

Hide the spies

In this **investigation** pupils determine where spies should sit in a park that has a square grid of benches, interspersed by bushes, so that they cannot see each other, and investigate how many different arrangements are possible.

Suitability Pupils working in pairs or as individuals

Time Up to 1 hour

Equipment

'Hide the spies' worksheet (Pupil stimulus)

Interactive software

Squared or dotty paper

Counters/cubes, cups

Resources

PUPIL STIMULUS

PDF INTERACTIVE

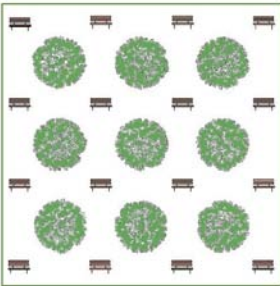
TEACHER SUMMARY

TEACHER GUIDANCE


PROGRESSION TABLE

SAMPLE RESPONSES

Hide the spies



Which park benches should the four spies use so that they cannot see each other?
How many different ways are there of doing this?



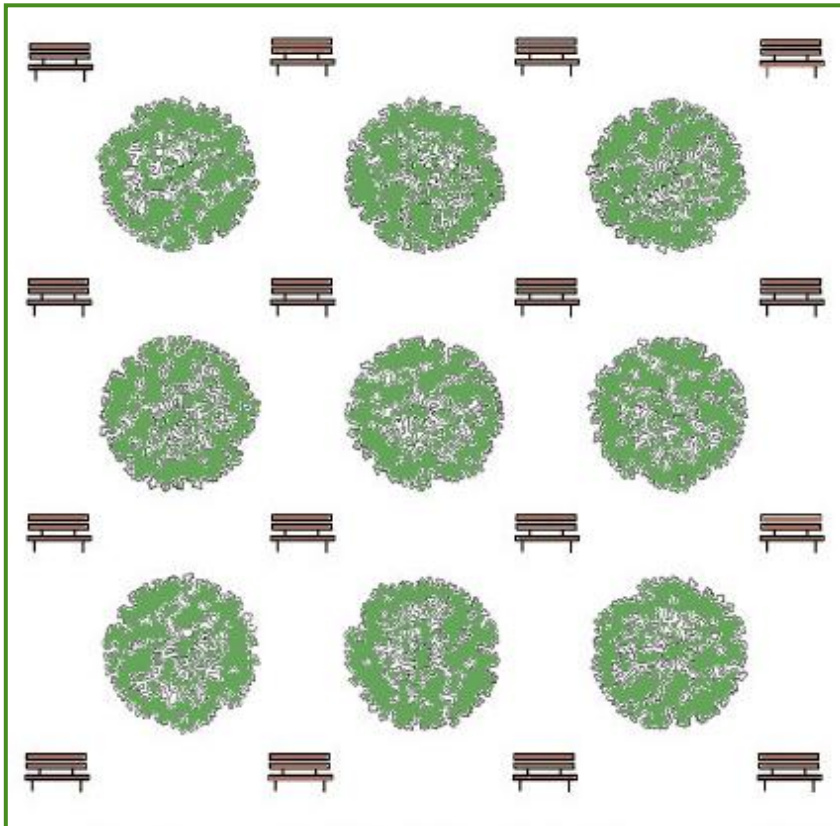
A park has 16 park benches and 9 very large bushes.
Four spies want to make secret phone calls in the park, at the same time.

Nuffield AMP: Pupil stimulus 'Hide the spies'
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Hide the spies



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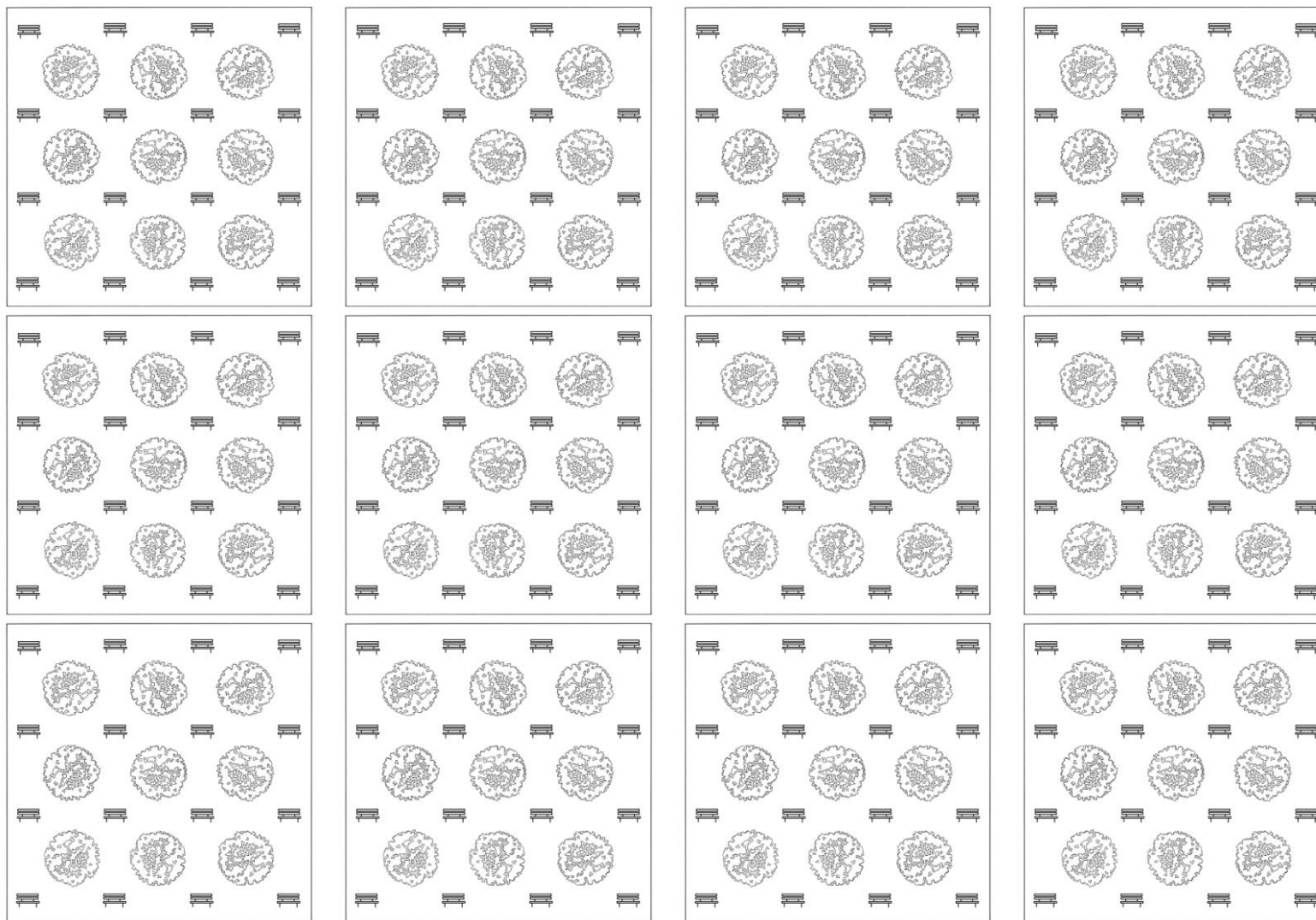
Which park benches should the four spies use so that they cannot see each other?

How many different ways are there of doing this?



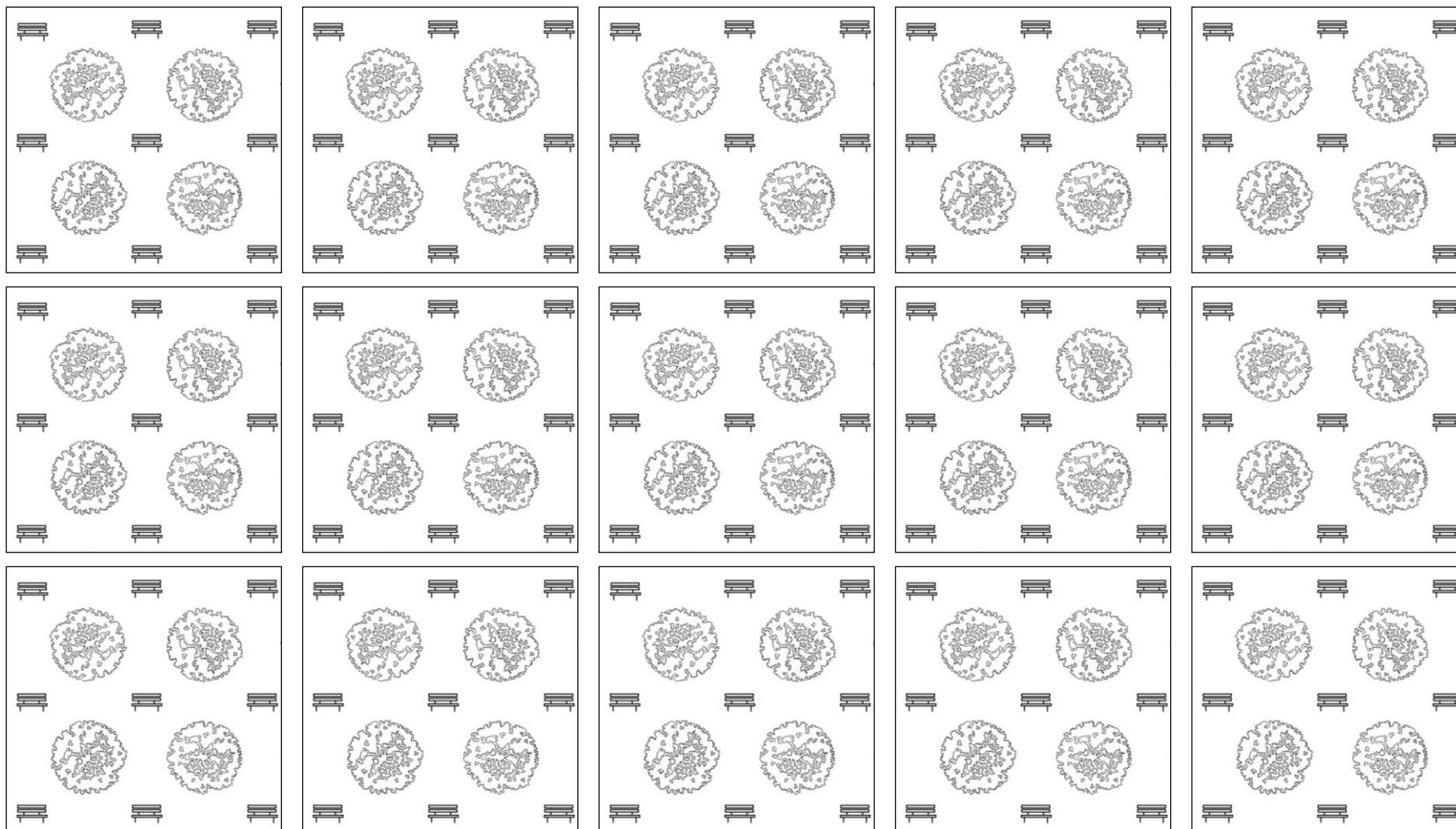


Hide the spies *worksheet*





Hide the spies *worksheet*



Hide the spies worksheet image/teacher notes

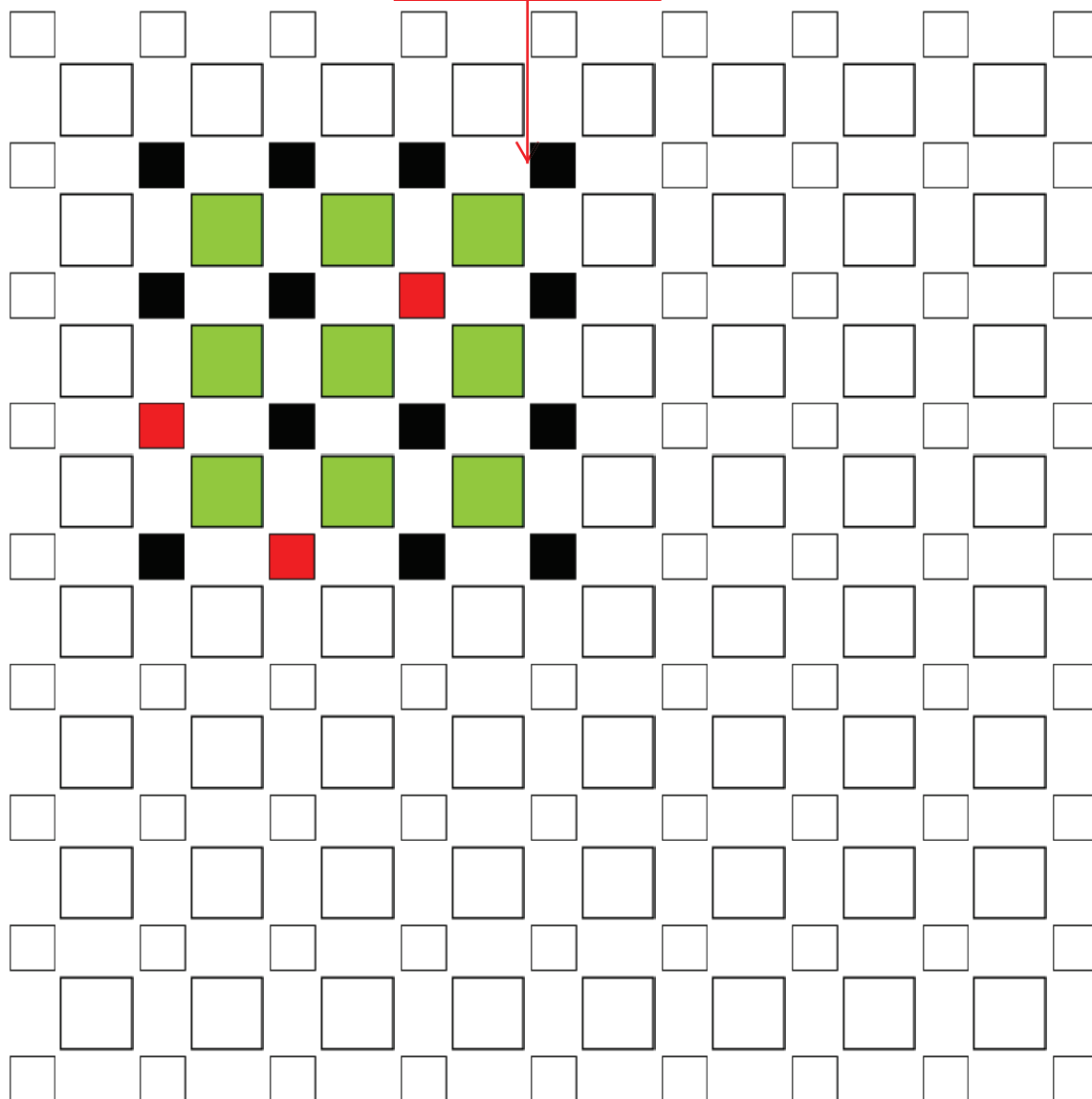
Click on the
squares to scroll
through the
different colours



Investigator

Class

Date



Description of investigative work

This text box can be used by pupils for the description of investigative work on the AMP activity Hide the spies. Teachers can use the comments box to give feedback.

The coloured boxes can be used to illustrate the benches, bushes, and spies.

Notes/comments

Download a version of this pdf for pupil use at www.nuffieldfoundation.org/AMP



NUFFIELD APPLYING MATHEMATICAL PROCESSES

TEACHER NOTES Hide the spies

Activity description

Pupils investigate where four spies should sit in a square park so that they cannot see each other, and how many such arrangements are possible.

Suitability

Pupils working in pairs or as individuals

Time

Up to 1 hour

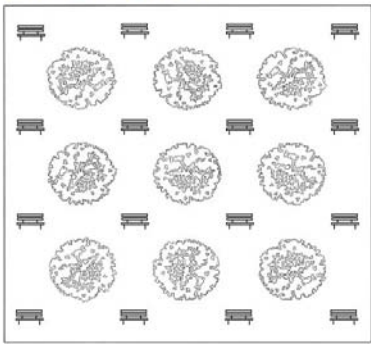
AMP resources

Pupil stimulus,
Print worksheets,
Interactive PDF
worksheet


Equipment

Counters, cubes,
cups, etc. to
simulate spies
and bushes

Hide the spies



Which park benches should the four spies use so that they cannot see each other?
How many different ways are there of doing this?



A park has 16 park benches and 9 very large bushes.
Four spies want to make secret phone calls in the park, at the same time.

Nuffield AMP Pupil stimulus 'Hide the spies'

Key mathematical language

lines of sight, arrangement/configuration, symmetry, reflection, rotation

Key processes

Representing Choosing a mathematical approach to finding arrangements.

Analysing Working systematically, generating arrangements, using symmetry.

Interpreting and evaluating Identifying patterns and making generalisations, considering findings to form convincing arguments.

Communicating and reflecting Explaining the approach taken and the outcomes achieved at each stage of the work.



Teacher guidance

Explain the context of the problem and emphasise that no spy should be able to see another spy. Preliminary discussion may be needed so that pupils understand how the very large bushes obscure vision. It might be worth investigating lines of sight within the school to help support understanding.

The interactive PDF provided can be used to introduce the task, or you may prefer to model the problem physically, perhaps using pupils and desks, or using counters and the worksheet provided.

Starting with one spy sitting on a bench, ask pupils to find all the benches where another spy could sit so that the spies cannot see each other. Invite one or two pupils to present their findings.

Pupils can use the interactive PDF, or printed worksheets, to investigate the problem. Pupils could also record solutions on squared paper.

During the activity

Encourage pupils to explore their ideas, generating their own systems for finding results, but highlight the need to work systematically. This could be done as a group discussion at some stage in the investigation, asking pupils to share how they have been organising their work.

Showing each spy in a different colour can help pupils to be more systematic, but it can lead to an unwieldy number of results if permutations are taken into account. Stress that the task is about which park benches the spies are sitting on, rather than the individual spies.

Probing questions and feedback

- Pupils who have two diagrams that are equivalent, say via a rotation, can be engaged in a discussion on 'equating' congruent/symmetric arrangements.
- How are you deciding what result to find next?
- Have you found all the different ways? How can you be certain?
- Could you have 5 spies for a '4x4' array of benches? Why not?

Extensions

- Exploring connections between *Hide the spies* and *Sudoku*.
- Using different sizes of rectangular parks, and/or different numbers of spies.

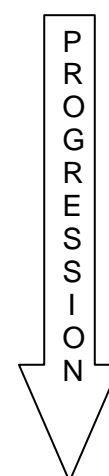
- Numbering the park benches as shown on the right, and adding together the numbers of the park benches that the spies are sitting on: What do pupils notice? Ask them to investigate and explain any patterns they find.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16



Progression table

Representing	Analysing	Interpreting and evaluating	Communicating and reflecting
<i>Choosing a mathematical approach</i>	<i>Logical ordering of the outcomes and accuracy of results</i>	<i>Identifying patterns and exceptions, and generalising</i>	<i>Explaining the approach taken and the outcomes achieved at each stage</i>
Shows understanding of the task by identifying benches where spies can or cannot sit	Produces arrangements that are correct Pupils A, B	Makes simple observations on arrangements found	Arrangements shown clearly
Recognises that there are mathematical principles that can be used to generate families of results Pupils A, B, C	Identifies patterns and uses them to find further arrangements and seeks all arrangements Pupil C	Makes observations about relationships between arrangements and how these can be used to produce further results Pupils A, C	Presents arrangements and describes some common features and differences, and/or gives some indication of approach Pupils A, B, C
Develops a mathematical approach to find arrangements Pupil D	Uses a mathematical approach accurately to generate all arrangements Pupil D	Seeks general principles within arrangements for a given size and/or across varying sizes Pupil D	Presents results systematically Communicates the approach taken and how conclusions were arrived at Pupil D
Chooses an effective and efficient strategy to explore all possibilities	Uses a well-defined mathematical approach systematically and accurately to find all arrangements Finds the number of possible arrangements	Makes insightful observations about arrangements and strategies found and relates these to the original problem, and possibly to extensions	Describes clearly and succinctly the systematic approach taken and how conclusions were reached Reflects on how approaches can be generalised to solve related problems



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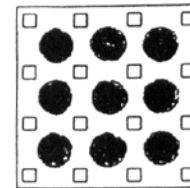
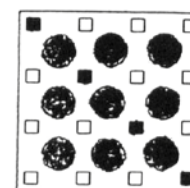
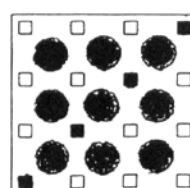
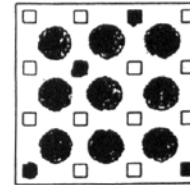
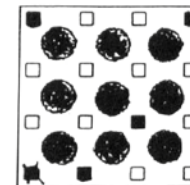
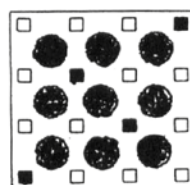
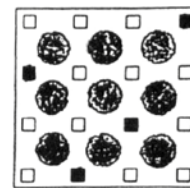
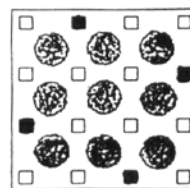
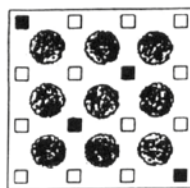
Sample responses

Pupil A

Pupil A has drawn 8 different results, and recognises the role of reflection and rotation in generating further results.

Probing questions

- What approach have you been using to generate new results?
- How can symmetry help you to find and classify other results?
- How will you determine that you have found all possible arrangements?



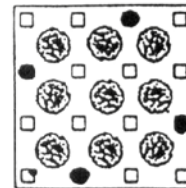
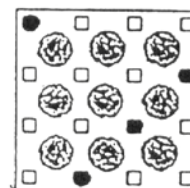
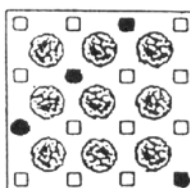
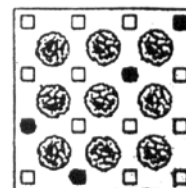
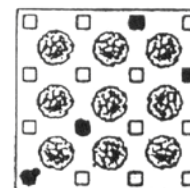
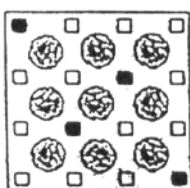
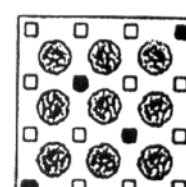
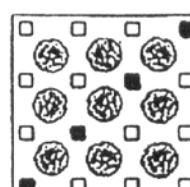
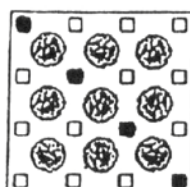
you can only have one spy in each line
there is a minimum of 1 plat & a max of 4 through rotation some of them work through reflection.

Pupil B

Pupil B used symmetries to generate results and explained '... means you don't miss out ones by mistake and you can concentrate on thinking where the next one is going to go'. But there is no evidence that the order of symmetry has been recognised for different arrangements.

Probing questions

- What did you do to diagram 3 (top right) to get diagram 4 (middle left)? Could you use these to get further arrangements?
- Can you tell how many more arrangements you can get from one particular arrangement by using symmetry?





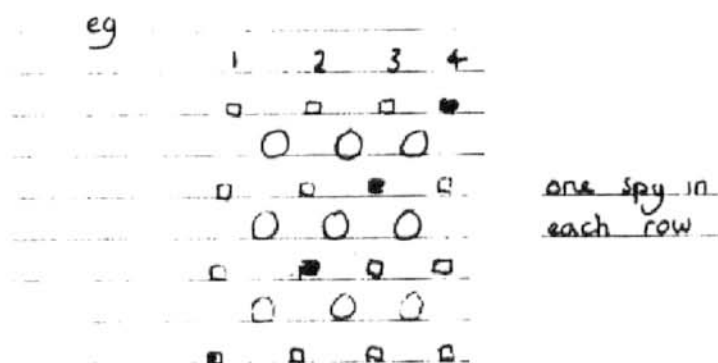
Pupil C

Pupil C has found all 24 results using a 'careful' analysis, but claims to have found 'no rules'. Not all the arrangements shown are those that work, and the method is unclear.

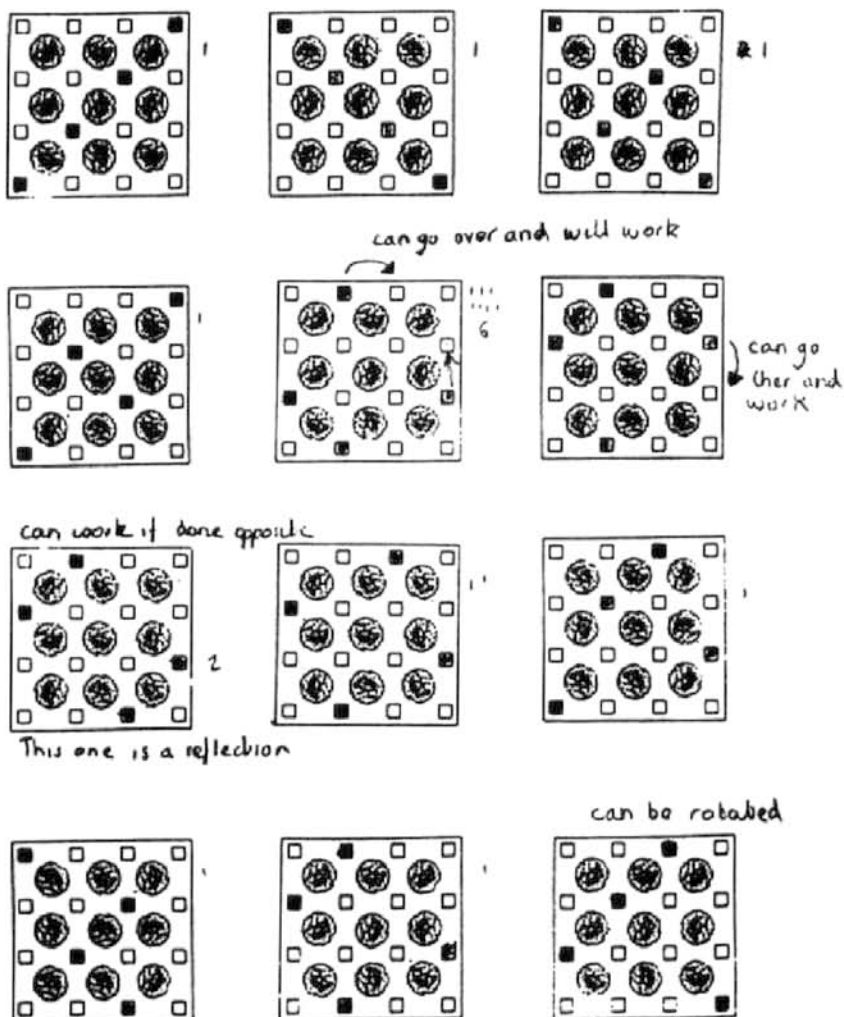
Probing questions

- Why doesn't each result have the same number of rotations?
- How would you explain your approach in greater detail? And how has it led you to 24 arrangements?

I thought that you can only have one spy in each row



I found 24 ways of doing it but no rules you had to be careful as there was rotations and reflections



For it to work it has to have one spy in each row. Some of the ones above are reflections and rotations.



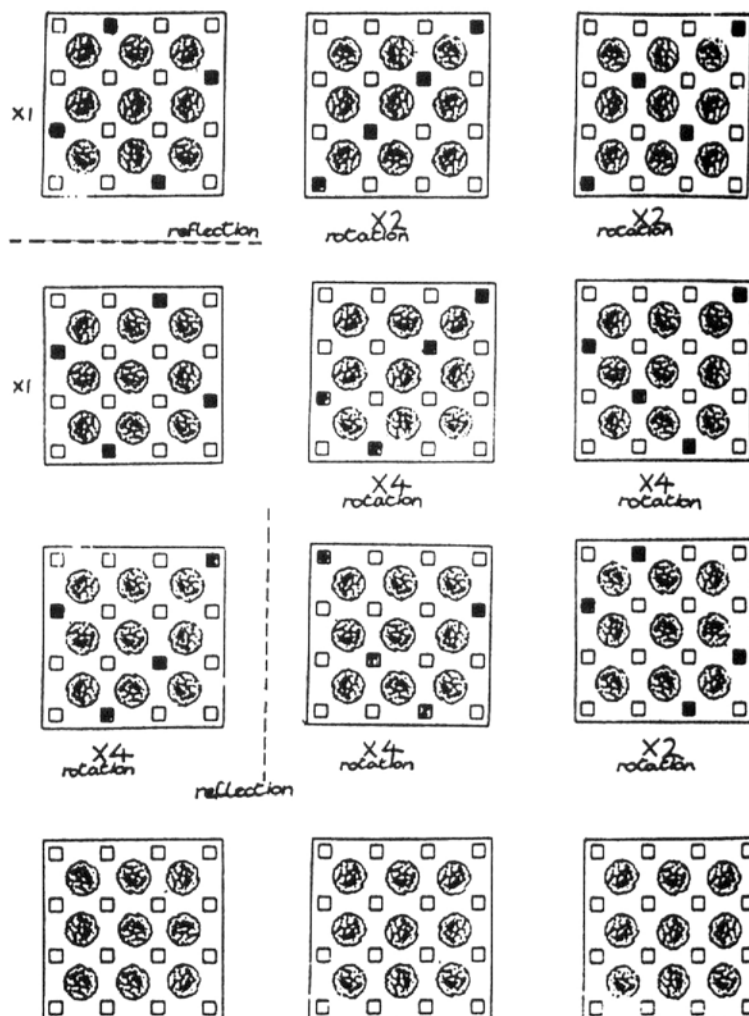
Pupil D

Pupil D shows all 24 results, some of which were found by controlling the positions of spies.

Different sized square parks have been investigated, but the result for a 5 by 5 park is incorrect.

Probing questions

- How can you be sure that you have all the results for the 5 by 5 park?
- What do you notice about the results for different sizes of park? Can you explain your observations?



Only one box can be used in each row and each column.

Results:

1x1 with 1 man = 1 way
 2x2 with 2 men = 2 ways
 3x3 with 3 men = 6 ways
 4x4 with 4 men = 24 ways
 5x5 with 5 men = 36 ways

From these results I cannot see any obvious rules (apart from number one)