Secondary National Strategy: Algebra study modules

Introduction for teachers

Aims of these study modules

These ten *Algebra study modules* are intended for teachers who would like to reinforce, confirm and extend their knowledge of teaching algebra in Key Stages 3 and 4. They are suitable for all teachers of mathematics, including supply teachers, trainee teachers and those who would like to return to teaching. They have been designed as guides for use either by an individual teacher working alone or by a group of teachers working together.

The main aims of the modules are:

- to develop understanding of some important aspects of the algebra in Key Stages 3 and 4;
- to strengthen the planning and teaching of mathematics.

Content

The modules have been adapted from two short courses written originally by the Key Stage 3 Strategy, *Constructing and solving equations* and *Planning and teaching mathematics*. The course materials have been extended to include aspects of the mathematics curriculum in Key Stage 4 and to take account of the *Revised learning objectives for mathematics* published by the Secondary National Strategy (2010).

Descriptions of the ten modules are as follows:

Module 1: Using a grid method to multiply expressions **40 minutes**

This discusses ways of using a grid method to help pupils to multiply a single term over a bracket and to find the product of two linear expressions.

Module 2: Transforming expressions and equations **40 minutes**

This considers ways of helping pupils to form equivalent expressions through 'clouding the picture' activities. It also discusses two of the teaching principles which help pupils to use and apply their algebraic skills with confidence:

- providing opportunities for pupils to express generality;
- asking pupils to 'find as many ways as you can'.

Module 3: Constructing expressions and equations **40 minutes**

This examines helpful ways of teaching pupils to construct algebraic expressions, equations and formulae, starting from pupils' existing knowledge of numerical examples and building up to algebraic examples.

Module 4: Rearranging equations and formulae **60 minutes**

This discusses ways of using arithmetic to introduce algebraic ideas, specifically of using pupils' understanding of the laws of arithmetic and inverse operations to help

© Crown copyright 2010

them to rearrange equations and formulae. It also introduces the idea of a 'mental computer screen' to support algebraic manipulation.

Module 5: Collecting like terms **20 minutes**

This short module considers some stimulating activities to help pupils to practise collecting like terms.

Module 6: Applying skills in the context of 'Pyramids' **60 minutes**

This explores how pupils can use and apply their algebraic skills in the context of 'Pyramid puzzles'. The work is developed in four stages: understanding the structure of a pyramid by using numbers, collecting like terms, constructing and solving equations intuitively and constructing and solving equations more formally.

Module 7: Applying algebraic reasoning **60 minutes**

This discusses ways of using magic squares and addition grids to help pupils to apply algebraic reasoning and, for pupils working confidently at level 5 or above, to prove their results.

Module 8: Generalising from patterns and sequences **40 minutes**

This examines ways of using patterns and sequences to help pupils to generalise.

Module 9: Linking sequences, functions and graphs **40 minutes**

This explores some interesting activities to help pupils to link sequences, functions and graphs. It also considers the importance of helping pupils to make connections between mathematical topics.

Module 10: Classroom approaches to algebra **20–30 minutes**

The purpose of this module is to conclude and help teachers to reflect on a sequence of one or more algebra modules which have been studied either earlier in the day or previously. Although it cannot be studied on its own, it may be studied on more than one occasion by the same teacher or group of teachers. Since it involves consideration of what has just been worked through, it can result in different conclusions and decisions each time it is used.

Planning your study time

There is no need to study all the modules unless you want to. Each is designed as a self-contained unit of work. Choose those that most interest you and that will give you the most help.

The approximate time needed for studying a single module is stated. They can be worked through as a sequence over time, e.g. as part of a regular departmental meeting, or they can be combined on a 'mix and match' basis if a longer study period is available, e.g. on a professional development day.

'Doing' the modules by reading through them is not enough. You will gain much more from them if you try out and evaluate ideas in the classroom and incorporate successful aspects into your teaching plans.

If you are an individual teacher who intends to work through the materials on your own, you should try to get some support or mentoring for your study, perhaps from your head of department or another experienced mathematics teacher who will act as a subject mentor. There may be points that you are unsure about and it is useful to have someone to ask or talk to. It also helps if you study the modules at about the

 © Crown copyright 2010

same time as another colleague so that you can discuss what you are learning as you go along.

In each module, time is allowed for you to reflect on your stage of development, to study sections of the *Revised learning objectives for mathematics* published by the Secondary National Strategy (2010), and to think about and note any action points arising from your reflections.

Aim to build up your own learning file as you study. You can then refer back to it to gauge your progress. You can also have it by your side when you are later planning, trying and refining your teaching approaches.

Resources you will need

Each module identifies for you the essential resources that you will require, including materials that can be downloaded as PDF files from the Internet. The relevant web references for downloadable files are given in each module.

In all the modules, you will need to equip yourself with:

- a folder for storing resource sheets as you work through the modules;
- a personal notepad;
- a copy of the algebra strand of the *Revised learning objectives for mathematics* for Key Stages 3 and 4 produced by the Secondary National Strategy (2010), which you can download from:

  nationalstrategies.standards.dcsf.gov.uk/secondary/framework/maths/fwsm/mlo

Before you begin

Read the Appendices to this introduction:

- **Appendix 1: Why algebra?**
- **Appendix 2: The teaching principles**

As you read, consider why and how we teach algebra, and how the views expressed in the articles compare with your own view.

Follow up in the classroom

After you have completed your study you may wish to read and use these excellent materials, which describe a number of different secondary algebra lessons.

- *Improving learning in mathematics: Mostly algebra* (sessions A1–A14)

The materials were originally published in 2005 by the Standards Unit of the then Department for Education and Skills (DfES). They can be downloaded from the Learning and Skills Improvement Service (LSIS) Excellence Gateway:

tlp.excellencegateway.org.uk/pdf/mat_imp_02.pdf

© Crown copyright 2010

Appendix 1: Why algebra?

This short extract is from *Key aspects of teaching algebra in schools*, by John Mason, John and Ros Sutherland (2002). London, UK: QCA 02 913.

> 'All of the summaries and many of the papers we reviewed agree, implicitly or explicitly, that algebraic thinking contributes to being a full citizen able to participate fully in the democratic process, and that algebra is the language in which the use of mathematics in economic activity is expressed.
>
> From a democratic point of view, any citizen who is unconfident with expression and manipulation of generality cannot function fully in the political and economic process, because modern society runs on the assertion and critique of generality, including the use of mathematical models to study and predict the effects of policy decisions. Citizens unable to engage in this debate are disenfranchised.
>
> In an industrial culture, owners, factors, and managers all need to deal with the general in formulating (note the etymology) and deciding amongst different policies and when determining procedures to be followed by employees, which is in essence, a form of algebra. By contrast, customers are interested only in the particular application of these rules to their situation. However, citizens need to be able to engage in thinking about the general in order to appreciate how those decisions are being made.
>
> In a knowledge-economy, everyone who participates is faced with assertions of generality concerning policy decisions and choices. Citizens need to be able to analyse and critique these assertions and the models which underlie them, and to assert their own versions. Algebra provides the basis, the language, the foundation for this participation.
>
> Today's society places considerable emphasis on the use of technological tools such as spreadsheets and databases. These have their roots in the early development of computer programming languages, which in their turn have their roots in mathematics generally and algebra in particular. Thus, it can be argued that today's citizens should both appreciate and become competent in the generalising and symbolising power of algebra, in order to be able to understand the potential and the constraints of these computational packages. Software only does what it has been 'programmed' to do.'

© Crown copyright 2010

Appendix 2: The teaching principles

The Algebra Study Modules focus on three main teaching principles.

1 Providing opportunities for pupils to express generality

Generality lies at the heart of mathematics. Learning to make algebraic generalisations begins with analysis of numerical examples and moves on to the use of letters to stand for unknown numbers or variables.

The first principle is to get pupils to generalise for themselves from examples that they have generated rather than having generalisations presented to them. The main advantages of this approach are that pupils:

- relate what they are doing in algebra to what they already know in arithmetic;
- begin to appreciate the purpose of algebra;
- are better able to understand algebraic expressions, equations and formulae, and sequences, functions and graphs, if they have generated some for themselves.

2 Asking pupils to 'find as many ways as you can'

This principle requires that pupils are regularly asked to write algebraic expressions in different ways: for example, to construct expressions, equations or formulae and to transform them, or to represent an algebraic relationship in as many different ways as possible. The many benefits of this are that pupils:

- appreciate that the same relationship can be expressed or represented in more than one way;
- gain confidence in manipulating and transforming expressions, equations and formulae into different equivalent forms;
- have opportunities to choose ways of representing such relationships, using their knowledge of equivalent forms (e.g. tables, functions and graphs), so that the context can be analysed and the solution communicated;
- have opportunities to discuss which transformations are the most efficient to use in a particular context, e.g. when they solve an equation;
- learn how expressions are built up and the related process of 'undoing', which they need when they simplify expressions or solve equations.

3 Creating connections between mathematical topics

Pupils may feel that mathematical learning is compartmentalised. Closely related ideas, notation or forms of representation stay unconnected in their minds. 'Linking activities' help to draw out connections across mathematical topics. The advantages are that pupils:

- make connections between arithmetical operations and equivalent algebraic forms when they transform expressions and equations;
- make connections between sequences, functions and graphs when they explore the effects of varying values;
- find it easier to transfer what they learn to similar situations.

© Crown copyright 2010

Appendix 2 [continued]

When the three teaching principles are applied consistently, pupils learn to construct and manipulate algebraic expressions, equations and formulae, and to represent algebraic relationships, based on their understanding. This provides a far stronger foundation for their learning than when they are given sets of rules to apply. As a result, when the steps in a procedure are not obvious they are much more able to resolve difficulties for themselves.

© Crown copyright 2010