

# Thinking proportionally

## (Year 8)

### Introduction

These two lessons, each 50 to 60 minutes long, could form part of the unit Solving problems (P1) in the *Sample medium-term plans for mathematics* (DfES 0504/2001) towards the end of the summer term in Year 8.

The lessons provide an opportunity to draw together pupils' understanding of the equivalence of fractions, decimals, percentages, ratio and proportion. They are key lessons within the unit and are intended to help teachers review the progress pupils have made in Year 8 in this crucial area of the teaching programme.

Assessment sheets P1d and P1e, when passed on to Year 9 teachers, can inform them of any errors and misconceptions that need to be addressed. They should also help teachers to pitch work at a suitable level when using the Year 9 proportional reasoning unit in the file *Interacting with mathematics in Key Stage 3* (Year 8: DfES 0220/2002; Year 9: DfES 0588/2002).

### Preparation

#### Lesson P1.1

- Mini-whiteboards
- Calculators (if needed)
- Resources P1a and P1b, cut into cards for sorting; one set per pair of pupils
- Cards enlarged from P1a and P1b, to support the whole-class introduction and for display

#### Lesson P1.2

- Sets of cards enlarged from P1a and P1b with one odd one out
- Resources P1a and P1b, cut into cards for sorting; one set per pair of pupils
- Resource P1c, cut into cards for sorting (extension material)
- Cards enlarged from P1a and P1b, to support the whole-class introduction and for display
- Assessment P1d 'Think links', one per pupil
- Assessment P1e 'How well am I doing?', one per pupil

# P1.1

## Thinking proportionally (Year 8)

### Objectives

- **Identify the necessary information to solve a problem; represent problems and interpret solutions in algebraic, geometric or graphical form**, using correct notation.
- **Use logical argument to establish the truth of a statement.**
- Suggest extensions to problems, conjecture and generalise.
- Consolidate understanding of the relationship between ratio and proportion; reduce a ratio to its simplest form, including a ratio expressed in different units, recognising links with fraction notation.

### Starter

#### Vocabulary

proportional  
compare  
equivalent  
fraction  
decimal  
percentage

#### Resources

mini-whiteboards  
calculators (if needed)

This starter should be short.

Remind pupils of the work they did in the Year 8 multiplicative relationships unit, generating proportional sets (see mini-pack prompts for phase 2, p. 11). Point to markers on the top and bottom of a counting stick, saying that the top markers reference multiples of 2 and the bottom markers multiples of 5. Chant the numbers aloud as a class:

Two, five; four, ten; six, fifteen; eight, twenty; ...

Extend beyond the limit of the counting stick.

**Q** If I point to 85 on the bottom, what number will be on top?

**Q** If the top number at this end of the counting stick is 18, what is the bottom number? What numbers will be at the other end of the counting stick?

### Main activity

#### Vocabulary

ratio  
proportion  
direct proportion  
rate  
per, for every, in every  
notation

#### Resources

P1a and P1b, cut into cards for sorting; one set per pair of pupils)  
enlarged cards (to support the whole-class introduction)

Explain that you are going to give out two sets of cards for pairs to sort into sets. Sorting activities help pupils to develop their reasoning skills as they have to justify their decisions. There are no predetermined answers, so encourage pupils to think creatively (*Training materials for the foundation subjects*, Module 11, 'Principles for teaching thinking'; ref: DfES 0350/2002). The cards are designed to help pupils to check their understanding of ratio and proportion.

Hold up three enlarged cards, D, M and 1, and pose questions for pupils to discuss in pairs:

**Q** Are these cards linked in any way? Try to explain your decision to your partner.

Give pupils a couple of minutes, then ask two or three pairs to explain their decisions.

Pupils might say that cards D and M are linked because they are fraction and percentage equivalences for  $\frac{1}{2}$  and that card 1 is linked because the ratio of the colours is 1 : 2.

Say that there are different ways to interpret the images and make links, so it is important that pupils can give clear reasons for their choice of classification. They should not be inhibited by having a classification that is different to that of another

pair. Encourage them to think about the rationale presented by other groups of pupils.

Ask pupils to work in pairs to identify sets of equivalent cards from resources P1a (cards A to X) and P1b (cards 1 to 18), justifying their decisions.

**Support:** Use fewer cards. For example, use resource P1a (cards A to X) only – this includes all three ratios but omits the extra images. Alternatively, use the first two columns of each of the resources (cards A, B, D, E, G, H, J, K, M, N, P, Q, S, T, V, W and 1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17) – this includes all images but only two of the ratios.

**Extension:** Ask pupils to create their own extra cards to fit the groupings.

After about 10 minutes, check on progress and ask each pair to join up with another pair to compare and explain their sets.

**Q Explain why you have put those cards together.**

**Q Does everyone understand the reasons for this grouping?**

Circulate to observe and note the different explanations being given. Probe pupils' understanding and help them to extend and refine their understanding of the links between ratio and proportion.

## Plenary

### Resources

cards enlarged from  
P1a on display

Display collection A, D, G, J, M, P and collection A, E, H, K, N, Q. Ask pupils to discuss these collections in pairs.

**Q Can you explain to each other why these collections are correct?**

**Q Find another card to add to the collection to help explain the links.**

Ask one or two pairs to explain which card they have decided to add to the collection.

Pupils may choose to add S to A, D, G, J, M, P as this illustrates use of a ratio to compare quantities, e.g. dimensions on a rectangle.

Pupils may choose to add V, the pie chart, to A, E, H, K, N, Q as this illustrates use of a ratio to compare part to whole. In this case the ratio 1 : 2 describes the two sectors relative to the whole circle. The smaller sector is  $\frac{1}{3}$  of the whole.

### Remember

- It is important to be clear about which values are being compared.
- Values can be compared using ratio, fraction, decimal and percentage notation.

# P1.2

## Thinking proportionally (Year 8)

### Objectives

- Identify the necessary information to solve a problem; represent problems and interpret solutions in algebraic, geometric or graphical form, using correct notation.
- Use logical argument to establish the truth of a statement.
- Suggest extensions to problems, conjecture and generalise.
- Consolidate understanding of the relationship between ratio and proportion; reduce a ratio to its simplest form, including a ratio expressed in different units, recognising links with fraction notation.

### Starter

#### Vocabulary

ratio  
proportion

#### Resources

sets of cards enlarged from P1a and P1b with one odd one out

Display enlarged cards A, D, R, S, or a similar collection on the board or OHP.

#### Q Which is the odd card out? Why?

Ask pupils to discuss in pairs and agree on an explanation. Ask two or three pairs to share their explanations.

Repeat with other sets of cards, or ask pupils to compose their own set of four with an odd card out.

### Main activity

#### Vocabulary

ratio  
equivalent  
notation  
fraction  
decimal  
percentage  
proportion  
direct proportion  
rate  
per, for every, in every

#### Resources

P1a and P1b, cut into cards for sorting; one set per pair of pupils  
P1c extension cards enlarged cards (to support the whole-class introduction)

Hold up enlarged cards A (1 : 2), B (1 : 3) and C (2 : 3). Tell pupils that they are going to classify the cards into just three groups, each linked to one of these ratios. Ask pupils to suggest other cards which might link to each of these ratios. Emphasise that pupils should make their own decisions which they must be able to justify.

#### Q Can you explain why you think this card is linked to this ratio?

#### Q Does everyone understand why these cards are linked?

Ask pupils to work in groups of four and to classify the cards from resources P1a (cards A to X) and P1b (cards 1 to 18) into three groups.

#### Q Can you explain why you have put these images with this ratio?

**Support:** Use resource P1a, cards A to X only – this includes all three ratios but omits the extra images. Alternatively, use the first two columns of resources P1a and P1b, cards A, B, D, E, G, H, J, K, M, N, P, Q, S, T, V, W and 1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17 – this includes all images but only two of the ratios.

**Extension:** Include extra cards 19 to 27 from resource P1c.

Circulate to observe and note the different explanations being used. Probe pupils' understanding of the links they have identified, helping them to extend and refine their reasoning.

Identify one or two groups to share their reasoning in the plenary.

## Plenary

### Resources

cards enlarged from  
P1a

P1d, one per pupil

P1e, one per pupil

Ask the identified groups to share their results and reasoning.

To introduce the questions on **assessment P1d**, 'Think links', show three enlarged cards – for example D, J and M – and ask pupils, in pairs, to think of a sentence they could write to describe the link between the cards.

**Q How would you explain the link between these cards?**

**Q How could you write that in a sentence?**

Take some suggestions.

Ask pupils to choose one set of three cards, with an obvious link and then to choose another set of three cards, where the link is harder to understand. Ask them to complete the sheet. This will help to identify suitable starting points on their next piece of work on ratio and proportion in Year 9.

Finally, ask pupils to complete **assessment P1e**, 'How well am I doing?'.

### Remember

- There are many different images that link to ratio and proportion.
- If we are comparing values we can do it in many different ways: ratio, fractions, decimals or percentages.