Module 9: Linking sequences, functions and graphs

Description

This module is for an individual teacher or group of teachers in secondary schools who are considering their teaching of algebra. It discusses some stimulating activities to help pupils to link sequences, functions and graphs.

Other activities which could be combined with this one, either to create a longer session, or to work through in a sequence over time are:

- Module 8: Generalising from sequences and patterns
- Module 10: Classroom approaches to algebra

Study time

About 40 minutes

Resources

Each teacher will need a personal notepad.

Each teacher or pair of teachers working together will need:

- copies of **Resources 9a, 9b and 9c** at the end of this module;
- a copy of the algebra strand of the *Revised learning objectives for mathematics* for Key Stages 3 and 4 produced by the Secondary National Strategy (2010), which you can download from:

  nationalstrategies.standards.dcsf.gov.uk/secondary/framework/maths/fwsm/mlo

Linking sequences, functions and graphs

1 Connections between sequences, functions and graphs are sometimes not given enough emphasis mathematics lessons.

Pupils are often introduced to functions through 'number machines' or 'function machines'. In this first example, the rule is given, along with some input numbers. Pupils soon learn to work backwards intuitively from the output numbers.

in $\longrightarrow$ $\boxed{\text{multiply by 4 then add 1}}$ $\longrightarrow$ out

in	1	7	200	2.5			$^{-}2$
out					121	2	

In Year 7 pupils are expected to begin to use algebraic notation:

$n \rightarrow 4n + 1$ or $y = 4x + 1$

The mapping can be separated out into two single operation machines:

$x \longrightarrow \boxed{\times 4} \longrightarrow \boxed{+ 1} \longrightarrow y$

By Year 8, pupils should be able to transform this using inverse operations:

$x \leftarrow \boxed{\div 4} \leftarrow \boxed{-1} \leftarrow y$ or: $x = \frac{y-1}{4}$

This links closely to the second form of machine, which is of the ‘What went in?’ type.

in $\rightarrow$ $\boxed{\text{multiply by 2 then add 3}}$ $\rightarrow$ out

in							
out	7	25	3	55	863	3.1	2

The function for this machine is $y = 2x + 3$ and its inverse is $x = \frac{1}{2}(y - 3)$.

Both types of machine, once introduced, can provide useful number practice in an oral and mental activity. The examples show how operations on different numbers can be targeted for practice, and so questions can be matched to the stage of development of individual pupils.

A third type of number machine has input and corresponding output numbers and the function has to be found.

in $\rightarrow$ $\boxed{}$ $\rightarrow$ out

in	3	4	6	10	1.7		$^{-}5$
out	9	13	21	37		0	

In this machine, the function is $y = 4x - 3$ and $x = \frac{1}{4}(y + 3)$.

- What number is unchanged by this machine? Or, to put this question another way, for what number are the input and output numbers the same? What are the ‘stay the same’ numbers for the first two types of number machines?

A possible investigation for pupils is to explore ‘stay the same’ numbers for different number machines.

2 Another context in which pupils will meet mappings such as these is in work on number sequences. Work on number sequences will have started in Key Stage 2. Look at the three questions from the Key Stage 2 national tests on **Resource 9a, Key Stage 2 test questions**.

Now look at the number sequence on **Resource 9b, Squares in a cross**, and answer the accompanying questions.

3 The function related to *Squares in a cross* is represented by the equation $y = 4x + 1$.

The large numbers in the table are beyond pupils’ capacity to draw and count. This encourages them to move from particular cases to the general. Pupils should first be asked to find the rule in words, to provide a step towards the algebra, and then to suggest a way of checking on its accuracy.

Two levels of generalisation emerge from these types of spatial patterns. For pupils, it is easier to spot links between successive terms, for example ‘It goes up in fours’, than to relate a term to its position in the sequence.

It helps pupils if you encourage them to justify and explain why a rule or relationship works in the context of the situation, relating back to the diagrams and not just to the pattern of numbers. One way of doing this is to ask them to calculate particular terms in the sequence. For example, in *Squares in a cross*, the 10th cross needs $10 \times 4 + 1$, or 41 squares, and the 100th cross needs $100 \times 4 + 1 = 401$ squares. From the particular examples, they are able to see that, in general, $y = 4x + 1$.

4 With number sequences based on spatial patterns like these, the values of the variables are whole-number values only. In a true algebraic relationship, the variables can take any values on a continuous scale. Graphs of number patterns should really be a set of separate points, but in order to look at the algebraic relationship we usually join the points as though they represent a continuous function.

Look at **Resource 9b, Graphs of linear functions**. One of the graphs represents the function $y = 4x + 1$. Identify which graph it is and then find the equations of the other graphs.

5 Pupils in Year 7 are expected to draw graphs such as those on **Resource 9c**. In Year 8, pupils are introduced to ideas of gradient and intercept.

Compare your answers to the questions on **Resource 9c** with those below.

A $x + y = 4$
B $y = 4x + 1$
C $y = 4x - 3$
D $y = \frac{1}{2}x + 3$
E $y = 2x + 3$

The graph for which the point of intersection with each line would be the 'stay the same' number for that function is the line $y = x$.

Linking elements of algebra together is part of the algebraic reasoning that needs to be developed throughout Key Stage 3. Pupils need to gain insight into the power and purpose of algebra as well as learning algebraic techniques.

6 Understanding the links between sequences, functions and graphs is a cornerstone of mathematics in Key Stages 3 and 4. Teaching has to help pupils to appreciate that algebra allows them to represent and explore general relationships and that this is more powerful than looking only at specific cases.

Pupils need opportunities to use their algebraic skills in problem solving in order to:

- increase their awareness of when and how algebra can be useful;
- improve their knowledge of algebraic conventions;
- deepen their understanding of algebraic rules;
- practise their use of algebraic techniques;

but most importantly they need opportunities to

- see how algebra can provide insights into the underlying situation that the algebra is modelling.

7 Now think about these questions, or discuss them with colleagues in your group.

- What opportunities do you currently provide for linking work on sequences, functions and graphs in mathematics lessons? Give some examples.

- What other opportunities could you provide for linking work on sequences, functions and graphs? You may wish to refer to the revised objectives for mathematics in Years 7 to 11 produced by the Secondary Strategy (2010).

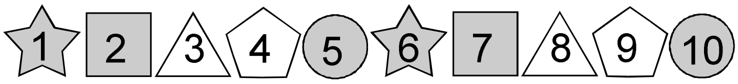
Note these examples on your note pad, together with points that you need to follow up and/or discuss with colleagues.

8 You could if you wish follow up this module by reading and using lesson A7 from the series *Improving learning in mathematics: Mostly algebra* (sessions A1–A14). These materials were originally published in 2005 by the Standards Unit at the then Department for Education and Skills (DfES). They can be downloaded from the Learning and Skills Improvement Service (LSIS) Excellence Gateway:

tlp.excellencegateway.org.uk/pdf/mat_imp_02.pdf

Resource 9a: Key Stage 2 test questions

1 Here is a repeating pattern of shapes.
Each shape is numbered.

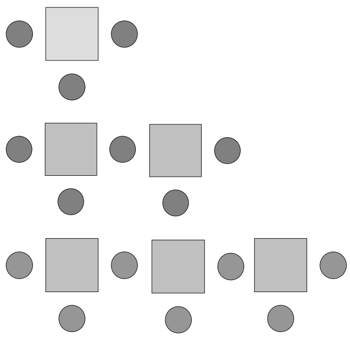


The pattern continues in the same way. Write the numbers of the next two stars in the pattern.

Complete this sentence.
Shape number 35 will be a circle because ...

KS2 2003 Paper A level 4

2 Here is a sequence of patterns made from squares and circles.

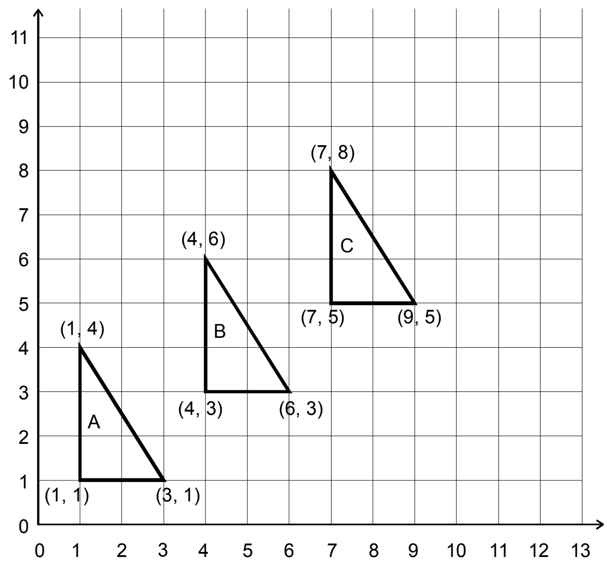


Number of squares	Number of circles
1	3
2	5
3	7

The sequence continues in the same way.
Calculate how many squares there will be in the pattern which has 25 circles.

KS2 2001 Paper A level 5

3

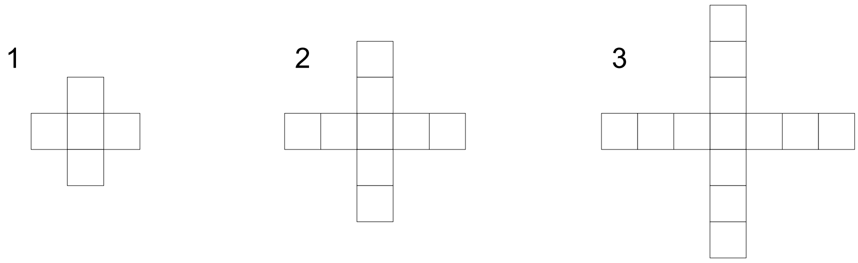


Write the co-ordinates of the next triangle in the sequence.

Y4 optional test 1999 Paper B level 4

Resource 9b: Squares in a cross

Fill in the missing values in the table by studying the patterns.



Number of cross	x	1	2	3	4	5	10	50		200
Number of squares	y								1001	

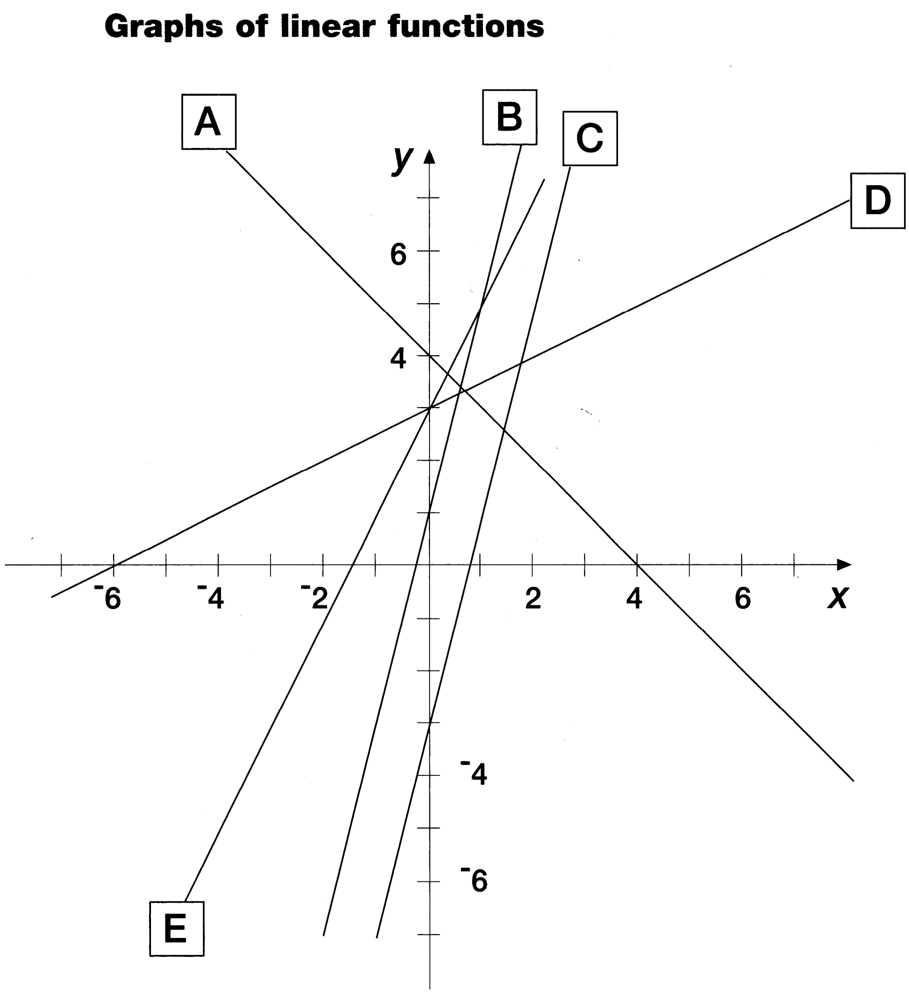
What is the rule which links the number of squares in any cross (y) to its position in the sequence (x)?

Explain why this is the rule by referring to the properties of the shapes.

Resource 9c: Graphs of linear functions

Which line represents the function $y = 4x + 1$?

A B C D E



Find the equations of the other lines.

Line	Equation
A	
B	
C	
D	
E	

What other graph would you need to draw so that the point of intersection with each line would be the 'stay the same' number for that function?

