Module 6: Applying skills in the context of 'Pyramids'

Description This module is for an individual teacher or group of teachers in secondary schools who are considering their teaching of algebra. It explores how pupils can use and apply their algebraic skills in the context of 'Pyramid puzzles'. The work is developed in four stages: understanding the structure of a pyramid by using numbers, collecting like terms, constructing and solving equations intuitively and constructing and solving equations more formally.

Other modules which could be combined with this one, either to create a longer session, or to work through in a sequence over time are:

- Module 5: Collecting like terms
- Module 10: Classroom approaches to algebra

Study time About 60 minutes

Resources Each teacher will need a personal notepad.

Each teacher or pair of teachers working together will need:

- copies of **Resources 6a, 6b, 6c and 6d** at the end of this module;
- a copy of *Key processes in teaching algebra*, published by the National Secondary Strategy (2009), which you can download from:

  nationalstrategies.standards.dcsf.gov.uk/node/47335
- a copy of the algebra strand of the *Revised learning objectives for mathematics* for Key Stages 3 and 4 produced by the Secondary National Strategy (2010), which you can download from:

  nationalstrategies.standards.dcsf.gov.uk/secondary/framework/maths/fwsm/mlo

Applying skills in the context of 'Pyramids'

1 Pyramids are constructed by adding adjacent cells to obtain the cell above. The simple structure lends itself to developing number and algebra skills (see **Stage 1** of **Resource 6a, Pyramids 1)**. Quickly work through the three phases in Stage 1.

Pupils need to work through the same three phases, using their knowledge of number facts to fill in the missing boxes. The first example ensures that they understand the structure. In the second and third examples, they may use inverse operations or trial and improvement, or may even introduce a variable.

Some prompts that you can use to help pupils to explain their solutions and compare their approaches are:

- Do you always use the same approach?
- In this example, why is that method the most efficient method? Is it always the most efficient method?
- How can you check your solution?

2 In the second stage, pupils use their understanding of the structure to collect like terms as they complete pyramids with algebraic expressions. The examples in **Stage 2** of **Resource 6a** increase in difficulty to support pupils as they develop and modify their skills. Work quickly through these examples.

When pupils work on problems like these, aim to make connections between approaches in stages one and two, in particular, the use of inverse operations and working backwards to check solutions. Pupils may also substitute a consistent value for the variable in a pyramid to check the value of the expression in each box and that the structure of the pyramid still works.

Some prompts to guide pupils' thinking are:

- Can you check your answer by substituting a value in the variable?
- Explain how you work backwards in examples like this (e.g. phase 2.2).

3 In the third stage, pupils construct and solve equations using informal methods. Study the examples shown on **Resource 6b, Informal methods of solving equations** and jot down an advantage and limitation of each method.

Now go back to the examples in **Stage 3** of **Resource 6a**. Quickly work through them using a different informal method for each example. After you have done this, consider this question:

- For each of the three puzzles in Stage 3, which of the informal methods of solving linear equations would you recommend for pupils in your classes?

In the first example, the equation $2n + 7 = 17$ lends itself to 'matching' (although some pupils may prefer to use other informal methods, such as 'balancing' or 'trial and improvement'). The 'matching' method should be explored with further examples so that pupils can 'match' less obvious cases, e.g. $4n + 8 = 13$. It may be helpful for some of them to demonstrate this as: $4n + 8 = 5 + 8$.

The second and third examples introduce the possibility of 'matching' both number terms and letter terms. The third example is more difficult but the early thinking in 'matching' may lead them to solve examples like the third one intuitively.

Pupils should be involved in lots of discussion while they find the missing expressions and solutions. They should be encouraged to explain and compare how they approached each problem, and why. Again, they should check their results by substitution.

4 In the fourth stage, the examples become more complex and are likely to require more formal methods of solving equations. Work quickly through the examples in **Stage 4** of **Resource 6c, Pyramids 2**.

In examples like these in the classroom, aim to build on the 'matching' method to inform the 'balancing' method, leading to more formal equation solving.

Pyramids can be adapted to the needs of groups of pupils and individual pupils by using larger numbers, decimal or fraction coefficients, expressions with brackets, or indices leading, say, to a quadratic equation to solve.

Try the examples in the **Further developments** section of **Resource 6c**.

Now make up your own examples of a pair of pyramids requiring the solution of:

- a pair of simultaneous linear equations in two variables;
- a quadratic equation with integer solutions.

5 To round off, look at the document *Key processes in teaching algebra*, published by the Secondary Strategy (2009), which you can download from:

nationalstrategies.standards.dcsf.gov.uk/node/47335

Read through the document and tick those key processes which a 'Pyramids' task could help to address.

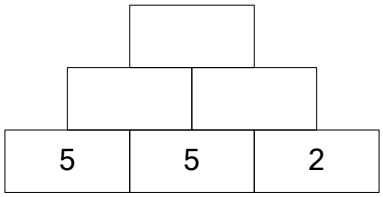
Then compare your selection with the processes listed on **Resource 6d, Developing process skills using 'Pyramids'**.

6 You could if you wish follow up this module by reading and using lesson A9 from the series *Improving learning in mathematics: Mostly algebra* (sessions A1–A14). These materials were originally published in 2005 by the Standards Unit at the then Department for Education and Skills (DfES). They can be downloaded from the Learning and Skills Improvement Service (LSIS) Excellence Gateway:

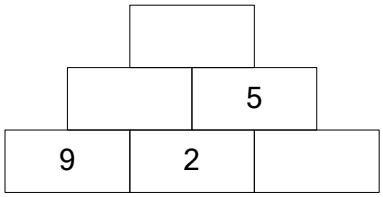
tlp.excellencegateway.org.uk/pdf/mat_imp_02.pdf

Resource 6a: Pyramids 1

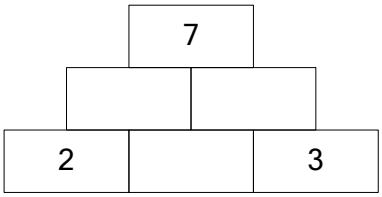
Stage 1: Understanding the structure of 'Pyramids' by using numbers



Phase 1.1

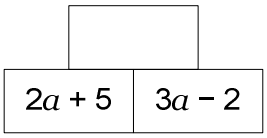


Phase 1.2

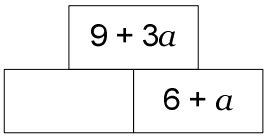


Phase 1.3

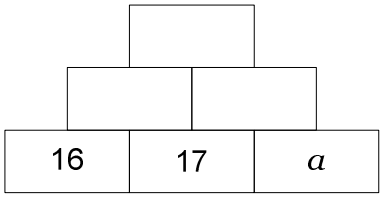
Stage 2: Collecting like terms



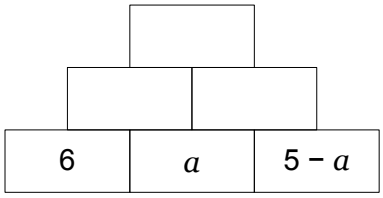
Phase 2.1



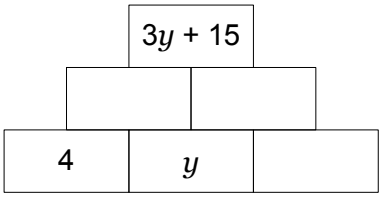
Phase 2.2



Phase 2.3

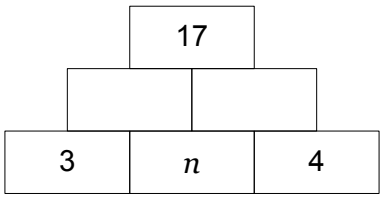


Phase 2.4

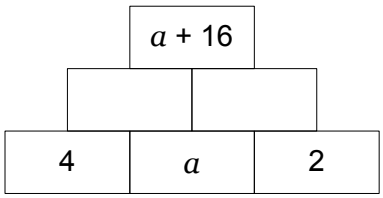


Phase 2.5

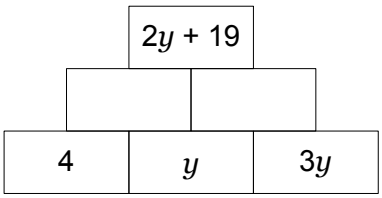
Stage 3: Constructing and solving equations using informal methods



Phase 3.1

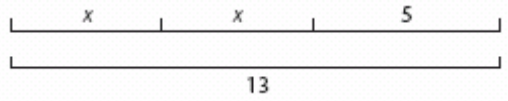
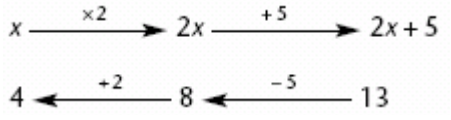


Phase 3.2



Phase 3.3

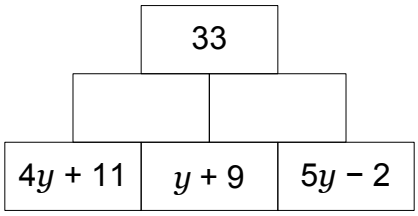
Resource 6b: Informal methods of solving equations

Informal method	Advantages and limitations
Trial and improvement $2x + 5 = 13$ Guess: x is 3 The left-hand side is too low. Guess: x is 4 $(2 \times 4 + 5 = 13)$ is correct.	
Matching method with a number line $2x + 5 = 13$  $2x + 5 = 13$ $2x + 5 = 8 + 5$ $2x = 8$ $x = 4$	
Balancing: doing same to both sides $2x + 5 = 13$ $2x + 5 = 13$ $2x + 5 - 5 = 13 - 5$ $2x = 8$ $2x \div 2 = 8 \div 2$ $x = 4$	
Write the equation in 'as many ways as you can' $2x + 5 = 13$ Using the commutative law and inverses, write the equation in as many ways as you can until you find an equation that you can solve in your head, e.g. $2x = 13 - 5$ $2x = 8$ So: $x = 4$	
Inverses $2x + 5 = 13$ 	

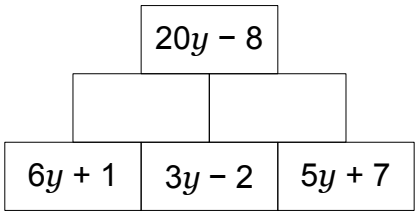
Resource 6c: Pyramids 2

Stage 4: Solving equations more formally

In each pyramid, find the value of y.

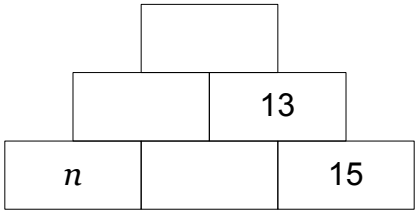


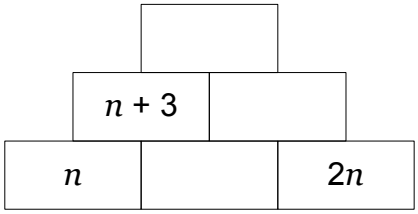
Phase 4.1



Phase 4.2

In these two pyramids, the values of the two top cells are equal.
Find the value of n.

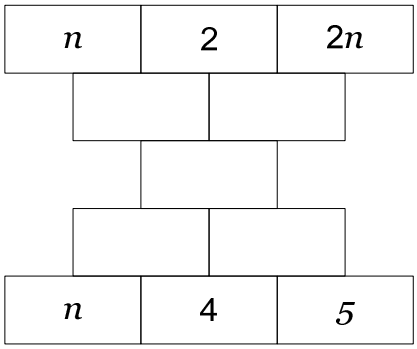




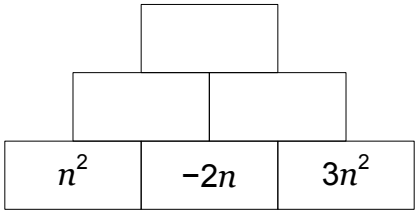
Phase 4.3

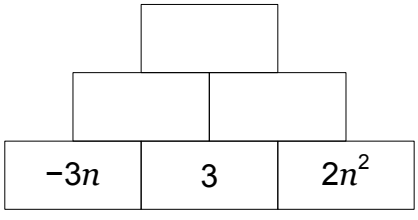
Further developments, e.g.

In this pyramid puzzle, find the value of n.



In these two pyramids, the values of the two top cells are equal.
Find the value of n.





Resource 6d: Developing process skills using 'Pyramids'

Representing:

- developing understanding of algebraic conventions, for example, conventions of writing terms;
- constructing algebraic expressions and equations.

Analysing-use mathematical reasoning (number and algebra):

- using different techniques to analyse a situation, evaluate an approach or check a calculation, for example, looking at simpler cases or working backwards;
- making connections with arithmetical operations and with equivalent algebraic forms when transforming expressions and equations.

Analysing-use appropriate mathematical procedures:

- developing increasing fluency with algebraic manipulation without being rule-bound and, when the steps in a procedure are not obvious, being able to resolve difficulties for themselves;
- generating equivalent expressions and equations, including a simplified form;
- substituting values into equations;
- solving equations exactly and approximately.

Interpreting and evaluating:

- considering and evaluating different approaches; for example, where another pupil has represented the problem or approached its solution in a different way.

Communicating and reflecting:

- making links to related problems or to different problems with a similar structure.