Module 2: Transforming expressions and equations

Description

This module is for an individual teacher or group of teachers in secondary schools who are considering their teaching of algebra. It discusses ways of helping pupils to form equivalent expressions through 'clouding the picture' activities. It also considers two of the teaching principles which help pupils to use and apply algebra with confidence:

- providing opportunities for pupils to express generality;
- asking pupils to 'find as many ways as you can'.

Other modules which could be combined with this one, either to create a longer session, or to work through in a sequence over time are:

- Module 5: Collecting like terms
- Module 7: Applying algebraic reasoning

Study time

About 40 minutes

Resources

Each teacher will need a personal notepad.

Each teacher or pair of teachers working together will need:

- copies of **Resources 2a, 2b, 2c and 2d** at the end of this module;
- a copy of the algebra strand of the *Revised learning objectives for mathematics* for Key Stages 3 and 4 produced by the Secondary National Strategy (2010), which you can download from:

  nationalstrategies.standards.dcsf.gov.uk/secondary/framework/maths/fwsm/mlo

It would be helpful to have available a copy of *Teaching mental mathematics from level 5: Algebra,* published by the Secondary National Strategy (2009), which you can download from:

nationalstrategies.standards.dcsf.gov.uk/node/241296

Transforming expressions and equations

1 It is not unusual for teachers to model the simplification of an expression of like and unlike terms and then to give pupils an exercise of similar questions of increasing complexity. A less common approach is to present pupils with an algebraic expression and ask them to find as many as possible different ways to write it. This activity supports an interactive teaching approach and can help you to identify pupils' misconceptions.

Look at the 'clouding the picture' diagrams on **Resource 2a, Equivalent expressions**. The aim in each case is to rewrite the central expression in many different ways by continuing a process along the same or a different branch, then to generalise by explaining succinctly what is happening along each branch.

Consider how you might use these prompts and suggestions if pupils were undertaking the activity in the classroom.

- Choose two branches. What is the same and what is different about them?
- Can you start a new branch that does something different to change the expression?
- Evaluate each expression by substituting a value for each variable, e.g. x = 1, a = 2.
- What can you say about the expressions in any pair of boxes? [They are equivalent.]

You can prompt pupils to be as creative as possible, e.g. introduce brackets or fractional terms, or a second variable, differentiating the level of challenge for different pupils. For example, pupils who are ready to ‘complete the square’ could start by working in this way.

Recording pupils’ results during lessons on a flipchart or OHT helps them to share their developments and generalisations.

2 The transformation of equations can be approached in a similar way. Traditionally, pupils are taught rules for solving equations such as ‘change the side, change the sign’ or ‘do the same to both sides’. If they try to use these rules without understanding them and why they work, pupils often make errors such as:

$4x = 20 \Rightarrow x = \frac{20}{-4}$ (change the side, change the sign)

$3(x - 2) = 6 \Rightarrow x - 2 = 3$ (take 3 from both sides)

‘Doing the same to both sides’ is the more meaningful method, but there are two difficulties:

- knowing how to change both sides of an equation so that equality is preserved;
- knowing which operations lead towards the desired goal.

Building equations is easier than solving them because it postpones the second difficulty and so is an easier place to start.

Look at the diagrams on **Resource 2b, Transforming equations** while you read the text below. The central box contains the equation x = 3. Along each branch of the tree, build an equation, step by step, using each of the four rules, +, –, ×, ÷ and whole numbers between 1 and 10.

3 When you work with pupils in the classroom you could do a similar activity on an OHT or whiteboard. As pupils suggest each operation, you could supply the notation and explain it carefully. For example, you could explain that we use brackets to show that a whole expression is being multiplied and that we use the fraction bar rather than the usual division symbol (÷).

Then ask pupils to work in pairs with another example, being as creative as they can. They get them to swap with another pair and analyse the work to see what is the same and what is different about their approaches.

Encourage pupils to verbalise what they are doing to maintain the balance of their equations, and to check that they are generating equivalent equations by substitution of the original value of the variable.

4 From here, the next step is to generate equivalent equations by changing the rule between each stage, as in the first diagram on **Resource 2c, Changing the rule for each step**.

If you were doing this in the classroom you could ask pupils to check that the original value of x still satisfies the final equation.

$$3\left(\frac{3+5}{4}-1\right)=3\left(\frac{8}{4}-1\right)=3(2-1)=3$$

Now hide all the steps except the final equation and ask pupils to recall each operation in sequence, using prompts such as:

- This equation tells the story of 'x'. What happened to x first? How can you tell by looking only at the equation?
- What then?
- What then?
- What was the last thing that happened?

In this way, you can show pupils that the final equation 'tells the story' of the operations used. You could follow this by asking them:

- Suppose you had started with this equation and you wanted to find the value of x. How could you do this?
- How can you undo what we have just done?

Gradually get pupils to unpick each step in reverse order. As they do this, uncover the preceding equations one by one and write the corresponding operation to the right of each equation (with upward arrows), as in the second diagram on Resource 2c.

You will probably need to work through more examples like this with the whole class, until they get the idea. It is worth changing the letter used each time, just to make the point that there is nothing special about the letter x.

Follow up by asking pairs of pupils to create equations starting from a simple equation such as $a = 6$. The pairs then give one of their equations to another pair to find the original value of the variable by working in reverse order.

5 For further practice in the creation of equations you could ask pupils to use their mini-whiteboards to write down algebraic equations that correspond to some 'think of a number' problems. For example, you might say:

Think of a number, call it n.
Double it.
Add 4.
Divide your answer by 7.
Multiply your answer by 2.
The result is 4.
Show me the equation.

And pupils might respond: $2\left(\frac{2n+4}{7}\right)=4$

6 You could also set pupils the task of generating simultaneous equations from using a chosen solution such as $a = 2$, $b = 3$ in the central box of a tree structure. Take a few minutes to sketch a tree structure in your notepad to show how they would do this.

You could then ask pupils to choose two of the equations from the end of branches to present to another pupil, who must then find the original solution. This can be done by transforming each equation by changing the rule at each step in order to match the coefficients of one of the variables. Again, sketch out a possible process in your notepad.

Now look at one way of generating then solving a pair of simultaneous equations on **Resource 2d, Simultaneous equations**.

7 To sum up, the 'clouding the picture' approach helps pupils to:

- generate equivalent expressions and equations, including a simplified form;
- substitute values into equations and formulae;
- consider different approaches, e.g. where another pupil has represented the problem or approached its solution in a different way;
- communicate solutions;
- gain confidence and develop increasing fluency in manipulating expressions into equivalent forms without being rule bound.

Now reflect on these questions.

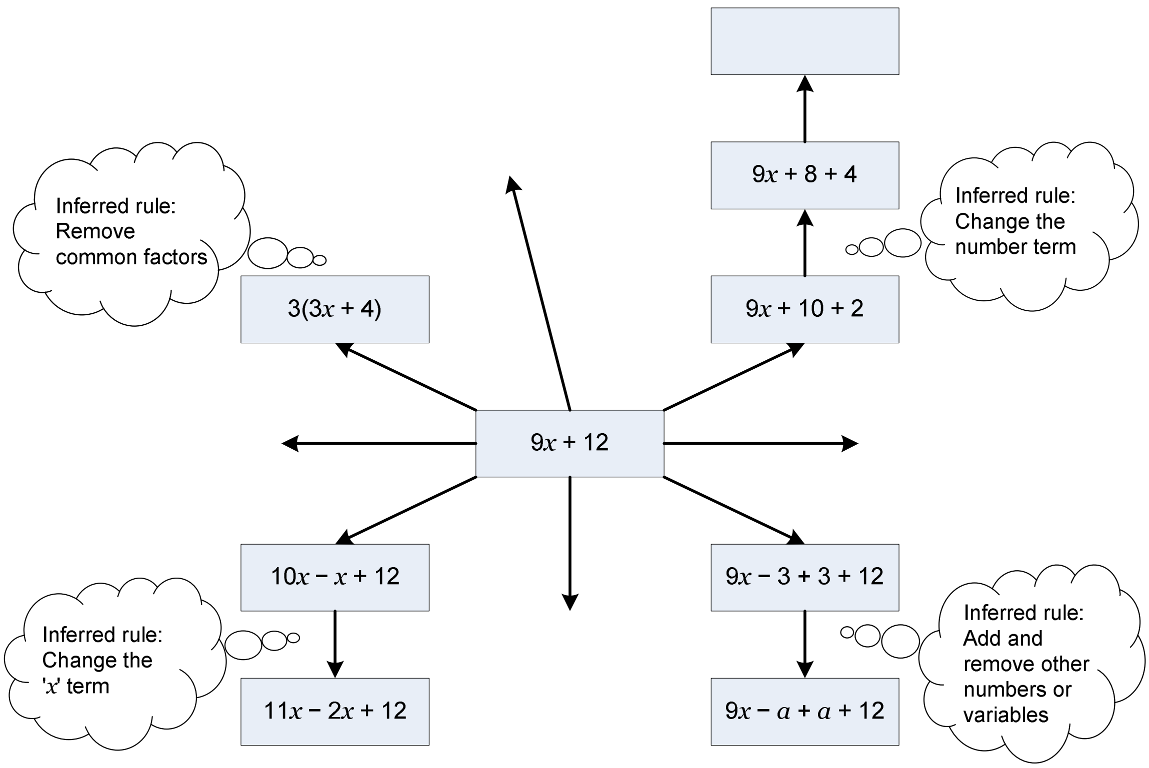
- Where could 'clouding the picture' activities for creating and solving equations fit in your mathematics lessons for the classes that you teach? To help, you may wish to refer to your copy of the *Revised learning objectives for mathematics* for Key Stages 3 and 4 produced by the Secondary National Strategy (2010).
- What action, if any, do you need to take?

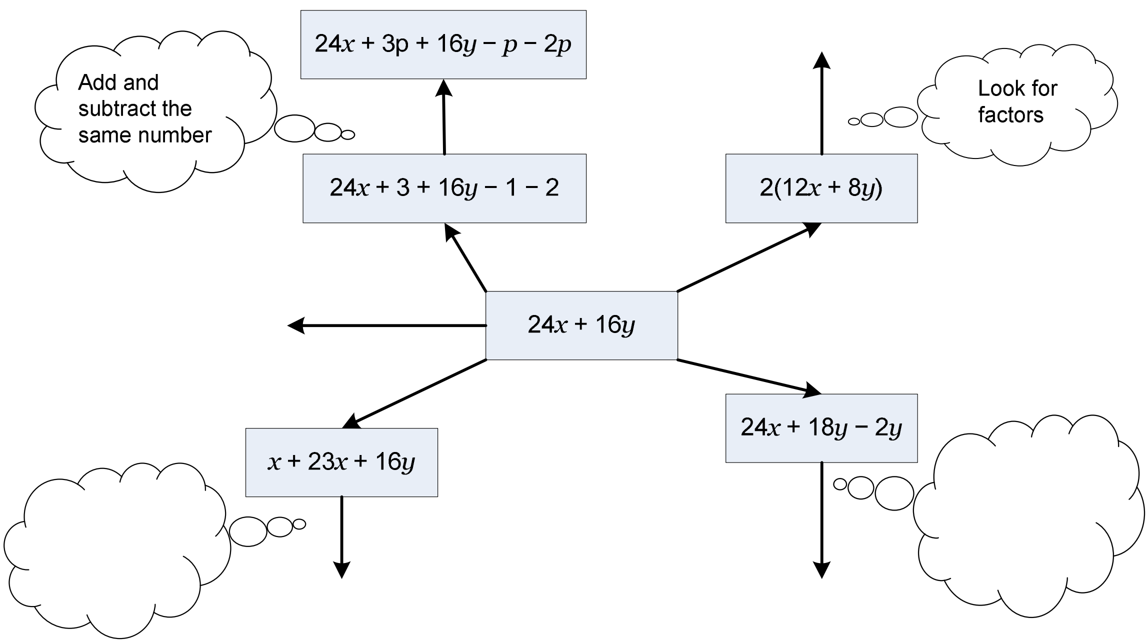
You can find more examples of 'Clouding the picture' tree structures for pupils to complete on pages 22–24 of *Teaching mental mathematics from level 5: Algebra,* published by the Secondary Strategy (2009), which you can download from:

nationalstrategies.standards.dcsf.gov.uk/node/241296

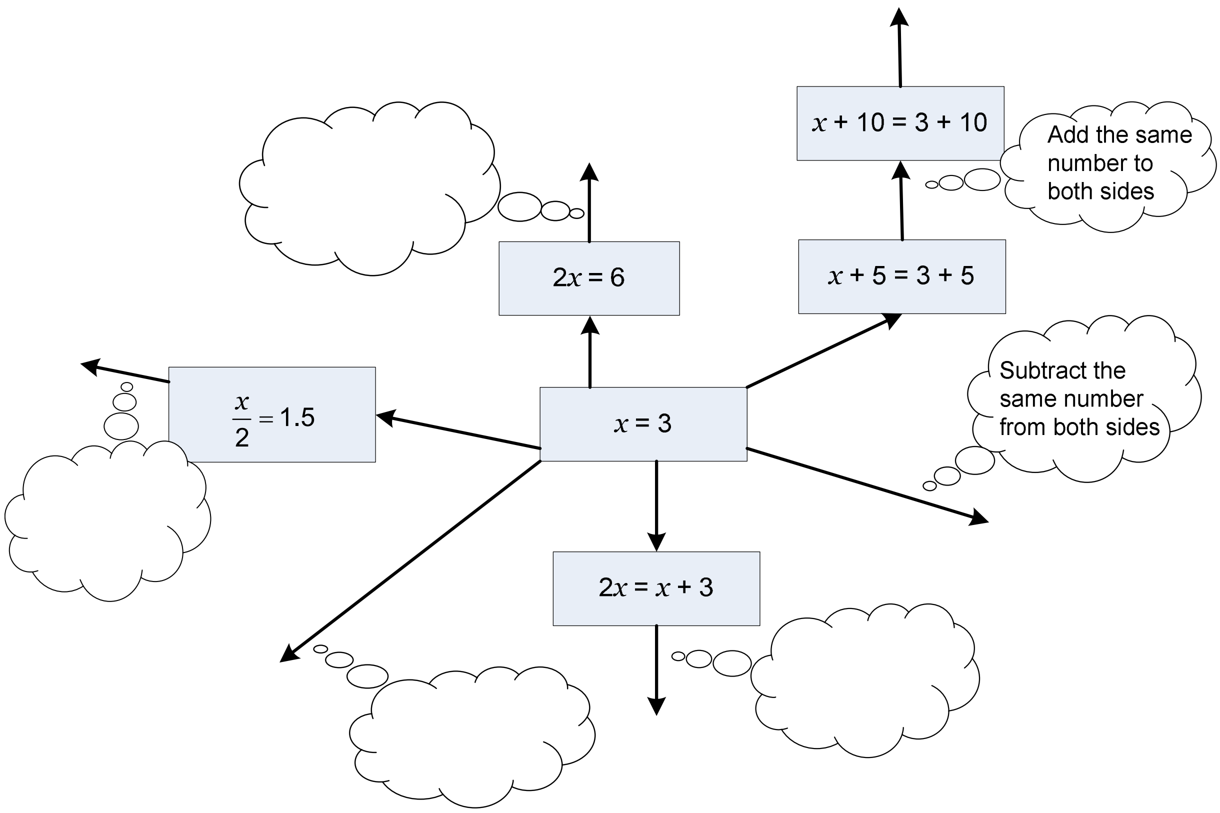
Finally, jot down any points to follow up and, if you are studying alone, any points you want to discuss with your head of department or other colleagues.

Resource 2a: Equivalent expressions





Resource 2b: Transforming equations



Resource 2c: Changing the rule at each step

Diagram 1

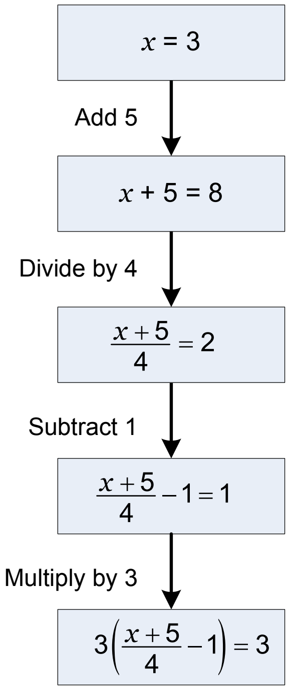
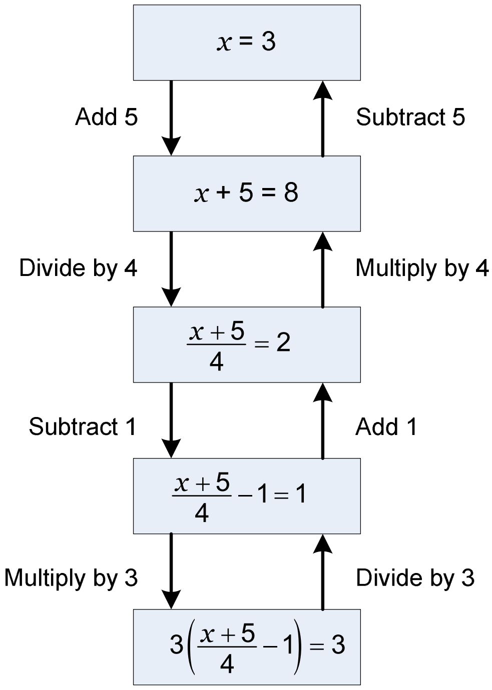
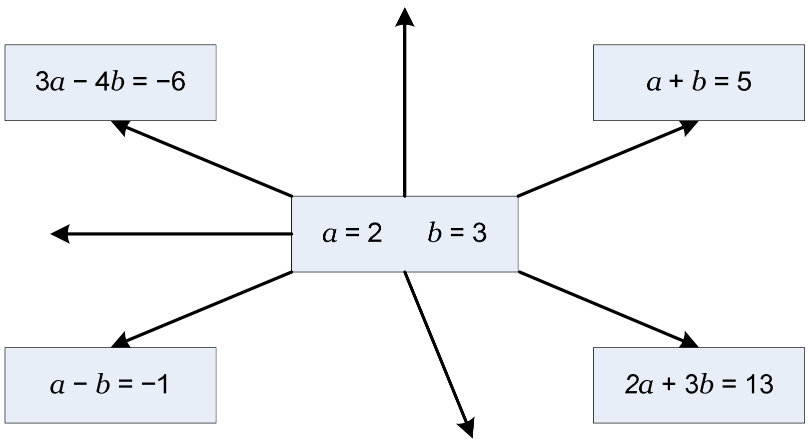


Diagram 2

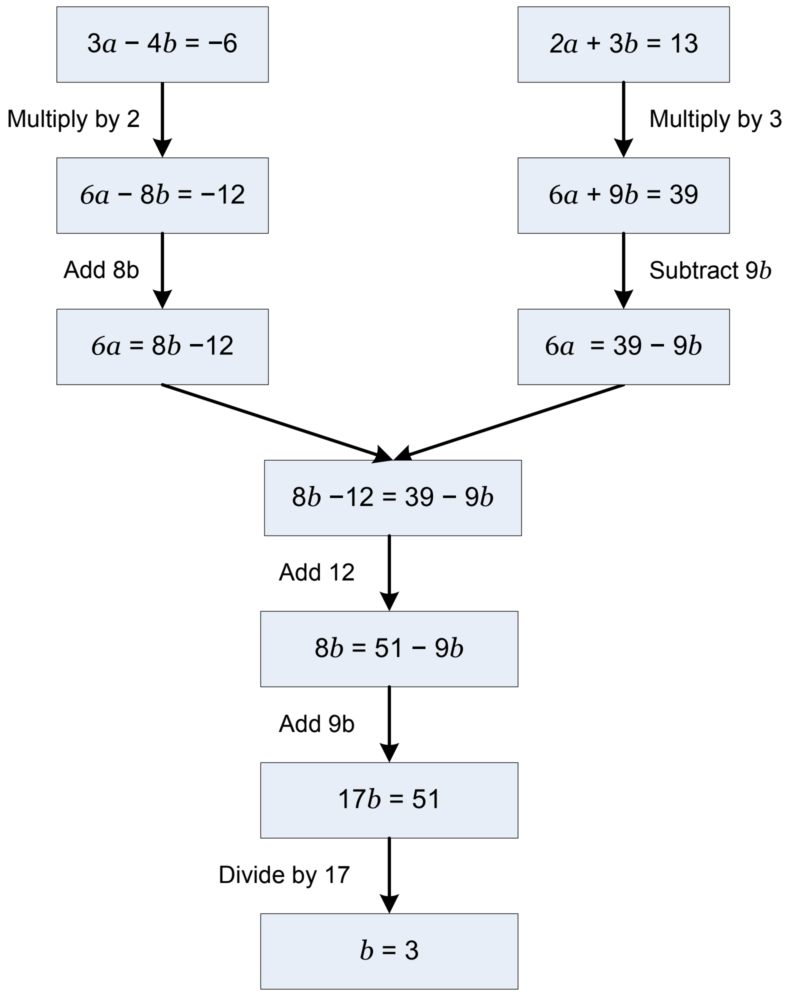


Resource 2d: Simultaneous equations

Generating simultaneous equations



Solving simultaneous equations



Substituting $b = 3$ in $2a + 3b = 13$ gives $2a + 9 = 13 \Rightarrow 2a = 4 \Rightarrow a = 2$

Solution $a = 2$, $b = 3$

Check by substituting $a = 2$, $b = 3$ in $3a – 4b = –6$, which gives $6 – 12 = –6$