Module 4: Rearranging equations and formulae

Description This module is for an individual teacher or group of teachers in secondary schools who are considering their teaching of algebra. It discusses ways of using arithmetic to introduce algebraic ideas, specifically of using pupils' understanding of the laws of arithmetic and inverse operations to help them to rearrange equations and formulae. It also introduces the idea of a 'mental computer screen' to support algebraic manipulation.

Other modules which could be combined with this one, either to create a longer session, or to work through in a sequence over time are:

- Module 8: Generalising from patterns and sequences
- Module 10: Classroom approaches to algebra

Study time About 60 minutes

Resources Each teacher will need a personal notepad.

Each teacher or pair of teachers working together will need:

- copies of **Resources 4a, 4b, 4c and 4d** at the end of this module;
- a copy of the algebra strand of the *Revised learning objectives for mathematics* for Key Stages 3 and 4 produced by the Secondary National Strategy (2010), which you can download from:

  nationalstrategies.standards.dcsf.gov.uk/secondary/framework/maths/fwsm/mlo

Rearranging equations and formulae

1 Most pupils arrive at secondary school with a sound understanding of additive and multiplicative relationships in number. Algebra can be regarded as generalised arithmetic. But pupils often find it difficult to link algebra to their previous learning in number so too often they see it as a new unrelated topic.

By building on pupils' understanding of number skills, principles and procedures it is possible to demystify algebra and improve pupils' understanding of how to use and apply algebra effectively.

The first step is to ask pupils to reflect on what they already know, particularly about the relationships between addition and subtraction and between multiplication and division. The next step is to help them to recognise and then express the generality which underlies the many particular examples that they have met.

Start by considering this question. If you are working in a group, do this in pairs.

- In which year groups, and for which topics, do pupils in your school learn a new technique in algebra by first considering the equivalent technique in arithmetic?

Write down examples on your notepad and keep them for the end of this module.

2 **Addition and subtraction**

A sound understanding of the ways in which a number sentence involving addition and/or subtraction can be manipulated is a pre-requisite for the understanding of algebraic equations and how to manipulate them.

Work through the questions on **Resource 4a, Addition and subtraction**.

If you are working in a group, when you have completed Part 1 of Resource 4a, you could share with colleagues your ideas on how the images can be represented on a number line.

3 Now try this visualisation activity, called 'Using a mental computer screen'.

If you are working alone, read the text below slowly and silently to yourself. As you do so, visualise the computer screen as you answer the questions.

If you are working in a group, one of you could read the text below for the rest of the group to visualise.

> *Imagine the equation $b + c = a$ written on a computer screen. How many symbols (letters and signs) are there altogether?*
>
> *Imagine that each symbol can be dragged around the screen. Imagine the symbols in a different order, but the equation is still true. What can you see on the screen now? Explain exactly how the symbols have moved.*
>
> *Can you do it another way?*
>
> *Imagine that you have a minus sign as well. Using this sign, what other ways of writing the equation are there?*

Now consider these questions.

- Could this type of visualisation activity be used with pupils and, if so, with which year groups? What advantages and disadvantages would the activity have?
- How could the activity be changed so that it could be used more widely?

In the classroom, the activity could be set up on an interactive white board and the symbols dragged around on the board.

4 Consider how the idea of a 'mental computer screen' can be extended by working through **Resource 4b, More complex + and – equations**.

5 Having tried some addition and subtraction activities to help pupils to manipulate equations, consider each of the statements below, and whether you agree with it.

- The progression from pictures, to symbols on paper, then to mental imagery will deepen pupils' understanding of and facility with algebraic manipulation.
- There is a second thread in the progression, which moves from numbers to variables represented by letters, then to more complex terms.
- The ability to manipulate equations 'in your head' is an important skill, and is developed by using a 'mental computer screen'. If necessary, a teacher can support this by setting it up on an interactive white board and dragging the symbols around on the board.
- Regularly substituting numbers for letters to check answers reinforces the pupils' understanding that the same rules apply to numbers and algebraic expressions.

6 **Multiplication and division**

The relationship between multiplication and division can be explored similarly.

Work through the questions on **Resource 4c, Multiplication and division**.

7 Now try another visualisation activity, using the ‘Mental computer screen’ as before.

Imagine the equation $a \times b = c$ *written on a computer screen. How many symbols (letters and signs) are there altogether?*

Imagine that each symbol can be dragged around the screen. Imagine the symbols in a different order, but the equation is still true. What can you see on the screen now? Explain exactly how the symbols have moved.

Can you do it another way?

Imagine that you also have a division sign (÷). Using this sign, what other ways of writing the equation are there?

In the classroom, this could be set up on an interactive white board and the symbols dragged around the board.

8 Extend the idea of a ‘mental computer screen’ by working through **Resource 4d, More complex × and ÷ equations**.

The progression from pictures, to symbols on paper, then to mental imagery mirrors the addition and subtraction activities, as does the progression from numbers to increasingly complex algebraic expressions.

Note the link with proportionality in the generalisation $ab = cd$ or $\frac{a}{c} = \frac{b}{d}$.
This link could be developed further with pupils, if appropriate.

9 The examples in this module, and adaptations of them, can help pupils to:

- construct algebraic equations;
- make connections with arithmetical operations and with equivalent algebraic forms when they transform expressions and equations;
- substitute values into equations and formulae;
- develop increasing fluency in manipulating expressions and equations into different equivalent forms, including a simplified form, without being rule-bound.

Now return to the question you considered at the very beginning of this module and the notes that you made.

- In which year groups do pupils in your school learn a new technique in algebra by first considering the equivalent technique in arithmetic?

Add as many as possible further examples which you could try in the future. Try to include some examples for each year group that you teach. You may wish to refer to your copy of the revised learning objectives for mathematics produced by the Secondary National Strategy (2010).

Jot down any other points you want to follow up in further study or raise with colleagues.

7 You could if you wish follow up this module by reading and using lesson A1 from the series *Improving learning in mathematics: Mostly algebra* (sessions A1–A14). These materials were originally published in 2005 by the Standards Unit at the then Department for Education and Skills (DfES). They can be downloaded from the Learning and Skills Improvement Service (LSIS) Excellence Gateway:

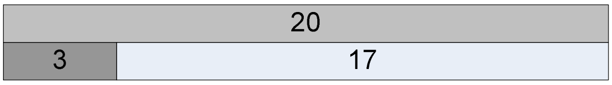
tlp.excellencegateway.org.uk/pdf/mat_imp_02.pdf

Resource 4a: Addition and subtraction

Part 1 Look at these arrangements of strips. For each set of strips:

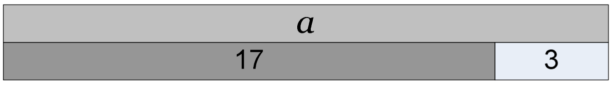
- What relationship can you write down?
  In how many different ways can you write this relationship?

1

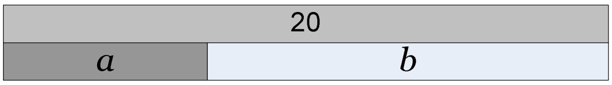


(Possible answers: 20 =17 + 3, 20 – 3 = 17, 20 – 17 = 3, 20 = 3 + 17 and the same equations with the left and right-hand sides reversed)

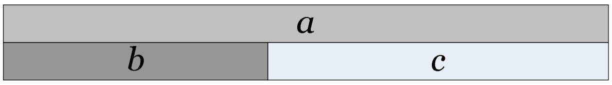
2



3



4



Substitute numbers for letters to see if you are correct.

- How would each of the images above be presented as steps on a number line?

Part 2 Use the structures above to write these number sentences or equations in as many ways as you can:

1 18 – 3 = 15

2 12 + a = 16

3 $q + t = 9$

4 $y - d$ = w

Substitute numbers for letters to check that you are correct.

Resource 4b: More complex + and – equations

Here are some ways to extend the 'mental computer screen' to more complex equations, again working first with numbers and then with letters.

Examples

Write the equation 3 + 4 = 5 + 2 **in as many ways as you can**. For example, • using the commutative law: 3 + 4 = 5 + 2 4 + 3 = 5 + 2 3 + 4 = 2 + 5, etc. • using inverse operations (bracket expressions to think of as a single number): (3 + 4) – 5 = 2 (3 + 4) – 2 = 5 3 = (5 + 2) – 4, etc.	Write the equation 3 + a = 5 + b **in as many ways as you can**: For example, • using the commutative law: 3 + a = 5 + b a + 3 = 5 + b 3 + a = b + 5, etc. • using inverse operations (bracket expressions to think of as a single number): (3 + a) – 5 = b (3 + a) – b = 5 3 = (5 + b) – a, etc.

More complex equations

Think of each of these equations as consisting of three terms.

Without removing any brackets, write them **in as many ways as you can**.

1 $7(s + 3) = 45 – 3(12 – s)$

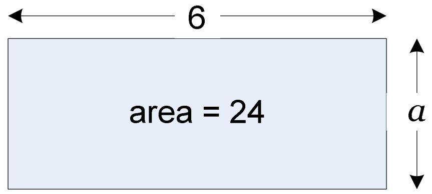
2 $3(2a – 1) = 5(4a – 1) – 4(3a – 2)$

3 $2(m – 0.3) – 3(m – 1.3) = 4(3m + 3.1)$

Substitute numbers for letters to see if you are correct.

Resource 4c: Multiplication and division

Part 1 This rectangle has an area of 24 square units.
Its length is 6 units and its width is a units.



- What relationship can you write down? What else can you write down?
  In how many different ways can you write this?

- What if the area was n square units, and the lengths were p units and q units?

- Substitute numbers for letters to check that you are correct.

Part 2 Now write each of these equations in **as many ways as you can.**

1 $6 \times 3 = 18$

2 $\frac{10}{2} = 5$

3 $2 \times \mathrm{a} = 17$

4 $\frac{8}{f} = 2$

5 $q \times t = 20$

6 $\frac{p}{g} = 6$

7 $j \times k = v$

8 $\frac{y}{d} = w$

Substitute numbers for letters to check that you are correct.

Resource 4d: More complex × and ÷ equations

Examples

Write the equation 3 × 4 = 2 × 6 **in as many ways as you can**.	Write the equation 3 × a = b × 6 **in as many ways as you can**.
For example, • using the commutative law: 3 × 4 = 2 × 6 4 × 3 = 2 × 6 3 × 4 = 6 × 2, etc. • using inverse operations (bracket expressions to think of as a single number): $\frac{4}{2} = \frac{6}{3}$ $3 \times \left(\frac{4}{2}\right) = 6$ $\frac{3 \times 4}{2} = 6$ or $\frac{1}{2}(3 \times 4) = 6$ $\frac{3 \times 4}{6} = 2$ or $\frac{1}{6}(3 \times 4) = 2$, etc.	For example, • using the commutative law: 3 × a = b × 6 a × 3 = b × 6 3 × a = 6 × b, etc. • using inverse operations (bracket expressions to think of as a single number): $\frac{4}{2} = \frac{6}{3}$ $3 \times \left(\frac{a}{b}\right) = 6$ $\frac{3 \times a}{b} = 6$ or $\frac{1}{b}(3 \times a) = 6$ $\frac{3 \times a}{6} = b$ or $\frac{1}{6}(3 \times a) = b$, etc.

Try other examples, working towards generality, e.g. $ab = cd$ or $\frac{a}{c} = \frac{b}{d}$.

More complex equations

Without expanding any brackets, write each of these equations **as many ways as you can**:

1 $2(p + 5) = 24$

2 $4(n+3) = 6(n - 1)$

3 $\frac{12}{(x+1)} = \frac{21}{(x+4)}$

Substitute numbers for letters to check that you are correct.

