

Resource sheet PR1

Proportion or not? Data sets

These examples can be used to raise the question 'Are these data sets in direct proportion?'

A	1	3	B	20	5
	2	6		28	7
	3	9		44	11
	7	21		4	1
				84	21
C	3	4	D	10	15
	7	8		12	20
	14	15		14	25
				16	30
E	3	10	F	77	11
	4	13		21	3
	5	16		672	96
	6	17			
G	411	611	H	3	9
	457	657		5	25
	429	629		8	64
				10	100
I	42	4	J	14.2	65.32
	84	8		6.9	31.74
	105	10		321	1476.6
	252	24		55.55	255.53
	357	34			

These sets are in direct proportion: A, B, F, I, J.

Resource sheet PR2

Proportion or not? Contextualised data sets

These examples can be used to raise the question 'Which sets of variables are in direct proportion?' by considering the sets of data in context.

Playgroup

The following table is designed to show staff at a playgroup how many adults are needed to look after different sized groups of pupils.

Number of adult staff	Maximum number of children
2	8
3	16
4	24
5	32

Although there is a constant difference in each column, the figures are not in direct proportion. The relationship could be described by $c = 8(a - 1)$.

Phone bill

A mobile phone bill shows the following details.

Calls to other networks

Duration (min:sec)	Cost (pence)
2:35	77.5
7:12	216
3:04	92
12:55	387.5
10:10	305
1:44	52

The cost is directly proportional to the duration (30p/min) but the times may need to be converted to seconds to make the relationship clear.

Belt prices

A clothing website allows customers to pay in pounds sterling (£) or euros (€)

These are the prices for four different belts:

£6.99	or	€11
£13.99	or	€22
£15.99	or	€25
£25	or	€39

The two prices are very nearly in direct proportion. A conversion rate of $£1 = €1.56$ has been used, but the euro prices have been rounded to the nearest whole number.

Clicko kits

Clicko building kits come in five sizes. Their components are listed below.

Kit	Base plates	Long rods	Short rods	L-joints	H-joints
Beginner	1	10	30	20	15
Designer	1	16	48	32	24
Advanced	1	24	72	48	36
Expert	2	42	126	84	63
Supreme	2	60	180	120	90

The number of base plates is not in proportion to the numbers of other components. However, the others are provided in the ratio 2 : 6 : 4 : 3.

Photographs

A photographic shop offers to reprint enlargements of photographs according to the following table.

Size of print	Cost
10" by 8" (25 cm by 20 cm)	£5.00
14" by 10" (36 cm by 25 cm)	£9.00
18" by 12" (46 cm by 30 cm)	£13.80

The size data are 'real'. Several comparisons are possible. The ratio of the height to width of the different prints is not consistent (either measured in inches or centimetres). The conversion rate from inches to cm is nearly consistent at $1'' = 2.5$ cm. The price is directly proportional to the area of the prints given in cm^2 .

Cooling coffee

In a science experiment, the temperature of a cup of coffee is measured over half an hour. The results are tabulated.

Elapsed time in minutes	Temperature in °C
0	98
5	73
10	56
15	42
20	34
25	30
30	28

The temperature is not directly proportional to the elapsed time.

Wire

A factory sells spools of wire cable by weight. The labels show the length of wire on each spool.

Weight (kg)	Length (m)
0.75	57
1.5	114
3.5	266
5	380

The weight and length are in direct proportion. The 'size' of the wire can be expressed in different units, for example 76 m/kg.

Beethoven's symphonies

A boxed set of Beethoven's nine symphonies provides the following information.

Symphony number	Duration of recording in minutes
1	36
2	30
3	48
4	31
5	29
6	33
7	32
8	24
9	64

Clearly, these figures show no arithmetic relationship.

Water drum

A large concrete drum holds water for cattle on an Australian farm. The farmer measures the depth of the water and uses this table to estimate its volume.

Depth of water	Volume
0.9 m	150 gallons
1.2 m	200 gallons
1.5 m	250 gallons
2.4 m	400 gallons

The depth and volume measures are in direct proportion.

Resource sheet PR3

Proportion or not? Problems

These closed questions require a decision about whether the variables are in direct proportion.

- 1** 2.5 litres of paint are sufficient to cover 80 square metres. How much paint do I need to cover 250 square metres?
- 2** A seaside harbour has a tide marker showing the depth of water inside the harbour. At midnight the depth is 4.2 m. At 2:00 am it is 4.9 m. What will the depth be at midday?
- 3** A garage sells diesel fuel at 73.9p per litre. How much can I buy for £20?
- 4** Henry the Eighth had six wives. How many wives did Henry the Fourth have?
- 5** My recipe for 9 scones uses 200 grams of flour. How much flour will I need for 24 scones? The nine scones need 8 minutes in a hot oven. How long will I need to cook 24?
- 6** A gardener has a lawn which is 15 m by 12 m. She decides to feed it with fertiliser applied at 1.5 grams per square metre. How much fertiliser does she need?
- 7** A sprinter can run 100 m in 11.2 seconds. How long will it take the sprinter to run 250 m?
- 8** A shop buys cans of soft drink in boxes of 24 for £1.99 per box. They sell the cans at 39p each. Is their total profit proportional to:
 - (a) the number of boxes they buy;
 - (b) the number of cans they sell;
 - (c) the money they take?
- 9** When Robyn was 1 year old she weighed 11 kg. When she was 2 years old she weighed 14 kg. How much did she weigh when she was 4 years old?
- 10** A 750 g box of cornflakes costs £2.19. How much does a 1 kg box cost?

Resource sheet PR4

Proportion or not? Situations

These examples can be used to raise the question 'Are the variables in direct proportion?' by considering the situation rather than sets of data.

True or false?

- 1 In the different countries of the world, the number of cars on the road is directly proportional to the population.
- 2 The weight of flour in a sack is directly proportional to the volume of flour.
- 3 The monthly electricity bill is directly proportional to the amount of electricity used.
- 4 The time an audio tape plays for is directly proportional to the length of tape.
- 5 The temperature of a saucepan of soup is directly proportional to the time it has been on the stove.
- 6 The cost of an article of clothing is proportional to how long it will last.
- 7 The time taken to read a maths problem and the time taken to solve it are in direct proportion.
- 8 The cost of a train journey is directly proportional to the distance travelled.

When could we reasonably assume the following to be true and when false?

- 9 The time taken to drive a journey is directly proportional to the distance covered.
- 10 The amount of money a waitress earns is directly proportional to the number of hours she works.
- 11 The cost of a phone call is proportional to the length of the call.
- 12 The amount of wallpaper I have to buy is directly proportional to the area of the walls I want to cover.
- 13 The time taken to read a book is directly proportional to the number of pages in the book.

Resource sheet PR5

Pupils' solutions to proportion problems: Stacking CDs

Kelsey's solution

9 CDs put side by side on a shelf measure 5 cm. How many centimetres would 14 CDs placed side by side measure?

Handwritten solution on grid paper:

0.55

9 | 5.00

1 CD \approx 0.55 cm wide

$$\begin{array}{r} \times 14 \\ 0.55 \\ \hline 7.7 \end{array}$$

14 CDs side by side would measure 7.7 cm

David's solution

9 CDs put side by side on a shelf measure 5 cm. How many centimetres would 14 CDs placed side by side measure?

Handwritten solution on grid paper:

$\frac{1}{2}$ of 9 = $4\frac{1}{2}$ cd's

$\frac{1}{2}$ of 5 cm = $2\frac{1}{2}$ cm

$$\begin{array}{l} 9 \text{ cds} + 4\frac{1}{2} \text{ cd's} \text{ about } 14 \text{ cd's} \\ 5 \text{ cm} + 2\frac{1}{2} \text{ cm} \text{ about } 7\frac{1}{2} \text{ cm} \end{array}$$

Anna's solution

9 CDs put side by side on a shelf measure 5 cm. How many centimetres would 14 CDs placed side by side measure?

Handwritten work for Anna's solution:

- Top left:** A rough calculation: $5 \div 9 = 0.555$ (with a dot over the 5) and $1.8 \times 9 = 16.2$. Below this, it says "Roughly 7.56 cm".
- Top middle:** A long division problem: $54 \div 32 = 1.6875$. The result is written as 1.6875 with a horizontal line under the last two digits.
- Top right:** A list of numbers: $0.54, 7.02, 1.68, 1.62, 2.16, 2.70, 3.24, 3.78, 4.32, 4.86, 5.40, 5.94, 6.48$. The number 7.56 is underlined.
- Bottom left:** A calculation: $0.55 \times 14 = 7.7$. Below this, it says $2.75 + 5 = 7.75$ cm.
- Bottom right:** A cloud-shaped bubble containing the text: "the length of 14 CD will be 7.75 cm."

Sam's solution

9 CDs put side by side on a shelf measure 5 cm. How many centimetres would 14 CDs placed side by side measure?

$$9 \div 5 = 1.8 = 1.8 \times 14 = 25.2 \text{ cm}$$

Joanne's solution

9 CDs put side by side on a shelf measure 5 cm. How many centimetres would 14 CDs placed side by side measure?

$$\begin{array}{r} \text{approx: } 0.5 \\ 9 \overline{) 5.0} \end{array}$$

$$1 \text{ cd} = \text{approx } 0.5$$

$$\begin{array}{r} 0.5 \times 14 \\ \hline 7.0 \end{array}$$

$$14 \text{ cd's} = 7.0 \text{ cm (approx.)}$$

Simba's solution

9 CDs put side by side on a shelf measure 5 cm. How many centimetres would 14 CDs placed side by side measure?

Ratio	9 CDs	5 cm
	14 CDs	more

$$\Rightarrow \frac{14 \text{ CDs}}{9 \text{ CDs}} \times 5 \text{ cm} = 7.77777778$$

$$\approx 7.8 \text{ cm (1-d.p.)}$$

Bina's solution

9 CDs put side by side on a shelf measure 5 cm. How many centimetres would 14 CDs placed side by side measure?

$$\begin{aligned}
 9 &= 5 \text{ cm} & 9 \div 5 &= 4 \text{ mm} \\
 1 \text{ cd} &= 4 \text{ mm} & 14 \times 4 \text{ mm} &= 56 \text{ cm}
 \end{aligned}$$

Lee's solution

9 CDs put side by side on a shelf measure 5 cm. How many centimetres would 14 CDs placed side by side measure?

$$\begin{aligned}
 & \begin{array}{r} 0.55 \\ 9 \overline{) 5.000} \\ \underline{45} \\ 50 \\ \underline{45} \\ 50 \\ \underline{45} \\ 50 \end{array} & 0.55 \times 2.75 & \begin{array}{r} 2.75 \\ 2 \overline{) 5.50} \\ \underline{55} \\ 0 \end{array} \\
 & 2.75 + 5 = 7.75 \text{ cm} & & \text{the length of 14 CD will be } 7.75 \text{ cm.}
 \end{aligned}$$

Resource sheet PR6

Pupils' solutions to proportion problems: Coursework

Asjad's solution

For a coursework task Ben gained $\frac{5}{8}$ of the available marks, while Natasha gained $\frac{2}{3}$. Who got the better mark?

$$\begin{aligned} 5/8 &= 100 \div 8 = 12.5 \times 5 = 62.5 \text{ ben's mark out of 100} \\ 2/3 &= 100 \div 3 = 33.\dot{3} \times 2 = 66.6 \text{ natasha's mark} \end{aligned}$$

natasha got a higher mark than Ben.

Daniel's solution

For a coursework task Ben gained $\frac{5}{8}$ of the available marks, while Natasha gained $\frac{2}{3}$. Who got the better mark?

$$\begin{aligned} \text{Ben got } 5/8 \\ 5 \div 8 &= 0.625 \\ \text{Natasha got } 2/3 \\ 2 \div 3 &= 0.66\bar{6} \\ \text{this means Natasha got the better mark} \end{aligned}$$

Zakir's solution

For a coursework task Ben gained $\frac{5}{8}$ of the available marks, while Natasha gained $\frac{2}{3}$. Who got the better mark?

Ben gained more marks.

$\frac{2}{3} = \frac{6}{9}$ bigger

$\frac{5}{8} = \frac{5}{8}$


Danny's solution

For a coursework task Ben gained $\frac{5}{8}$ of the available marks, while Natasha gained $\frac{2}{3}$. Who got the better mark?

they are the same.
~~Natasha~~ Because

$\frac{2}{3}$

$\frac{5}{8}$



Natasha's solution

For a coursework task Ben gained $\frac{5}{8}$ of the available marks, while Natasha gained $\frac{2}{3}$. Who got the better mark?

Natasha
~~Ben~~, because she has $\frac{2}{3}$ which is $\frac{1}{3}$ away from 1, whereas Ben has $\frac{5}{8}$ which is $\frac{3}{8}$ away from 1, so therefore Natasha has the highest mark.

Sam's solution

For a coursework task Ben gained $\frac{5}{8}$ of the available marks, while Natasha gained $\frac{2}{3}$. Who got the better mark?

Common denominator of 8 & 3

8	16	24	32	Common
3	6	9		denominator 15 =
24				3

$8 \overline{) 24} = 3$

$\frac{2}{3}$ is the better mark

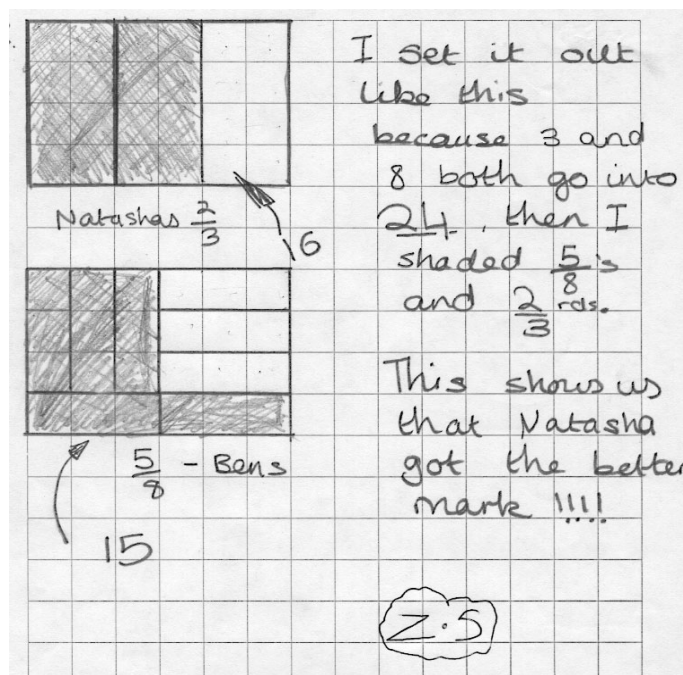
$\frac{2}{3}$	$\frac{15}{24}$	13
---------------	-----------------	----

$\frac{5}{8}$	$\frac{16}{24}$	N
---------------	-----------------	---

= Natasha =
15:16

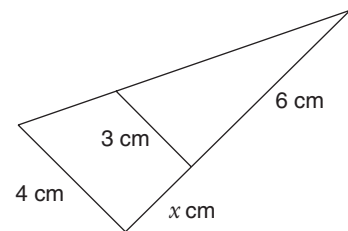
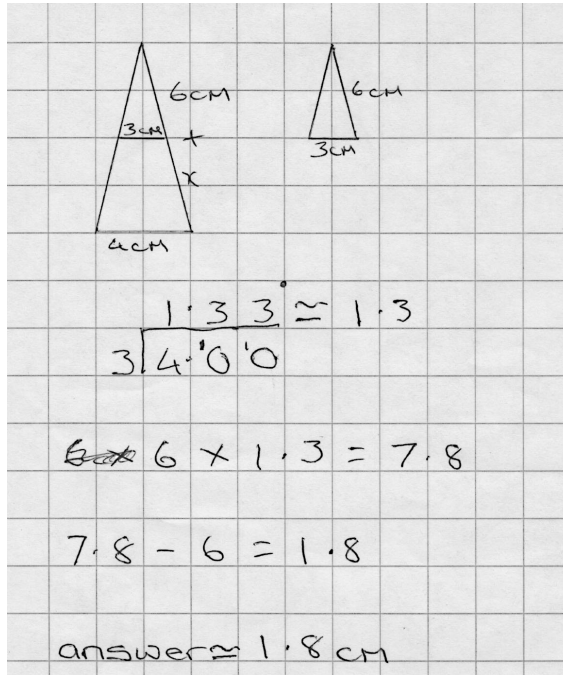
Sarah's solution

For a coursework task Ben gained $\frac{5}{8}$ of the available marks, while Natasha gained $\frac{2}{3}$.
Who got the better mark?

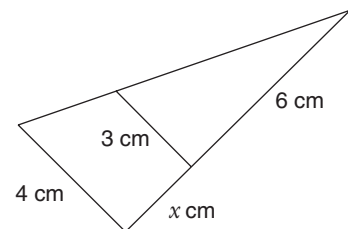


Pupils' solutions to proportion problems: Similar triangles

The two triangles are similar. Calculate x .



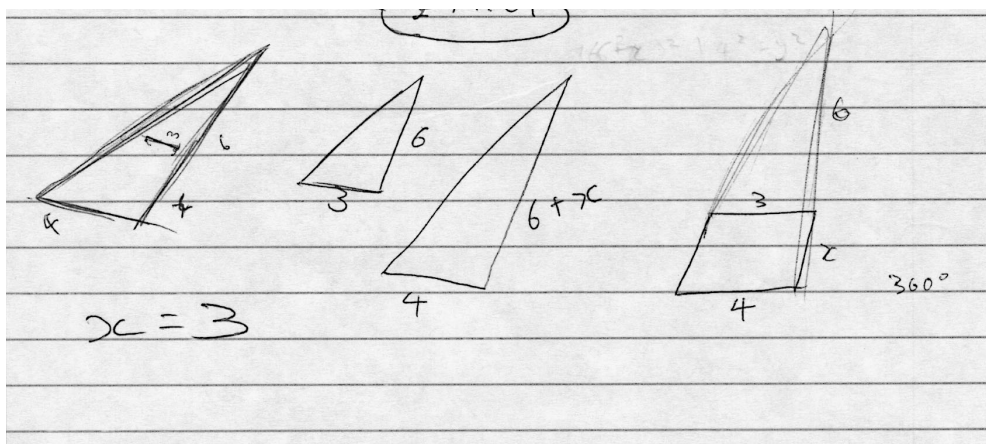
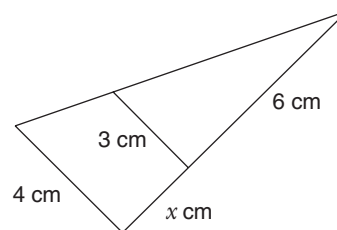
The two triangles are similar. Calculate x .



$$\begin{aligned} 3 \times 6 &= 18 & = 9 \div 2 & = 18 \div 2 \\ &= 18 \div 2 & = 4.5 & x = 9 \text{ cm} \\ &= 9 \text{ cm} & = 4.5 \times 4 & \\ & & = 18 & \end{aligned}$$

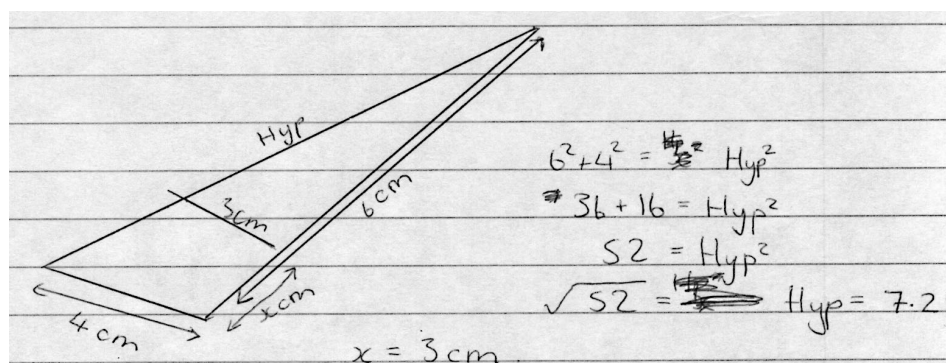
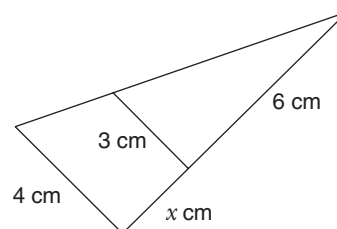
Kwon Joo's solution

The two triangles are similar. Calculate x .



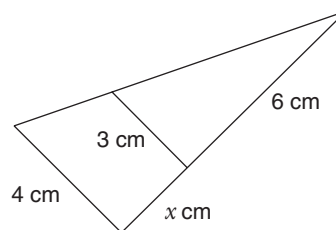
Michael's solution

The two triangles are similar. Calculate x .



Zoe's solution

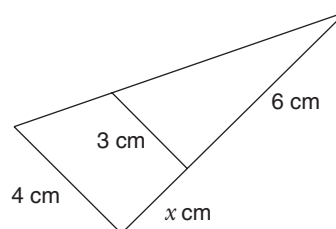
The two triangles are similar. Calculate x .



$$\begin{array}{l} \text{s/s} \quad 3 : 6 \\ \text{b/b} \quad 4 : 8 \end{array} \quad \frac{4}{8} = \frac{3}{6} \quad x = 2 \text{ cm}$$

Jack's solution

The two triangles are similar. Calculate x .



at 3 cm the height is doubled to 6 cm so therefore
at 4 cm the length will be 8 cm

Resource sheet PR8

Pupils' solutions to proportion problems: Gold brooch

Vamezy's solution

A gold brooch weighs 58.75 g. The density of gold is 19.3 g/cm³. What volume of gold is used to make the brooch?

$$58.75 \text{ g} \div 19.3 \text{ g/cm}^3 = 3.044041451$$

James's solution

A gold brooch weighs 58.75 g. The density of gold is 19.3 g/cm³. What volume of gold is used to make the brooch?

$$\begin{aligned} B &= 58.75 \text{ g} \\ 1 \text{ cm}^3 &= 19.3 \text{ g} \\ \frac{58.75}{19.3} &= 3.04 \text{ cm}^3 \end{aligned}$$

Kieran's solution

A gold brooch weighs 58.75 g. The density of gold is 19.3 g/cm³. What volume of gold is used to make the brooch?

57.9 g of gold is used to make the brooch

$$\begin{array}{r} 19.3 \\ \times 3 \\ \hline 57.9 \end{array}$$

$$\begin{array}{r} 19.3 \\ 19.3 \\ + 19.3 \\ \hline 57.9 \end{array}$$

~~57.9~~ 3.5 cm³

Lee's solution

A gold brooch weighs 58.75 g. The density of gold is 19.3 g/cm³. What volume of gold is used to make the brooch?

$$\begin{array}{r} 58.75 \\ - 19.30 \\ \hline 39.45 \end{array}$$

Gold = 39.45 g Gold = 39.45 g

Xenan's solution

A gold brooch weighs 58.75 g. The density of gold is 19.3 g/cm³. What volume of gold is used to make the brooch?

$$\begin{aligned}
 &\cancel{3.5} \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm} = 27 \text{ cm} \\
 &3.5 \text{ cm} \times 3.5 \text{ cm} \times 3.5 \text{ cm} = 42.875 \\
 &3.75 \text{ cm} \times 3.75 \text{ cm} \times 3.75 \text{ cm} = 52.73 \\
 &3.8 \text{ cm} \times 3.8 \text{ cm} \times 3.8 \text{ cm} = 54.87 \\
 &3.9 \text{ cm} \times 3.9 \text{ cm} \times 3.9 \text{ cm} = 59.319 \\
 &\cancel{3.85 \text{ cm} \times 3.8} \\
 &3.85 \text{ cm} \times 3.85 \text{ cm} \times 3.85 \text{ cm} = 57.07 \\
 &3.87 \text{ cm} \times 3.87 \text{ cm} \times 3.87 \text{ cm} = 57.96 \\
 &3.89 \text{ cm} \times 3.89 \text{ cm} \times 3.89 \text{ cm} = 58.86 \\
 &\text{estimate. } \cancel{3.8} 3.9 \text{ cm}^3
 \end{aligned}$$

Percy's solution

A gold brooch weighs 58.75 g. The density of gold is 19.3 g/cm³. What volume of gold is used to make the brooch?

$$\begin{aligned}
 \text{Volume} &= \frac{\text{Weight}}{\text{Density}} \\
 &= \frac{58.75}{19.3} \\
 &= \underline{3.04}
 \end{aligned}$$