g. $3x^2 + 4$ or $2x^2$); derive simple formulae.

Algebra 4

- Begin to distinguish the different roles played by letter symbols in equations, formulae and functions; know the meanings of the words *formula* and *function*.
- Construct and solve linear equations with integer coefficients (unknown on either or both sides, without and with brackets) using appropriate methods (e.g. inverse operations, transforming both sides in the same way).
- Use formulae from mathematics and other subjects; **substitute integers into simple formulae**, including examples that lead to an equation to solve; derive simple formulae.

Continuum diagram



Task 6.2B

Select one curricular target your pupils experience difficulty in progressing towards. As a department use the following steps to help you produce 'staged success criteria' for this curricular target (see handout 6.2.6).

1. For one year group or class, select between five and ten pieces of work which focus on the identified curricular target. Select pupils from across the ability range. Video or audio recordings can be used for performance-based outcomes.
2. Rank the pupil outcomes then analyse them to identify two or three important factors to explain why each is better than the one below it.
3. Produce staged criteria for the curricular target in question along the lines of those in handout 6.2.6. These should describe progress towards the curricular target in question. Make them as 'pupil friendly' as possible.
4. Use the criteria in a lesson to inform peer and self assessment which helps pupils to improve their work. You may find it helpful to watch the geography lesson sequence which supports Unit 6.2 again or to use handout 6.2.7 for ideas.

Context

Following the work on **task 6.2A** the department was able to send two teachers (the head of department and the teacher with responsibility for Key Stage 3 mathematics) on the core training *Constructing and solving linear equations*. Having attended the training and considered the materials available, the head of department decided to focus on exploring the progression and teaching approaches for the curricular target:

- you will be able to simplify linear expressions by collecting like terms.

This was chosen as the department felt it was a key precursor to the other curricular target identified as being cause for concern:

- you will be able to solve linear equations, including those with unknowns on either or both sides, without and with brackets.

Process

After the training, the two teachers met to consider how the materials and approaches would help address their curricular target. They decided to trial 'clouding the picture' with their Year 8 classes, focusing on expressions, and to use pupils' work for discussion at a department meeting. Six pieces of work were selected to share with the department at the meeting.

At the department meeting the head of department introduced the principle of 'find as many ways as you can' using the resource sheets 'clouding the picture' from the *Constructing and solving linear equations: Year 8 booklet* (DfES 0084-2004 G). She asked everyone to rewrite each of the three given expressions in as many ways as they could. **Appendix 6.2B.1** contains the resource sheets they used as a starting point. The head of department posed the following questions to prompt discussion.

- What mathematics is being used to work on this task?

- What potential does this approach have in developing pupils' understanding of simplifying linear expressions by collecting like terms?

The department felt that this approach would be beneficial to pupils as it would:

- be different from current ways of teaching simplifying linear expressions;
- encourage pupils to bring together the different ways they know of manipulating expressions rather than working on them one at a time;
- promote discussion on the meaning of equivalent expressions;
- move away from there being a single correct answer.

The department then looked at the materials trialled with the Year 8 class (see **appendix 6.2B.2**) and a sample of six pieces of pupils' work (see **appendix 6.2B.3** which lists pupils' responses in bold). They worked in pairs to rank the pieces of work according to the complexity and accuracy of the mathematics that each pupil had used; the pairs were also asked to justify their ranking. After the activity the department agreed what each pupil had demonstrated and recorded this on each sheet (shown in *italics*).

The department then discussed the steps in the progression and produced a learning path for the curricular target 'you will be able to simplify linear expressions by collecting like terms'. The learning path is included as **appendix 6.2B.4**. This was also produced as a poster for use in the classroom.

The department decided that teachers would all trial the activity with their Year 8 groups to identify pupils' current level of understanding of simplifying expressions and collecting like terms. They then used this work to identify where pupils were on the learning path. In the following lesson, the learning path was used with pupils to help them locate their own position on the path and the next stages in their learning.

Evaluation

Before the next department meeting teachers were asked to reflect on this work and consider the following.

- Where were the majority of the class on the learning path after the initial activity? Is this what you expected? What anomalies were there?
- Where were the majority of the class on the learning path after the second piece of work? What anomalies were there?
- How did pupils engage with the activity?

The discussions at the department meeting highlighted the following points.

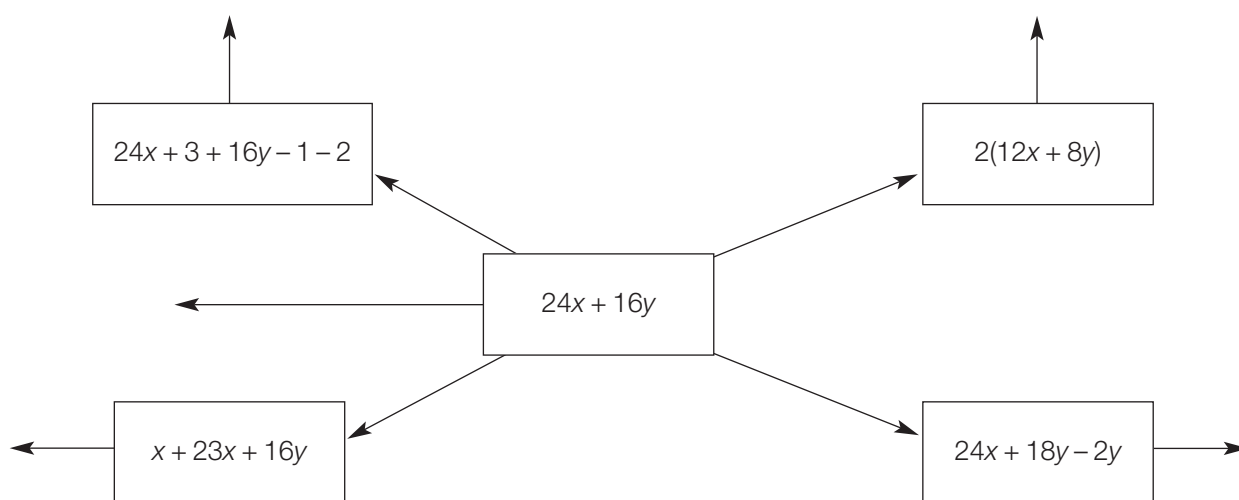
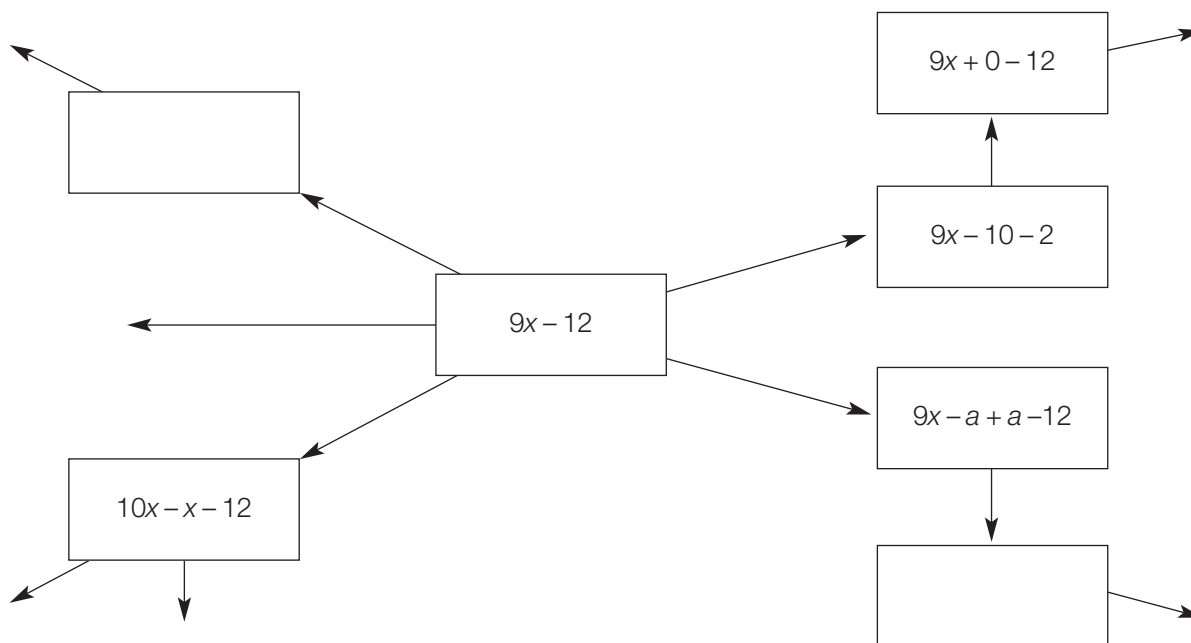
- The majority of pupils enjoyed the different style of activity.
- Some pupils were noticeably motivated by the learning path.
- The majority of revisited work illustrated a positive shift on the learning path.
- A few pupils tried to create expressions to meet the higher-level outcomes without some of the intermediary steps. This often resulted in expressions that reflected misconceptions.
- Teachers were positive about the activity and the learning path and felt both had strengthened teaching and learning.

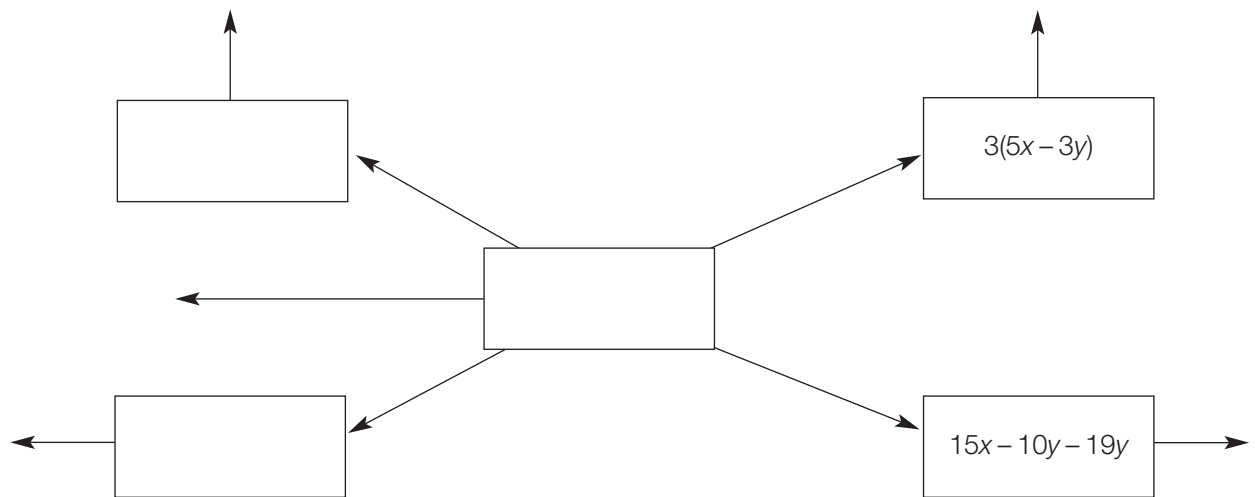
Teachers decided to incorporate the approach to teaching linear expressions into the Year 8 scheme of work. They also started to explore other developments of the 'clouding the picture' approach, including solving linear equations.

In the future, staff want to develop the idea of learning paths in order that:

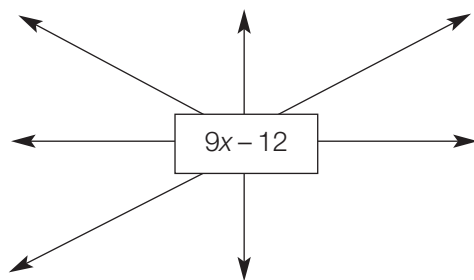
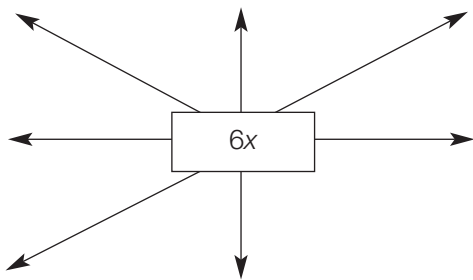
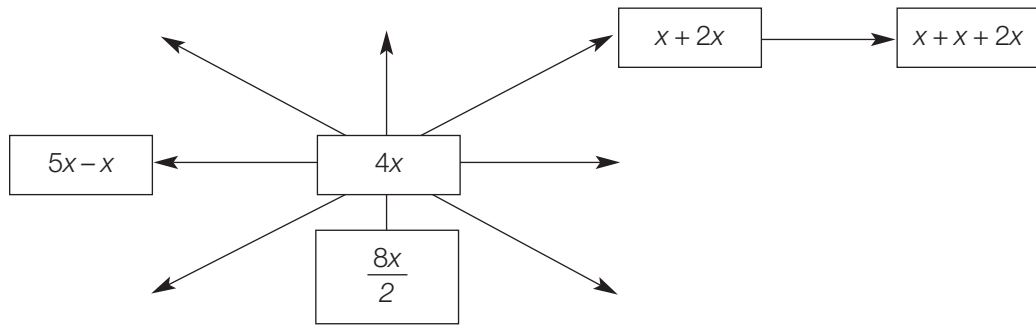
- they have a common understanding of the progression for crucial learning outcomes;
- they have a tool that enables them to discuss aspects of mathematics with pupils in a meaningful way;
- they have a framework to help support pupils with peer and self assessment.

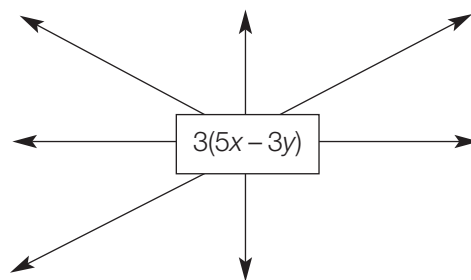
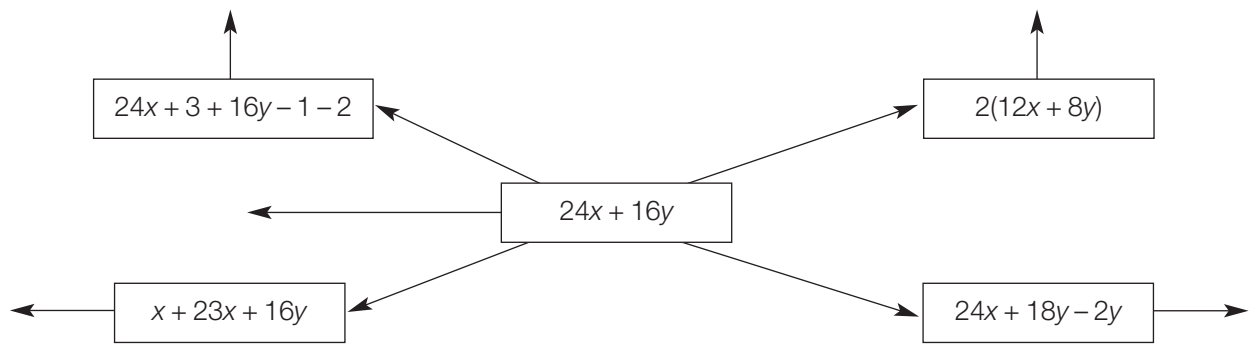
Expressions

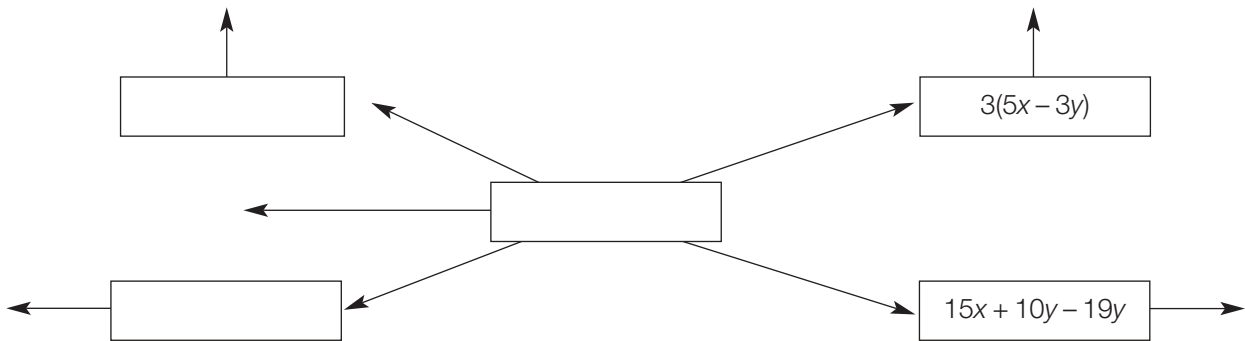




Clouding the picture: expressions







Pupil 1

4x

$5x - x$	$x + 3x$	$\frac{8x}{2}$	$3x + x$
	$x + x + 2x$		$x + x + 2x$
	$x + x + x + x$		$2x + 2x$

6x

$x + x + x + x + x + x$	$5x + x$	$10x - 4x$
$x + x + x + x + 2x$	$4x + 2x$	
$x + x + x + 3x$	$3x + 3x$	
$x + x + 4x$	$2x + 4x$	
$x + 5x$	$x + 5x$	

Pupil 1 did not attempt any more of the problems.

Pupil 1 can expand expressions with one variable using addition.

Pupil 2

4x

$5x - x$	$x + 3x$	$\frac{8x}{2}$	$3x + x$
$6x - 2x$	$x + x + 2x$	$\frac{16x}{4}$	$x + x + 2x$
$7x - 3x$	$x + x + x + x$	$\frac{32x}{8}$	$2x + 2x$
$8x - 4x$		$\frac{64x}{16}$	

6x

$x + x + x + x + x + x$	$5x + x$	$7x - x$
$x + x + x + x + 2x$	$4x + 2x$	$8x - 2x$
$x + x + x + 3x$	$3x + 3x$	$9x - 3x$
$x + x + 4x$	$2x + 4x$	$10x - 4x$
$x + 5x$	$x + 5x$	

9x - 12

$8x + x - 12$	$9x - 10 - 2$	$10x - x - 12$
$7x + 2x - 12$	$9x - 11 - 1$	$11x - x - x - 12$
$6x + 3x - 12$	$9x - 12 - 0$	

Pupil 2 did not attempt any more of the problems.

Pupil 2 can expand expressions with one variable using addition and subtraction. The pupil recognises the difference between the x term and numbers.

Pupil 3

4x

$5x - x$	$x + 3x$	$\frac{8x}{2}$	$3x + x$	$4 \times x$
$6x - 2x$	$x + x + 2x$	$\frac{16x}{4}$	$x + x + 2x$	$x \times 2x$
$7x - 3x$	$x + x + x + x$	$\frac{32x}{8}$	$2x + 2x$	
$8x - 4x$		$\frac{64x}{16}$		
$9x - 5x$, etc				

6x

$x + x + x + x + x + x$	$5x + x$	$7x - x$	$\frac{12x}{2}$	$6 \times x$
$x + x + x + x + 2x$	$4x + 2x$	$8x - 2x$	$\frac{24x}{4}$	
$x + x + x + 3x$	$3x + 3x$	$9x - 3x$	$\frac{48x}{8}$	
$x + x + 4x$	$2x + 4x$	$10x - 4x$		
$x + 5x$	$x + 5x$	$11x - 5x$		

9x - 12

$8x + x - 12$	$9x - 10 - 2$	$10x - x - 12$
$7x + 2x - 12$	$9x - 11 - 1$	$11x - 2x - 12$
$6x + 3x - 12$	$9x - 12 - 0$	$12x - 3x - 12$
$5x + 4x - 12$		

24x + 16y

$x + 23x + 16y$	$24x + y + 15y$	$24x + 18y - 2y$
$2x + 22x + 16y$	$24x + 2y + 14y$	$24x + 17y - y$
$3x + 21x + 16y$	$24x + 3y + 13y$	
$4x + 20x + 16y$		

Pupil 3 did not attempt any more of the problems.

Pupil 3 can expand expressions with two variables using addition and subtraction and is beginning to expand expressions with one variable using multiplication and division. The pupil recognises the difference between the x term and numbers.

Pupil 4

4x

$5x - x$	$x + 3x$	$\frac{8x}{2}$	$3x + x$	$4 \times x$	$\frac{4x}{1}$
$6x - 2x$	$x + x + 2x$	$\frac{16x}{4}$	$x + x + 2x$	$2 \times 2 \times x$	$\frac{8x}{2}$
$7x - 3x$	$x + x + x + x$	$\frac{32x}{8}$	$2x + 2x$	$2 \times 2x$	$\frac{12x}{3}$
$8x - 4x$		$\frac{64x}{16}$			$\frac{16x}{4}$
$9x - 5x$, etc					etc

6x

$x + x + x + x + x + x$	$5x + x$	$7x - x$	$\frac{12x}{2}$	$6 \times x$
$x + x + x + x + 2x$	$4x + 2x$	$8x - 2x$	$\frac{24x}{4}$	$2 \times 3 \times x$
$x + x + x + 3x$	$3x + 3x$	$9x - 3x$	$\frac{48x}{8}$	
$x + x + 4x$	$2x + 4x$	$10x - 4x$		
$x + 5x$	$x + 5x$			

9x - 12

$8x + x - 12$	$9x - 2 - 10$	$10x - x - 12$
$7x + 2x - 12$	$9x - 1 - 11$	$11x - 2x - 12$
$6x + 3x - 12$	$9x - 10 - 2$	$12x - 3x - 12$
$5x + 4x - 12$	$9x - 11 - 1$	$12x - 3x - 10 - 2$

24x + 16y

$x + 23x + 16y$	$24x + y + 15y$	$24x + 18y - 2y$	$2(12x + 8y)$
$2x + 22x + 16y$	$24x + 2y + 14y$	$24x + 17y - y$	$4(6x + 4y)$
$3x + 21x + 16y$	$24x + 3y + 13y$	$24x + 16y - 0$	
$4x + 20x + 16y$		$24x + 15y + y$	
		$24x + 14y + 2y$	

Pupil 4 did not attempt any more of the problems.

Pupil 4 can expand expressions with two variables using addition, subtraction, multiplication and division. The pupil recognises the difference between the x term and numbers. The pupil is beginning to factorise expressions by taking a numerical factor outside a bracket.

Pupil 5

4x

$5x - x$	$x + 3x$	$\frac{8x}{2}$	$3x + x$	$4 \times x$	$\frac{4x}{1}$
$6x - 2x$	$x + x + 2x$	$\frac{16x}{4}$	$x + x + 2x$	$2 \times 2 \times x$	$\frac{8x}{2}$
$7x - 3x$	$x + x + x + x$	$\frac{32x}{8}$	$2x + 2x$	$2 \times 2x$	$\frac{12x}{3}$
$8x - 4x$		$\frac{64x}{16}$			$\frac{16x}{4}$
$9x - 5x$, etc					etc

6x: pupil 5 did not have to do this task.

9x – 12

$8x + x - 12$	$9x - 2 - 10$	$10x - x - 12$	$9x - a + a - 12$
$7x + 2x - 12$	$9x - 1 - 11$	$11x - 2x - 12$	$9x - 2a + 2a - 12$
$6x + 3x - 12$	$9x - 10 - 2$	$12x - 3x - 12$	
$5x + 4x - 12$	$9x - 11 - 1$		

24x + 16y

$x + 23x + 16y$	$24x + 18y - 2y$	$24x + 19y - 3y$	$2(12x + 8y)$	$\frac{48x + 32y}{2}$
$2x + 22x + 16y$	$24x + 17y - y$	$24x + 20y - 4y$	$4(6x + 4y)$	$\frac{96x + 64y}{4}$
$3x + 21x + 16y$	$24x + 16y - 0$	$24x + 21y - 5y$	$8(3x + 2y)$	
$4x + 20x + 16y$	$24x + 15y + y$			
$5x + 19x + 16y$	$24x + 14y + 2y$			
etc	etc			

Pupil 5 did not attempt any more of the problems.

Pupil 5 can expand expressions with two variables using addition, subtraction, multiplication and division. The pupil recognises the difference between terms and numbers and can introduce a new variable to make an equivalent expression. The pupil is beginning to factorise expressions by taking a numerical factor outside a bracket.

Pupil 6

4x

$5x - x$	$x + 3x$	$\frac{8x}{2}$	$3x + x$	$4 \times x$	$\frac{4x}{1}$
$6x - 2x$	$x + x + 2x$	$\frac{16x}{4}$	$x + x + 2x$	$2 \times 2 \times x$	$\frac{8x}{2}$
$7x - 3x$	$x + x + x + x$	$\frac{32x}{8}$	$2x + 2x$	$2 \times 2x$	$\frac{12x}{3}$
$8x - 4x$		$\frac{64x}{16}$			$\frac{16x}{4}$
$9x - 5x$					$\frac{20x}{5}$
$100x - 96x$					$\frac{100x}{25}$
$234x - 230x$					

6x: pupil 6 did not have to do this task.

9x - 12

$8x + x - 12$	$9x - 10 - 2$	$10x - x - 12$	$9x - a + a - 12$	$9x + a - a - 12$	$\frac{18x + 24y}{2}$
$7x + 2x - 12$	$9x - 11 - 1$	$11x - 2x - 12$	$9x - 2a + 2a - 12$	$9x + 2a - 2a - 12$	
$6x + 3x - 12$	$9x - 12 - 0$	$12x - 3x - 12$	$9x - 3a + 3a - 12$	$9x + 3a - 3a - 12$	
$5x + 4x - 12$	$9x - 13 + 1$		$9x - 4a + 4a - 12$		
	$9x - 14 + 2$				

24x + 16y

$x + 23x + 16y$	$24x + 18y - 2y$	$24x + 19y - 3y$	$2(12x + 8y)$	$24x + 3 + 16y - 1 - 2$
$2x + 22x + 16y$	$24x + 17y - y$	$24x + 20y - 4y$	$4(6x + 4y)$	$24x + 4 + 16y - 1 - 3$
$3x + 21x + 16y$	$24x + 16y - 0$	$24x + 21y - 5y$	$8(3x + 2y)$	$24x + 5 + 16y - 1 - 4$
$4x + 20x + 16y$	$24x + 15y + y$			$24x + 6 + 16y - 1 - 5$
$5x + 19x + 16y$	$24x + 14y + 2y$			
etc	etc			

3(5x - 3y) and 15x + 10y - 19y

$15x + 10y - 19y$	$15x - 9y$	$3(5x - 3y)$
$15x + 11y - 20y$	$10x + 5x + 10y - 19y$	$6(2.5x - 1.5y)$
$15x + 12y - 21y$	$9x + 6x + 9y - 18y$	

Pupil 6 can expand expressions with two variables using addition, subtraction, multiplication and division. The pupil recognises the difference between terms and numbers and can introduce a new variable to make an equivalent expression. The pupil can factorise expressions by taking a numerical factor outside a bracket.

Learning path

You will be able to simplify linear expressions by collecting like terms

Use addition to simplify expressions with one term, e.g. $3b + 2b + b + 5b$	Use subtraction to simplify expressions with one term, e.g. $7y - 3y$
Use multiplication to simplify expressions with one term, e.g. $3 \times 2q$	Use division to simplify expressions with one term, e.g. $\frac{15x}{3}$
Recognise different terms in an expression, including number terms, e.g. $10a + 5y - 6a + 4 - 2$ has two terms and a number term	
Collect like terms using addition to simplify expressions in one term and a number term, e.g. $6c + 2a + c + 4 + 2a + 3 + 1 + c$	
Collect like terms using addition and subtraction to simplify expressions with more than one term, e.g. $5h + 10 - 3h + 4 - 12$ $4t + 3d - d + 5t - 5d$	
Multiply out a single bracket, e.g. $3(4a + 6)$ $2(3x - 4)$	Factorise simple expressions, e.g. $12x + 8$
Collect like terms using addition and subtraction to simplify more complex expressions, e.g. $3ab + 4xy - 2ab + 7ab + xy - ab$ $4(3x + 2) + 3y + 5(3 - 2y)$	
Simplify expressions involving addition, subtraction, brackets and complex terms, e.g. $3a(a - 4b) - 2b(3 + 4b)$	

Task 6.2C

Select a curricular target your pupils experience difficulty in progressing towards. If you have already carried out task 6.2B consider choosing a different curricular target. Ensure that all teachers in the department are secure in their understanding of progression towards this curricular target. If they are not, follow steps 1 to 3, as described in task 6.2B, so you agree the progression necessary to achieve the curricular target.

Select two or three strategies from handout 6.2.9 to trial in the classroom and use these over the course of several lessons, to support pupils' progress towards the curricular target.

As a department, evaluate the successes and issues for the strategies trialled. Consider:

- which enabled most pupils to make most progress;
- which built the greatest pupil independence;
- which generated most pupil motivation.

Context

Following their successful development work for the curricular target on simplifying linear expressions by collecting like terms, the department decided to focus on the other curricular target identified as being cause for concern.

- You will be able to solve linear equations including those with unknowns on either or both sides, without and with brackets.

From previous test paper analysis the department identified that pupils had two key misconceptions when solving equations: one being related to the understanding of simplifying expressions, and the other relating to the role of inverse operations when solving equations.

Process

The department selected two strategies from **handout 6.2.9**: 'analysing model answers to identify success criteria' and 'modelling or explaining'.

The head of department identified a Key Stage 3 test question (see **appendix 6.2C.1**) that focused explicitly on the misconceptions pupils had shown. At a department meeting teachers shared ideas on how the question could be used with pupils to get them to analyse answers and identify successful strategies to solve equations. They produced two tasks (see **appendix 6.2C.2**).

It was agreed that Year 9 teachers with pupils working at level 5 or above would try the tasks with their pupils and involve pupils in evaluating the outcomes. The key focus of the department was the explanations pupils gave for why the working was incorrect. To aid this, pupils would produce a poster and be expected to explain one of their responses to the rest of the class.

Evaluation

From the work pupils produced, the department identified that pupils working at a high level 5 and above had made progress with the curricular target to simplify linear expressions by collecting like terms. The focus on analysing the first steps to solving an equation helped pupils to recognise errors and explain the mistakes relating to inverse operations. However, teachers were disappointed that, in a number of cases, pupils chose equations that were very simple for their own example. As a result, the department decided that they would work on developing a learning path for solving equations, similar to the one produced for expressions in **task 6.2B**, in order to raise pupils' awareness of the progression within this aspect of mathematics.

Subject-specific references

Referenced strategy materials

Constructing and solving linear equations (DfES 0084/2004)

Framework for teaching mathematics: Years 7, 8 and 9 (DfEE 0020/2001)

Other strategy materials of interest

The 'Using teacher assessment to track pupils' progress in mathematics' materials have been produced by the mathematics strand to support targeted assessment of intervention groups in Year 7. They include a set of key indicators to help track pupils' progress towards level 4, with associated probing questions that can be used in lessons and plenaries. The materials also include suggestions on identifying curricular targets.

Key Stage 3 test question: 2003 tier 5–7 paper 1 question 18

1. (a) Pupils started to solve the equation $6x + 8 = 4x + 11$ in different ways.

For each statement below, tick (✓) True or False.



$6x + 8 = 4x + 11$	
so	$14x = 15x$
<input type="checkbox"/>	True <input type="checkbox"/> False

$6x + 8 = 4x + 11$	
so	$6x + 14x = 11 + 8$
<input type="checkbox"/>	True <input type="checkbox"/> False

$6x + 8 = 4x + 11$	
so	$6x = 4x + 3$
<input type="checkbox"/>	True <input type="checkbox"/> False

$6x + 8 = 4x + 11$	
so	$2x + 8 = 11$
<input type="checkbox"/>	True <input type="checkbox"/> False

$6x + 8 = 4x + 11$	
so	$2x = 3$
<input type="checkbox"/>	True <input type="checkbox"/> False

$6x + 8 = 4x + 11$	
so	$-3 = -2x$
<input type="checkbox"/>	True <input type="checkbox"/> False

3 marks

- (b) A different pupil used trial and improvement to solve the equation $6x + 8 = 4x + 11$

Explain why trial and improvement is not a good method to use.

1 mark



Tasks

Task A

Pupils started to solve the equation $6x + 8 = 4x + 11$ in different ways.

In pairs, sort the cards into groups reflecting whether you think the working is:
correct/incorrect/not sure

For cards that you have said are incorrect, identify and explain where the pupil has gone wrong and then solve the equation.

$$6x + 8 = 4x + 11$$

so $14x = 15x$

$$6x + 8 = 4x + 11$$

so $6x + 14x = 11 + 8$

$$6x + 8 = 4x + 11$$

so $6x = 4x + 3$

$$6x + 8 = 4x + 11$$

so $2x + 8 = 11$

$$6x + 8 = 4x + 11$$

so $2x = 3$

$$6x + 8 = 4x + 11$$

so $-3 = -2x$

Task B

In pairs:

- choose your own equation; it could include unknowns on both sides, brackets, subtraction, etc;
- show two methods for solving your equation correctly, explaining your decisions for each step;
- identify two ways someone may make a mistake when solving your equation, explaining what the mistake is that is being made.