Module 8: Generalising from patterns and sequences

Description This module is for an individual teacher or group of teachers in secondary schools who are considering their teaching of algebra. It examines some ways of using patterns and sequences to help pupils to generalise.

Other activities which could be combined with this one, either to create a longer session, or to work through in a sequence over time are:

- Module 9: Linking sequences, functions and graphs
- Module 10: Classroom approaches to algebra

Study time About 40 minutes

Resources Each teacher will need a personal notepad.

Each teacher or pair of teachers working together will need:

- copies of **Resources 8a, 8b and 8c** at the end of this module;
- a copy of the algebra strand of the *Revised learning objectives for mathematics* for Key Stages 3 and 4 produced by the Secondary National Strategy (2010), which you can download from:

  nationalstrategies.standards.dcsf.gov.uk/secondary/framework/maths/fwsm/mlo

Generalising from patterns and sequences

1 You are probably familiar with the definitions of a sequence, a function and a graph. By the end of Key Stage 3, pupils need to be aware of and understand these definitions. Check your understanding against the definitions on **Resource 8a, Sequences, functions and graphs: definitions**.

2 Pupils have been developing their ideas about pattern in number throughout Key Stages 1 and 2. One aspect of this work relates to number properties and sequences, such as:

- one more than a multiple of 3;
- numbers based on spatial patterns such as square numbers and triangular numbers;
- Fibonacci numbers.

In the problems on **Resource 8b, Fibonacci chains**, the number sequences have similar properties to the Fibonacci sequence – that is, each term is the sum of the previous two terms. Solve the problems and make a few notes on a method that can be used to solve them.

3 While the answer to the first problem on Resource 8b is easy to spot, you probably used algebraic methods to solve the other problems.

All the sequences can be generalised in this form:

$a,\ \ b,\ \ a + b,\ \ a + 2b,\ \ 2a + 3b,\ \ 3a + 5b,\ \ 5a + 8b,\ \ \ldots,$

where a is the first term and b is the second term. This allows the following equations to be formulated and solved:

$6 + 3b = 18$	3	4	7	11	18	
$5.66 + 3b = 25.91$	2.83	6.75	9.58	16.33	25.91	
$12 + 5b = 36$	4	4.8	8.8	13.6	22.4	36
$30 + 8b = 4$	6	–3.25	2.75	–0.5	2.25	1.754

The four examples show that changing the first and last terms in sequences of this type alters the level in difficulty, so that an activity for pupils that is based on this task can readily be differentiated. For example, the first problem on Resource 8b would be suitable for pupils working confidently at level 4, whereas the other three problems are more suitable for pupils working at level 5 or level 6.

4 In the Fibonacci sequence problems, it was possible to express each of the sequences in a general form by using letters to stand for numbers.

One way of introducing pupils to algebraic generalisation is to ask them to extend number patterns. Answer the questions on **Resource 8c, Generalising**, which are typical of the problems that can be given to pupils working at level 5.

5 In the first problem on **Resource 8c**, you have probably described the nth line in the pattern as:

$(n - 1)(n + 1) = n^2 - 1$

or as:

$n(n + 2) = (n + 1)^2 - 1$

The pattern can be extended backwards to explore multiplication of negative numbers, since any integer, positive or negative, can be substituted for n.

$1 \times 3 = 2^2 - 1$
$0 \times 2 = 1^2 - 1$
$(-1) \times 1 = 0^2 - 1$
$(-2) \times 0 = ({}^{-}1)^2 - 1$
$(-3) \times (-1) = (-2)^2 - 1$

- What happens if fraction or decimal values are substituted for n? Is it still the case that $(n - 1)(n + 1) = n^2 - 1$?

An equation like $n^2 - 1 = (n - 1)(n + 1)$ that holds true for all possible values of the variables is called an identity.

The second problem on **Resource 8c** has a connection with the first problem, in that each of the four numbers 899, 3599, 10 403, 359 999 is 1 less than a perfect square. It can therefore be expressed in the form $n^2 - 1$, which factorises as $(n - 1)(n + 1)$. This helps to find the solutions:

$899 = 30^2 - 1 = 29 \times 31$
$3599 = 60^2 - 1 = 59 \times 61$
$10\,403 = 1022 - 1 = 101 \times 103$
$359\,999 = 600^2 - 1 = 599 \times 601$

- What happens with other values? Check that:

  $(5.816)^2 - 1 = 4.816 \times 6.816$

  and that:

  $(-15.216) \times (-13.216) + 1 = (-14.216)^2$

6 Giving pupils opportunities to explore generalisations like these helps them to develop an understanding of the power of algebra.

Now consider these questions.

- In which year groups do pupils in your school learn to generate sequences, use term-to-term and position-to-term rules, and find the nth term of a sequence?
- How does this compare with the revised mathematics learning objectives produced by the Secondary Strategy (2010)?
- What other opportunities to generalise do pupils have in their learning of number and algebra? In which year groups?
- Are there ways of extending these opportunities?

7 To round off, reflect on these two questions.

- What have you learned?
- What action will you take as a result?

Jot down any points you want to follow up and any modifications you will make to your planning or teaching.

If you are studying alone, jot down any points you want to discuss with your head of department or colleagues.

8 You could if you wish follow up this module by reading and using lesson A8 from the series *Improving learning in mathematics: Mostly algebra* (sessions A1–A14). These materials were originally published in 2005 by the Standards Unit at the then Department for Education and Skills (DfES). They can be downloaded from the Learning and Skills Improvement Service (LSIS) Excellence Gateway:

tlp.excellencegateway.org.uk/pdf/mat_imp_02.pdf

Resource 8a: Sequences, functions and graphs: definitions

Sequences

A **sequence** is an ordered succession of terms formed according to a rule. There can be a finite or infinite number of terms.

The sequences most commonly considered in mathematics in Key Stages 3 and 4 have:

- an identifiable mathematical relationship between the value of a term and its position in the sequence; and/or
- an identifiable mathematical rule for generating the next term in the sequence from one or more existing terms.

Examples
The squares of the integers: 1, 4, 9, 16, 25, …
The Fibonacci sequence: 1, 1, 2, 3, 5, 8, …

Functions

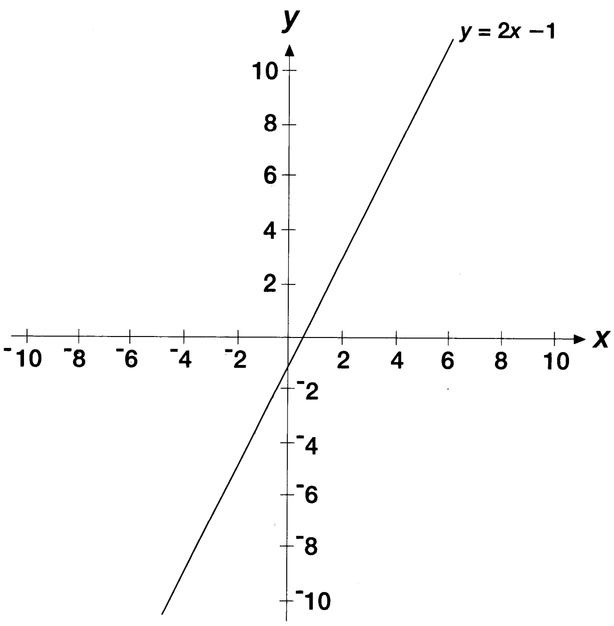
A **function** is a rule that associates each term of one set of numbers with a unique term in a second set. The relationship can be written in different ways.

Example
$x \to 2x - 1$
$y = 2x - 1$

Graphs

A **graph** of a function is a diagram that represents the relationship between two variables or sets of numbers.

Example



Resource 8b: Fibonacci chains

All these number chains have similar properties to the Fibonacci sequence – that is, each term is the sum of the previous two.

Find the missing terms.

3	…	…	…	18

2.83	…	…	…	25.91

4	…	…	…	…	36

6	…	…	…	…	…	4

Use this space for any working that you want to do.

Explain how to solve problems like these.

Resource 8c: Generalising

1 Consider this pattern:

$1 \times 3 = 2^2 - 1$

$2 \times 4 = 3^2 - 1$

$3 \times 5 = 4^2 - 1$

$4 \times 6 = 5^2 - 1$

a What will the next two lines be?

b What will the 10th line be?

c What will the 100th line be?

d If I wanted to know what a particular row will be, say the nth row, how could you tell me?

2 Find a pair of factors of:

a 899

b 3599

c 10 403

d 359 999

Make up two similar questions.