



Guidance

Curriculum and
Standards

Key Stage 3

National Strategy

Interacting with mathematics in Key Stage 3

Constructing and solving linear equations

Year 7 booklet

Teachers of mathematics

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Introduction

This booklet is to be used with the *Framework for teaching mathematics: Years 7, 8 and 9*. It provides additional guidance on developing progression in the teaching of *constructing and solving linear equations*. Although specific in this focus, it illustrates an approach that is designed to serve the broader purpose of developing the teaching of all aspects of algebra. The booklet:

- supports the training session ‘Constructing and solving linear equations in Year 7’;
- provides a resource for mathematics departments to use in collaborative planning for the teaching of algebra.

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Key teaching principles

Principle 1: Providing opportunities for pupils to express generality

Generality lies at the heart of mathematics. The teaching principle is to get pupils to *generalise* for themselves rather than just having generalisations presented to them. The advantages of this approach include the following.

- Pupils appreciate the purpose of algebra.
- Pupils are better able to understand the meaning of expressions if they have generated some for themselves.
- Knowing how expressions are built up helps to clarify the process of 'undoing', needed when solving equations.

Principle 2: Asking pupils to 'find as many ways as you can'

This teaching principle requires that pupils are regularly asked to write algebraic expressions in different ways – to *construct* expressions or equations and to *transform* them. The benefits of this are many.

- Pupils appreciate that the same general relationship can be expressed in more than one way.
- They manipulate expressions to demonstrate that one expression is equivalent to another.
- They experience forming and transforming expressions in different ways.
- They have opportunities to discuss which transformations are the most efficient to use in a particular context, e.g. when solving an equation.

In addition, by paying careful attention to the 'using and applying' objectives set out in section 2.4 below, pupils are provided with opportunities to:

- *represent* problems in *symbolic* form;
- develop *algebraic reasoning*.

Through consistent application of these principles, pupils learn to construct and manipulate algebraic expressions and equations on the basis of their understanding of mathematical relationships, rather than being given a predetermined set of rules. This helps them to choose the methods and the sequence of operations needed to solve an equation.

Key Stage 2: laying the foundations for algebra

Algebra is a compact language which follows precise conventions and rules. Formal algebra does not begin until Key Stage 3 but the foundations are laid in Key Stages 1 and 2 by providing early algebraic activities from which later work in algebra can develop. These activities include:

- **Forming equations** Ask pupils to give more than single word or single number answers. For example, you might sometimes expect the response to short questions such as: 'What is 16 add 8?' to be expressed as a complete statement: 'sixteen add eight equals twenty-four'. You might also invite a child to the board to write the same equation in symbolic form: $16 + 8 = 24$.
- **Solving equations** By asking questions such as: 'Complete $3 + \square = 10$ ' you can introduce the idea that a symbol can stand for an unknown number. You can also ask questions in the form: 'I double a number, then add 1, and the result is 11. What is the number?' By considering equations with two unknowns, such as $\square + \triangle = 17$, or inequalities like $1 < \square < 6$, you can lead pupils towards the idea that the unknown is not necessarily one fixed number but may also be a variable.
- **Using inverses** Another important idea in both number and algebra is the use of an inverse to 'reverse' the effect of an operation. The inverse of doubling is halving, of adding 7 is subtracting 7, and of multiplying by 6 is dividing by 6. Pupils can use their knowledge of an addition fact such as $4 + 7 = 11$ to state a corresponding subtraction fact: $11 - 7 = 4$. Similarly, pupils should be able to use their knowledge of a multiplication fact such as $9 \times 6 = 54$ to derive quickly a corresponding division fact: $54 \div 6 = 9$.
- **Identifying number patterns** Encourage pupils to look for and describe number patterns as accurately as they can in words and, in simple cases, to consider why the pattern happens. For example, they could explore the patterns made by multiples of 4 or 5 in a 10 by 10 tables square, or extend and describe simple number sequences such as 2, 7, 12, 17... and, where appropriate, describe and discuss how they would set about finding, say, the 20th term.
- **Expressing relationships** When discussing graphs drawn, say, in science, ask children to describe in their own words the relationships revealed: for instance, 'every time we added another 20 grams the length of the elastic increased by 6 centimetres'. They can also be asked to use and make their own simple word equations to express relationships such as:

$$\text{cost} = \text{number} \times \text{price}$$

By Year 6, pupils should be ready to express relationships symbolically: for example, if cakes cost 25p each then $c = 25 \times n$, where c pence is the total cost and n is the number of cakes.

- **Drawing graphs** Teach older pupils to draw and use graphs which show mathematical relationships, such as those of the multiplication tables, or conversions from pounds to foreign currency. Games like Battleships can be used to introduce the idea of coordinates to identify spaces and, later, single points. It is then possible to record graphically, for example, pairs of numbers that add up to 10.
- **Developing ideas of continuity** Another foundation stone for algebra is laid in Years 5 and 6 when pupils appreciate that between any two decimal numbers there is always another, and that the number line is continuous. They also need to understand that quantities like heights and weights are never exact. In growing from 150 cm to 151 cm, say, every possible value in that interval has been attained because measures too are continuous.

- **Finding equivalent forms** You should emphasise from the very beginning the different ways of recording what is effectively the same thing. For example:
 - $24 = 20 + 4 = 30 - 6$;
 - $30 = 6 \times 5 = 3 \times 2 \times 5$;
 - $15 + 4 = 19$ implies that $15 = 19 - 4$, and $3 \times 4 = 12$ implies that $12 \div 3 = 4$;
 - $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} \dots$ and each of these is equivalent to 0.5 or 50%.
- **Factorising numbers** Factorising 30 as $2 \times 3 \times 5$ is a precursor of the idea of factorising in algebra. It is also a useful strategy for multiplication and division. For example, since $12 = 6 \times 2$, the product 15×12 can be calculated in two steps, first $15 \times 6 = 90$, then $90 \times 2 = 180$. Similarly, $273 \div 21$ can be worked out by using the factors of 21, first $273 \div 3 = 91$, then $91 \div 7 = 13$.

Encourage pupils to factorise numbers as far as is possible. To factorise 24 as 6×4 is not as complete as $2 \times 2 \times 2 \times 3$.

- **Understanding the commutative, associative and distributive laws** You need to discuss the ideas behind these laws thoroughly since they underpin strategies for calculation and, later on, algebraic ideas.

Pupils use the commutative law when they change the order of numbers to be added or multiplied because they recognise from practical experience that, say:

$$4 + 8 = 8 + 4 \quad \text{and} \quad 2 \times 7 = 7 \times 2$$

The associative law is used when numbers to be added or multiplied are regrouped without changing their order: for example,

$$(4 + 3) + 7 = 4 + (3 + 7) \quad \text{and} \quad (9 \times 5) \times 2 = 9 \times (5 \times 2)$$

An example of the distributive law is a method for 'long multiplication' in which each part of the first number is multiplied by each part of the second, and then the products are added to find their total. So 35×24 is split up as:

| | | | |
|----|-----|-----|--|
| | 30 | 5 | |
| 20 | 600 | 100 | |
| 4 | 120 | 20 | |

$$600 + 100 + 120 + 20 = 840$$

As well as illustrating clearly how the multiplication method works, this method provides a foundation for the later idea of multiplying out a pair of brackets:

$$(30 + 5)(20 + 4) = (30 \times 20) + (5 \times 20) + (30 \times 4) + (5 \times 4)$$

Pupils who have a secure understanding of all these important ideas by the age of 11 will be in a sound position to start work on more formal algebra in Key Stage 3.

Adapted from: *Framework for teaching mathematics from Reception to Year 6*, Introduction, pages 9–10 (DfEE, 1999; ref: NNFT; ISBN 0 85522 922 5)

Key Stage 2 test questions

- 1 n stands for a number.

$$n + 7 = 13$$

What is the value of $n + 10$?

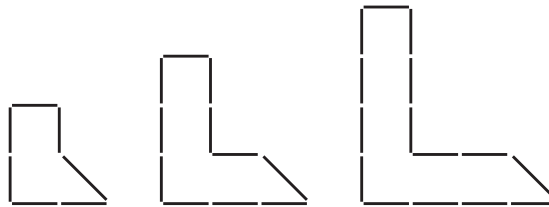
- 2 Jemma thinks of a number. She says:

'Add 3 to my number and then multiply the result by 5.

The answer is 35.'

What is Jemma's number?

- 3 Ann makes a pattern of L-shapes with sticks.



| | | | |
|-------------------|---|----|----|
| Shape number: | 1 | 2 | 3 |
| Number of sticks: | 7 | 11 | 15 |

Ann says: 'I find the number of sticks for a shape by first multiplying the shape-number by 4, then adding 3.'

Work out the **number** of sticks for the shape that has shape-number **10**.

Ann uses **59 sticks** to make another L-shape in this pattern.

What is its shape-number?

Here is Ann's rule again: **'I find the number of sticks for a shape by first multiplying the shape-number by 4, then adding 3.'**

Write a formula to work out the number of sticks for any L-shape.

Use S for the number of **sticks** and N for the **shape-number**.

Teaching objectives

Progression in the solution of linear equations

The following table sets out objectives from the yearly teaching programmes that are addressed in the training session.

| OBJECTIVES | | |
|--|--|--|
| Year 7 | Year 8 | Year 9 |
| Solving equations <ul style="list-style-type: none"> Construct and solve simple linear equations with integer coefficients (unknown on one side only) using an appropriate method (e.g. inverse operations). | <ul style="list-style-type: none"> Construct and solve linear equations with integer coefficients (unknown on either or both sides, without and with brackets) using appropriate methods (e.g. inverse operations, transforming both sides in the same way). | <ul style="list-style-type: none"> Construct and solve linear equations with integer coefficients (with and without brackets, negative signs anywhere in the equation, positive or negative solution), using an appropriate method. |
| Using and applying <ul style="list-style-type: none"> Represent problems mathematically, making correct use of symbols. Present and interpret solutions in the context of the original problem; explain and justify methods and conclusions. Suggest extensions to problems by asking 'What if...?'; begin to generalise. | <ul style="list-style-type: none"> Represent problems and interpret solutions in algebraic form. Use logical argument to establish the truth of a statement. Suggest extensions to problems and generalise. | <ul style="list-style-type: none"> Represent problems and synthesise information in algebraic form. Present a concise, reasoned argument, using symbols. Suggest extensions to problems and generalise. |
| Precursors <ul style="list-style-type: none"> Use letters or symbols to represent unknown numbers; know the meanings of the words <i>term</i>, <i>expression</i> and <i>equation</i>. Understand that algebraic operations follow the same conventions and order as arithmetic operations. Simplify linear algebraic expressions by collecting like terms; begin to multiply a single term over a bracket. | <ul style="list-style-type: none"> Know that algebraic operations follow the same conventions and order as arithmetic operations. Simplify or transform linear expressions by collecting like terms; multiply a single term over a bracket. Add, subtract, multiply and divide integers. | <ul style="list-style-type: none"> Simplify or transform algebraic expressions by taking out single-term common factors. |

Print these cards on acetate, cut them up and use them on an OHP to build equations.

| | | |
|----------|----------|----------|
| 23 | 45 | 68 |
| + | − | = |
| <i>m</i> | <i>p</i> | <i>r</i> |
| 16 | 9 | 144 |
| × | ÷ | |

Additive and multiplicative relationships

Addition and subtraction

In Key Stage 1 pupils begin to appreciate that they can know a 'family of facts'. Given a number sentence it is possible to generate three other related number sentences. For example:

$$2 + 3 = 5 \text{ leads to } 3 + 2 = 5, \quad 5 - 3 = 2 \text{ and } 5 - 2 = 3$$

As pupils gain experience they should be able to start to generalise. Using number and symbol cards (as in section 2.5), you can introduce pupils to an equation and develop the equivalent equations (see Ginty's lesson on video sequence 5).

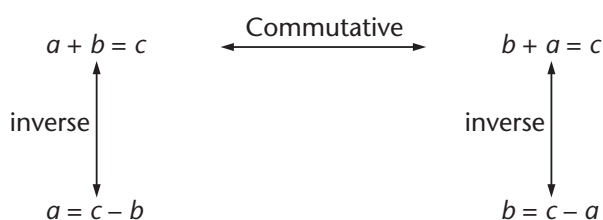
For example:

$$a + 3 = 7 \text{ leads to } 3 + a = 7, \quad 7 - a = 3 \text{ and } 7 - 3 = a$$

From this group of statements, pupils can see that one of them, $a = 7 - 3$, leads to the direct solution of the equation $a + 3 = 7$.

This process can be extended to $a + b = c$ and the associated statements, and eventually to the complete generalisation:

$$\text{if } a + b = c \text{ then } b + a = c, \quad c - a = b \text{ and } c - b = a$$



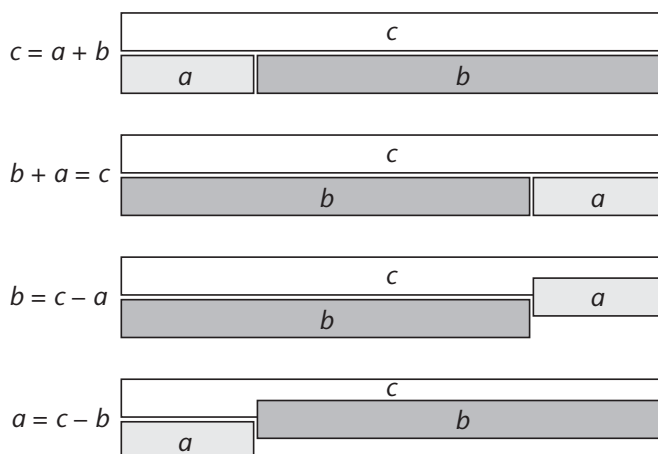
NOTE

This section can be used as the basis of a teaching sequence with pupils, drawing from and developing the range of questions suggested.

In the classroom you may wish to model the process by using number and symbol cards or different coloured rods where the length of the rod represents a number.

From pictures to symbols

Using coloured rods, linking blocks or line segments drawn on squared paper, set out these arrangements of strips of fixed lengths a , b and c :



The first diagram shows that the length c is the sum of the lengths a and b . This can be written as $c = a + b$.

- Q** What can you see in the other diagrams? Write a sentence connecting a , b and c .
- Q** Which diagrams make a *commutative* pair?
- Q** Which diagrams form an *inverse* pair?

The diagram/strips give a visual representation of the four related equations:

$$a + b = c, \quad b + a = c, \quad c - a = b, \quad c - b = a$$

Use the above structure to write the following number or symbol sentences in *as many ways as you can*.

1 $18 - 3 = 15$

2 $7 + 4 = 11$

3 $12 + a = 16$

4 $8 - f = 3$

5 $q + t = 9$

6 $p - g = 24$

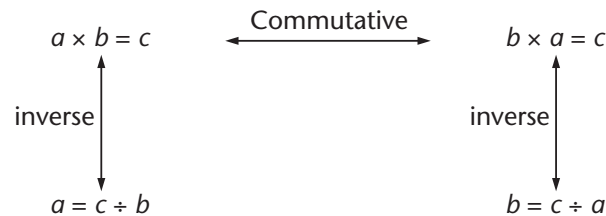
7 $j + k = v$

8 $y - d = w$

Multiplication and division

Use sets of cards with the symbols \times , \div and $=$ and:

- numbers – for example:
 $2 \times 3 = 6$ leads to $3 \times 2 = 6$, $6 \div 2 = 3$ and $6 \div 3 = 2$
- a mixture of numbers and letters representing numbers – for example:
 $2 \times a = 6$ leads to $a \times 2 = 6$, $6 \div 2 = a$ and $6 \div a = 2$
- just letters to develop the links between multiplication and division:
if $a \times b = c$ then $b \times a = c$, $c \div b = a$ ($b \neq 0$) and $c \div a = b$ ($a \neq 0$)
or if $ab = c$ then $ba = c$, $c/b = a$ ($b \neq 0$) and $c/a = b$ ($a \neq 0$)



Together with additive relationships, these constitute the basic transformations of elementary algebra. Pupils will gain confidence with these transformations if they are given opportunities to:

- practise transforming number sentences;
- use the four related facts in number and symbol sentences.

Use the above structure to write the following number or symbol sentences in *as many ways as you can*.

1 $6 \times 3 = 18$

2 $10 \div 2 = 5$

3 $2 \times a = 17$

4 $8 \div f = 2$

5 $q \times t = 20$

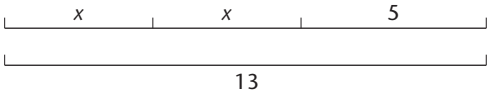
6 $p / g = 6$

7 $j \times k = v$

8 $y \div d = w$

Methods of solving equations in Year 7

| Method | Scope and limitations |
|--|-----------------------|
| <p>Undoing</p> <p>$2x + 5 = 13$</p> <p>'I think of a number, double it and then add 5. The answer is 13. What is my number?'</p> <p>Solution: 'The number is 4, because I subtract 5 from 13 to get 8 and then halve 8 to get 4.'</p> | |
| <p>Undoing with jottings</p> <p>$5x - 7 = 43$</p> <p>Problem: 'I think of a number, multiply it by 5 and then subtract 7. The answer is 43.'</p> <p>Solution: 'To find the solution I add 7 to 43 and then divide the answer by 5.'</p> <p>Jottings: $43 + 7 = 50$, $50 \div 5 = 10$</p> | |
| <p>Inverses</p> <p>$2x + 5 = 13$</p> <p> $x \xrightarrow{\times 2} 2x \xrightarrow{+ 5} 2x + 5$ $4 \xleftarrow{\div 2} 8 \xleftarrow{- 5} 13$ </p> | |
| <p>Write the equation in 'as many ways as you can'</p> <p>Using the commutative law and inverses, write the equation in as many ways as you can until you find an equation that you can solve in your head or with jottings.</p> <p>Solve the equations: $2x + 5 = 13$ $2x - 5 = 13$</p> <p>Write in the form: $2x = 13 - 5$ $2x = 13 + 5$</p> <p>So: $2x = 8$ $2x = 18$</p> <p>$x = 4$ $x = 9$</p> | |

| Method (continued) | Scope and limitations (continued) |
|---|-----------------------------------|
| <p>Matching method, supported by using a number line</p> <p>$2x + 5 = 13$</p>  <p style="margin-left: 100px;"> $2x + 5 = 13$ $2x + 5 = 8 + 5$ $2x = 8$ $x = 4$ </p> | |
| <p>Balancing</p> <p>$2x + 5 = 13$</p> <p style="margin-left: 40px;"> $2x + 5 = 13$ $2x + 5 - 5 = 13 - 5$ $2x = 8$ $2x/2 = 8/2$ $x = 4$ </p> | |
| <p>Trial and improvement</p> <p>$2x + 5 = 13$</p> <p>Guess: x is 3 The left-hand side is too low.</p> <p>Guess: x is 4 $(2 \times 4 + 5 = 13)$ is correct.</p> | |

2.1–2.3

Objectives

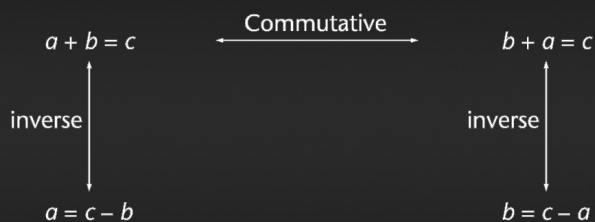


- To consider Year 7 teaching objectives in constructing and solving linear equations
- To outline an effective progression and teaching approaches to help pupils construct and solve linear equations
- To develop lessons that incorporate activities from the session

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2.1

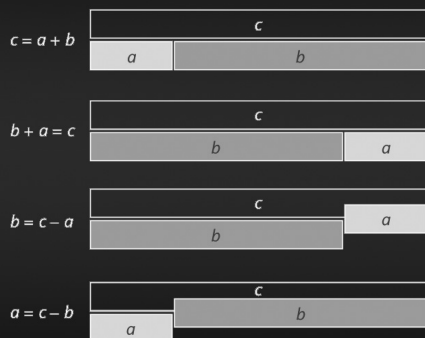
Algebra as generalised arithmetic
(addition and subtraction)

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2.2

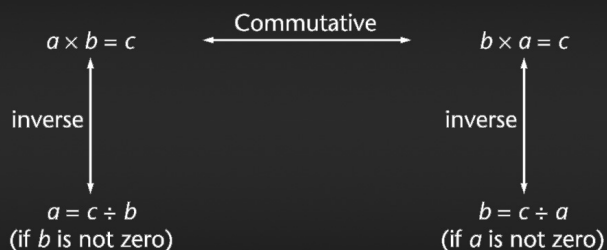
From pictures to symbols



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2.3

Algebra as generalised arithmetic
(multiplication and division)

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2.4

Year 7 lesson plans



- 7A.1** Pyramids (adapted from Ginty's lesson – video sequence 5)
- 7A.2** Equations from word problems
- 7A.3** Solving simple equations (algebra lesson from unit A4.2 in *Targeting level 4 in Year 7*)

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2.5

Examining a Year 7 algebra lesson plan



Working in pairs, consider a class for whom the lesson might be suitable.

- How could you adapt the lesson for your class?
- How might you follow it up in a subsequent lesson or lessons?

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2.6

LESSON

7A.1

Pyramids

This lesson is based on the video of Ginty's lesson.

OBJECTIVES

- Understand that algebraic operations follow the same conventions and order as arithmetic operations (addition and subtraction only).
- Use letters or symbols to represent unknown numbers.
- Construct and solve simple linear equations.
- Represent problems mathematically, making correct use of symbols.

STARTER

Vocabulary

symbols
operations
inverse
commutative
equation

Resources

Resource 7A.1a,
cut into cards; one
set per pupil

Give each pupil a set of number and symbol cards (**resource 7A.1a**).

Write on the board:

$$3 + 4 = 7$$

Ask pupils to use their cards to rework the number sentence in *as many different ways as they can*. Invite pupils to share their answers on the board, encouraging them to use both the commutative rule and subtraction as the inverse of addition.

Now pose the questions:

- Q** Can you write an equation using a mix of numbers and symbols, for example $3 + a = 7$?
- Q** Can you rearrange the equation you have written, keeping the same numbers and symbols? You can change the operation.

Discuss pupils' suggestions and model on the board how to find the four related sentences associated with the initial equation.

- Q** What if you start with the equation $a + m = 3$?

MAIN ACTIVITY

Vocabulary

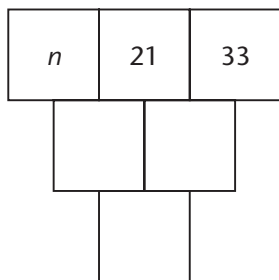
expression
equation
symbol
value

Resources

Resources 7A.1b and
7A.1c as handouts

Briefly recap how number pyramids are constructed (Framework supplement of examples, page 122).

Draw a pyramid on the board with an unknown number represented by the symbol n in the top left box and 21 and 33 in the other two boxes in the top layer. Explain how the pyramid is formed: add adjacent cells to get the expression in the cell below.



Invite pupils to tell you what expressions will go in the next layers of the pyramid. Verify that the mathematical expressions are all written correctly and then ask:

- Q** For this pyramid I was thinking of a number n and my number gave a value of 100 in the bottom box. What number was I thinking of?

When pupils offer an answer ask them:

Q How did you work the number out?

Q How can I write an equation using n ?

Now say to pupils that you want them to design some similar examples. Tell them that first, on their own, they are going to draw a three-layer number pyramid at the back of their book, keeping their pyramid a secret from their partner. Then they will hand over to their partner a partially completed version of the pyramid with the top layer filled with two known numbers and an unknown number.

Ask each pupil to complete their partner's pyramid with the correct mathematical expressions. When a pupil has completed the pyramid their partner (the originator of the pyramid) will ask:

Q I will tell you the number that goes in the box in the bottom layer. What is my unknown number?

Encourage higher-ability pupils to set challenges, for example to place the unknown number in the middle box of the top layer.

Circulate to observe pupils' strategies and select two examples to illustrate easier and harder cases. Discuss these examples in a mini-plenary, asking:

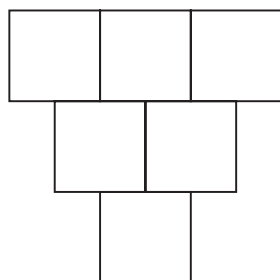
Q How do you work out the unknown number n ?

Set pupils to work in pairs on further examples using **resource sheets 7.1b** and **7.1c**. The pyramids on these resources span a range of difficulty from simple numbers and no unknowns to pyramids with four layers and decimal number inputs. Select different starting points for individual pupils.

PLENARY

Review the vocabulary used by asking pupils to define and give an example of an expression and an equation.

Draw a pyramid with three layers on the board.



Set the following puzzle pyramid as a challenge.

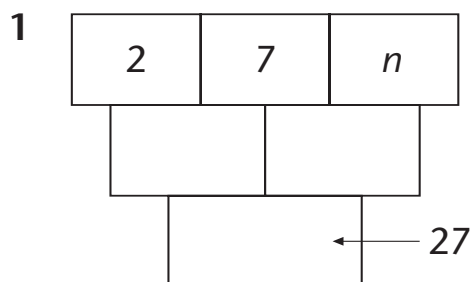
- In the top layer:
 - Box 1 contains an unknown number.
 - Box 2 contains a number that is 5 more than the number in the first box.
 - Box 3 contains a number that is 2 less than the number in the second box.
- The number in the box in the bottom layer of the pyramid is 28.

Invite pupils to suggest an expression that represents the number in each box and help them to construct and solve the equation that will give the value of the unknown number.

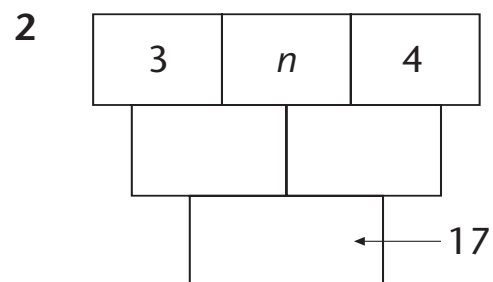
KEY IDEAS FOR PUPILS

- Algebra uses symbols to represent unknown numbers.
- Each side of an equation has the same value.
- The solution of an equation is the number that makes the equation true.

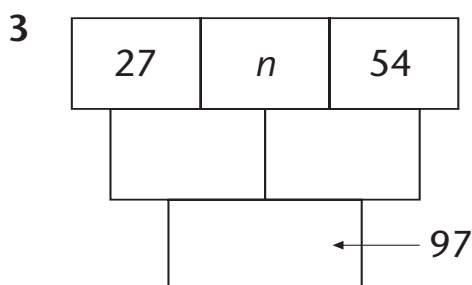
| | | |
|----------|----|----------|
| <i>m</i> | 1 | 2 |
| 3 | 4 | 5 |
| 6 | 7 | 8 |
| 9 | 10 | + |
| — | = | <i>a</i> |



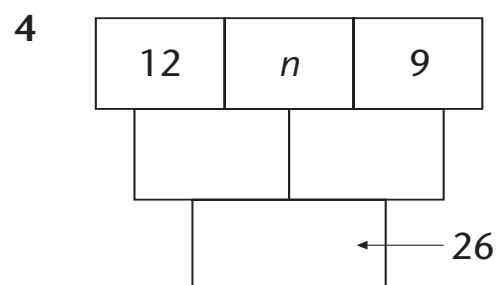
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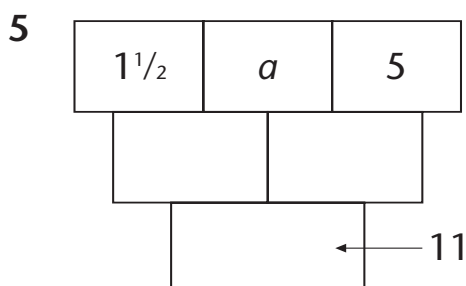
$n =$



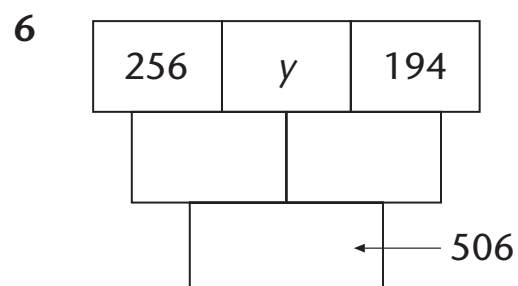
$n =$



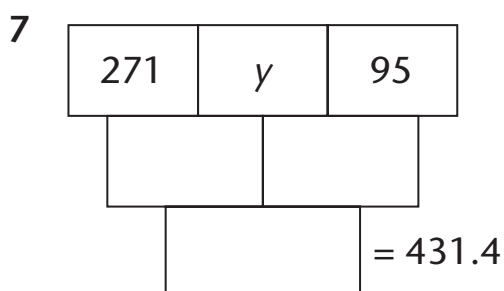
$n =$



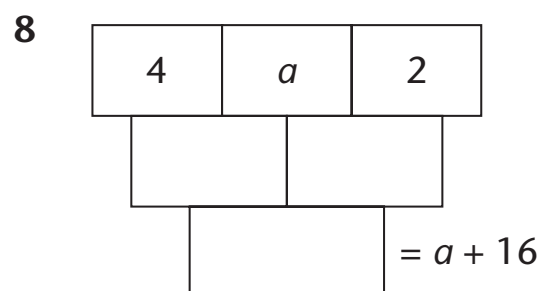
$a =$



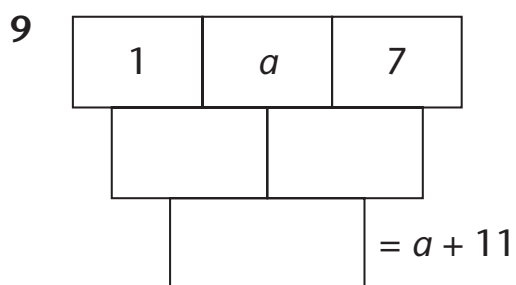
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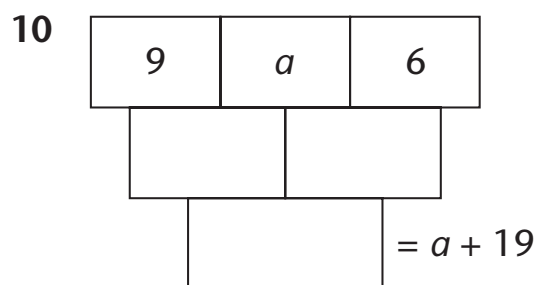
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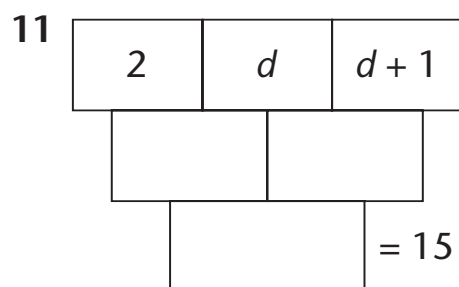
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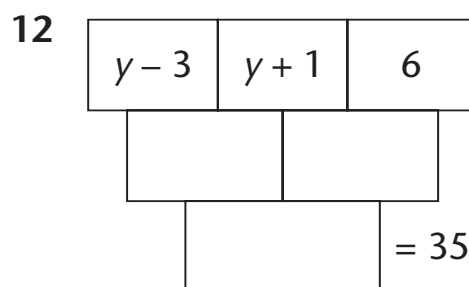
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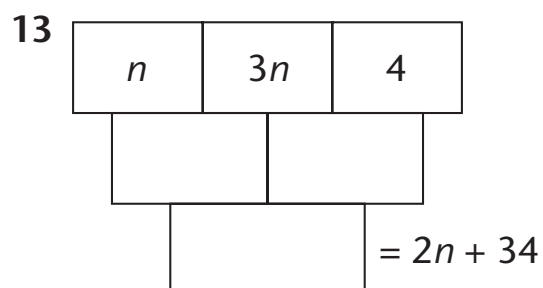
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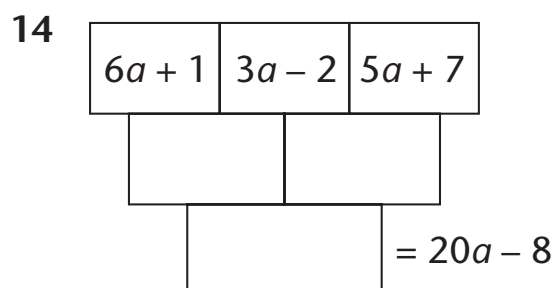
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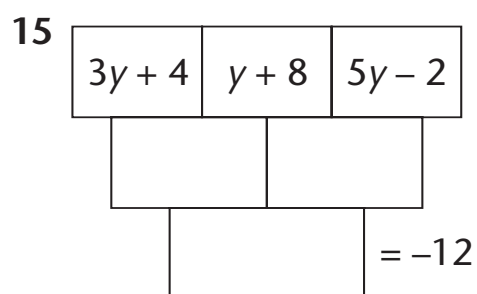
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$n =$



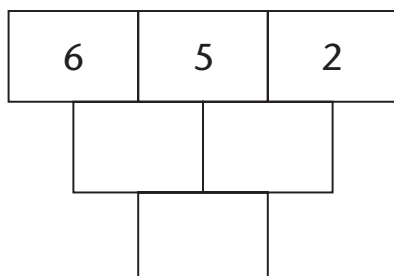
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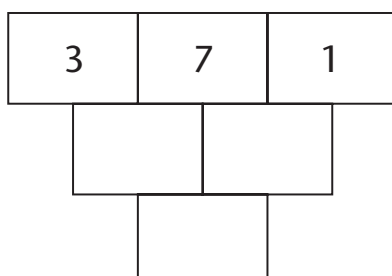
$y =$

Complete these pyramids.

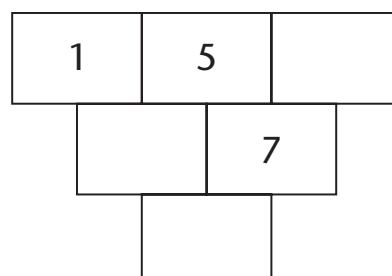
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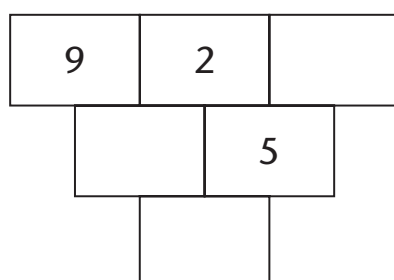
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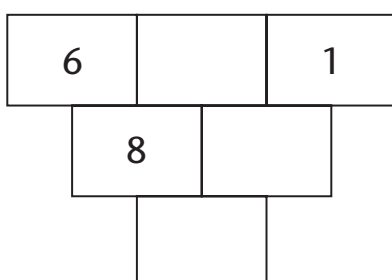
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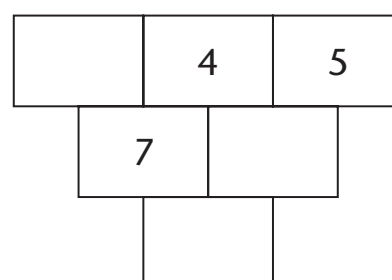
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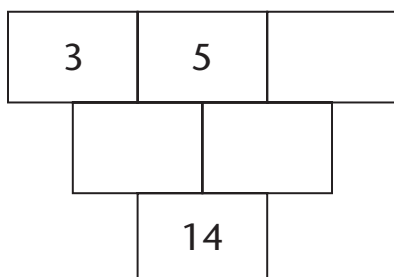
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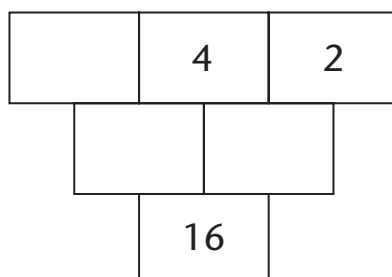
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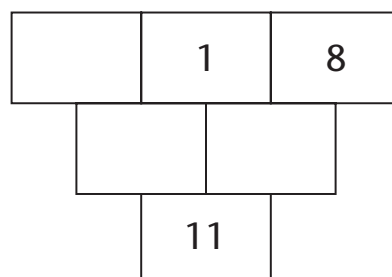
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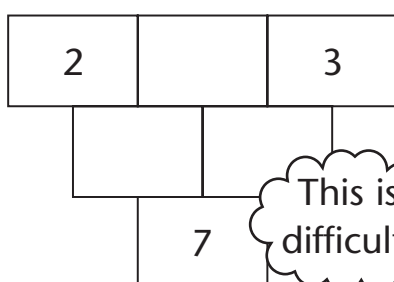
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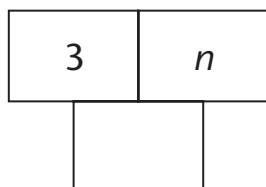


This is difficult.

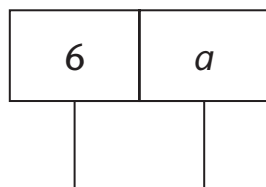
Pyramids (2)

These pyramids use letters.

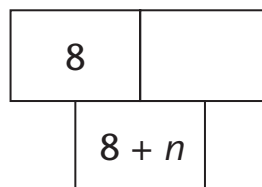
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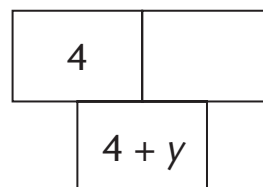
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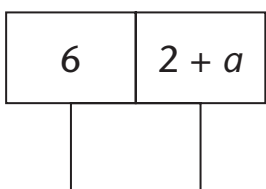
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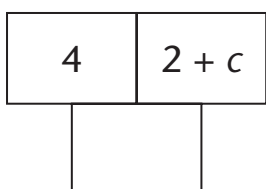
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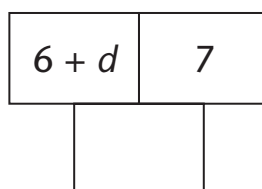
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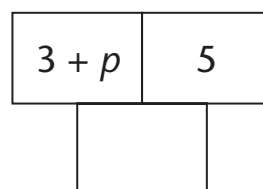
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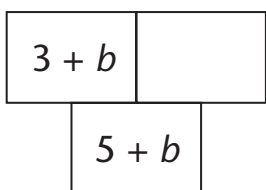
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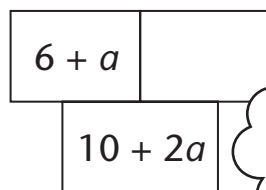
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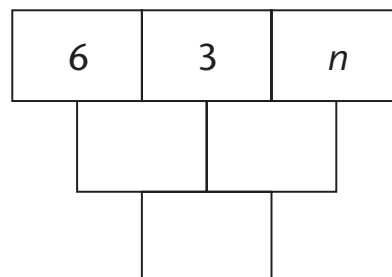


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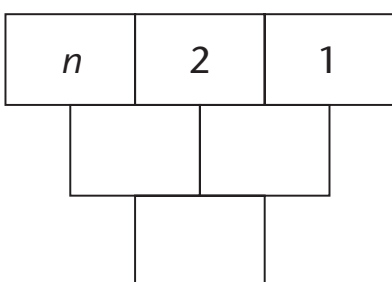


Another tricky one.

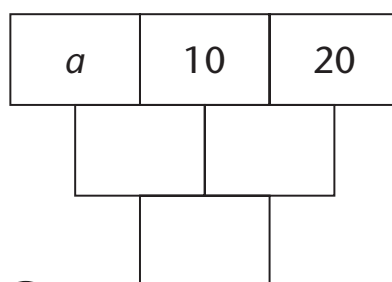
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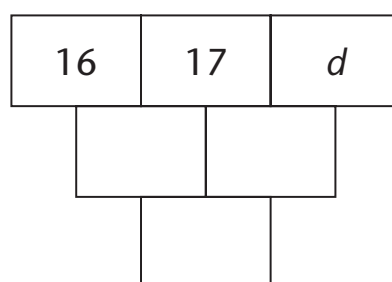
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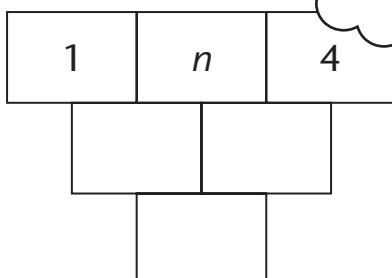
23



24



25



Another difficult one.

Try making up some of your own for your partner to solve. You could even try using the number from the bottom box to make up some equations.

Equations from word problems

OBJECTIVES

- Use letters or symbols to represent unknown numbers.
- Construct and solve simple linear equations.
- Represent problems mathematically, making correct use of symbols.

STARTER

Vocabulary

symbol
sequence
interval

Resources

Counting stick

The starter should be short.

Use a counting stick to encourage pupils to say out loud the terms of a sequence that increases in intervals of a . Chant the sequence aloud as a class:

$0, a, 2a, 3a, 4a, 5a, 6a, \dots$

Now ask pupils to chant a sequence that goes up in steps of $10a$, but this time move your finger up and down the counting stick:

$0, 10a, 20a, 30a, 40a, 30a, 20a, \dots$

Chant, as a model for the class, the sequence that goes up in steps of $(a + 1)$:

$0, a + 1, 2a + 2, \dots$

Ask pupils to count in their heads a sequence that goes up in steps of $(a + b)$, then point to the counting stick and say:

Q If I start at 0 and count up in intervals of $(a + b)$ the next term is $(a + b)$.
What is the next term? And the next term? And the next?

MAIN ACTIVITY

Vocabulary

expression
equation
symbol
value

Resources

Mini-whiteboards
Set of word problems
that lead to simple
equations (from
textbook)

Tell pupils that they are going to solve problems using algebra.

Pose the problem:

*I think of a number, multiply it by 5, then add 42. The answer is 57.
What is my number?*

Ask pupils to write their solutions on their whiteboards. Ask pupils to show their whiteboards. Pick one or two pupils to share their results asking:

Q Can you explain how you found out what my number was?

Once the class has agreed on an answer, set up a second problem:

*Zoe goes to a café with a friend and buys two hot chocolates and one muffin.
She pays £6.10.*

Ask pupils to work in pairs on a whiteboard to answer the following question:

Q Can you find out the price of the muffin? What possibilities are there?

After a few minutes draw the class together and ask two pupils to talk through their thinking.

Q Is it easier to answer the question if you are told that the cost of a cup of hot chocolate is £1.75?

Explain that it is often helpful to use algebra to solve a word problem. Model for the class the process of turning the statement into an equation. Emphasise that the letters represent numbers (of pounds or pence), not chocolate or muffins:

$$2c + m = 610 \quad (\text{alternatively, } 2c + m = 6.10)$$

$$350 + m = 610$$

Find the solution and check that it is correct. Remind the class that when a letter is used for a number, then the multiplication sign is often left out, so that $2c$ means the same as $2 \times c$.

Ask pupils, in pairs, to make up their own problems and explain how they solved the problem using algebra. Alternatively, choose a set of examples from a textbook for pupils to work on.

In a mini-plenary, use pupils' work to illustrate two different examples, one that is easy to solve without algebra and one where algebra will help.

PLENARY

Resources
OHT 7A.2a

Say that the solution to a word problem can sometimes be found mentally. However, in many practical problems, it is tricky to find the answer mentally; it is sometimes easier to write and solve the problem using algebra.

Display the following word problem (**OHT 7A.2a**).

There are 376 stones in three piles. The second pile has 24 more stones than the first pile. The third pile has twice as many stones as the second pile. How many stones are there in each pile?

Ask the question:

Q What is the best way of finding all the solutions?

Consider some suggested methods including trial and improvement. Then model writing the problem as an equation using symbols and suggest ways of finding a solution by means of 'undoing'.

For homework, ask pupils to make up a similar problem, writing it in words and then in numbers and letter symbols. Ask them to solve their own problem and to explain their working.

KEY IDEAS FOR PUPILS

- Algebra uses symbols to represent unknown numbers.
- Each side of an equation has the same value.
- $5a$ means $5 \times a$.
- The solution of an equation is the number that makes the equation true.

There are 376 stones in three piles.

The second pile has 24 more stones than the first pile.

The third pile has twice as many stones as the second pile.

How many stones are there in each pile?

Solving simple equations

This Year 7 intervention lesson is based on lesson A4.2 from the *Level 3 to level 4 lessons in Targeting level 4 in Year 7: mathematics* (folder: DfES 0085/2003; number and algebra lessons: DfES 0142/2003).

OBJECTIVES

- Use letters or symbols to represent unknown numbers or variables.
- Solve simple linear equations with integer coefficients (unknown on one side only) using an appropriate method.
- Solve problems and investigate in number and algebra.

STARTER

Vocabulary
problem

Resources
OHT 7A.3a

Show **OHT 7A.3a**, a set of menu problems. Explain to the class that they are to work out the cost of each menu option with no price shown.

Q How can we work out the cost of milk? (find the difference between the costs of pasta, salad and milk, and pasta and salad)

Q How can we find the cost of a salad? (subtract the cost of pasta)

Ask pupils to work in pairs to find the costs of the last two options on the first menu.

Q How shall we start to solve the second problem?

Discuss pupils' suggestions, then ask them to work in pairs to solve the problem. Invite one or two pairs to explain their solutions to the rest of the class.

Repeat with the third problem.

MAIN ACTIVITY

Vocabulary
equation
solution
value
inverse
substitute
trial and improvement

Resources
Mini-whiteboards
Computer with data projector and spreadsheet

Write on the board 8 and $3 + 5$. Tell the class that the two numbers that you have written are the same.

Now write $4 + \square$ and 9. Explain that this time a number is missing, and that there is a box symbol in its place. To make the pair of numbers $4 + \square$ and 9 the same, you need to put 5 in place of the box, like this: $4 + 5$ and 9.

Write a few more examples on the board, one by one. For example:

$$\begin{array}{lll} 6 + \square \text{ and } 11 & 15 \text{ and } 10 + \square & 4 \times \square \text{ and } 12 \\ 10 \text{ and } \square \div 2 & \square - 3 \text{ and } 1 & 11 - \square \text{ and } 9 \end{array}$$

Ask pupils to use their whiteboards. Ask:

Q What number should replace the box to make this pair of numbers the same?

Say that a letter is often used instead of a box. To show that a pair of numbers is the same, the equals sign is used. So instead of $4 + \square$ and 9, we write:

$$4 + n = 9$$

Say that this is called an *equation*. In this equation, putting 5 instead of n makes the equation true. So the *solution* to the equation is $n = 5$. Check by substituting 5 back into the original equation.

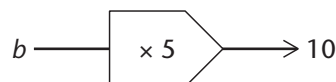
Write a few examples of simple equations on the board, one by one. For example:

$$\begin{array}{lll} n + 7 = 12 & a - 3 = 20 & 100 - x = 80 \end{array}$$

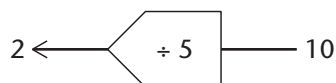
Remind the class that, when a letter is used for a number, the multiplication sign is often left out, so that $2b$ means the same as $2 \times b$. Ask pupils to use their whiteboards to write the solutions to these equations:

$$4x = 4$$

Solve the equation $5b = 10$.



The inverse machine is:



Solution: $b = 2$

Tell the class that another way to solve equations is by trial and improvement. Using a computer with a data projector, show a simple prepared spreadsheet, such as the one below. Set the size of the font to 28 pt or larger so that the whole class can see the text.

Microsoft Excel - A4-79present1.xls

File Edit View Insert Format Tools Data Window Help Acrobat

Tab: a question for help

| | A | B | C | D | E | F |
|----|-------------------|----|---|---|---|---|
| 1 | Value of n | 7 | | | | |
| 2 | Value of $3n + 5$ | 26 | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |
| 7 | | | | | | |
| 8 | | | | | | |
| 9 | | | | | | |
| 10 | | | | | | |

100% Zoom | A4-79 present1 | Page 1 of 1

For example, to solve the equation $3n + 5 = 17$, enter the formula $3*B2 + 5$ in cell B4. Ask pupils to suggest values for n , and enter these into cell B2. Tell the class to observe what happens in cell B4, and whether the result is too big or too small. Use the feedback to refine suggestions and home in on the correct solution $n = 4$.

Unit 6 section 6: Division

Unit 15 section 5: Brackets

page 488

Springboard 7

Units 6 and 15

PLENARY

Resources
OHT 7A.3b

Tell the class that equations with missing numbers can be solved by making sure that each side remains balanced. Show **OHT 7A.3b** and discuss the first problem:

$$400 + 300 = 600 + \square$$

Say that the left-hand side of the equation $400 + 300$ can be written as:

$$(400 + 200) + (300 - 200) = 600 + 100$$

Explain that 200 has been added in one place and then subtracted in another place.

The overall value of the left-hand side of the equation remains unchanged. By comparing the result with the right-hand side of the original equation we can see that the number that the box represents is 100.

Work through the second problem: $14 + 6 = 4 + \square$. Show that the left-hand side of the equation can be written as:

$$(14 - 10) + (6 + 10) = 4 + 16$$

In this case, the box represents the number 16.

Ask pupils to work in pairs to solve the remaining equations. After a few minutes, draw the class together and give the answers. Choose two or three of the equations and select a pair of pupils to explain their solutions to the class.

KEY IDEAS FOR PUPILS

- Algebra uses letters and symbols to replace words and numbers.
- $5a$ means 5 times a .
- Each side of an equation is the same.
- The inverse returns you to where you started.
- One way to solve an equation is to use an inverse function machine.
- Another way to solve an equation is by trial and improvement.

Menu 1

| | |
|-----------------------------|-----|
| Pasta | 55p |
| Pasta and salad | 75p |
| Pasta, salad and milk | 90p |
| Pasta and milk | — |
| Salad and milk | — |

Menu 2

| | |
|------------------------------------|-----|
| Egg | 30p |
| Egg and toast | 60p |
| Egg, tomato and toast | 70p |
| Egg, tomato, beans and toast | 90p |
| Tomato, beans and toast | — |
| Beans and toast | — |

Menu 3

| | |
|------------------------------------|-------|
| Curry | £2 |
| Curry and rice | £2.50 |
| Curry and bhaji | £2.40 |
| Curry and kebab | £2.80 |
| Curry, rice and bhaji | — |
| Curry, rice, bhaji and kebab | — |
| Kebab and rice | — |

Write a number in the box at the end of each equation to make it correct.

1 $400 + 300 = 600 + \square$

2 $14 + 6 = 4 + \square$

3 $23 + 2 = 13 + \square$

4 $37 - 20 = 27 - \square$

5 $40 + 17 = 30 + \square$

6 $40 - 17 = 30 - \square$

7 $6 \times 5 = 3 \times \square$

8 $40 \times 10 = 4 \times \square$

9 $7000 \div 100 = 700 \div \square$

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