Build-up scripts (dynamic geometry software versions)

Contents

Build-up 1: To prove that vertically opposite angles are equal

This version of the build-up uses dynamic geometry with diagrams A and B.

Show	**Say**	**Note on paper or board as the layers are built up**
Diagram A on screen. Rotate the arm, using the blue control ray so that it moves from 0° to 180° degrees. Rotate partway.	We know that angles on a straight line add to 180°. I am giving you that fact. How far have we gone? How much further to go?	Given: Angles on a straight line add to 180°.
Rotate a different amount.	If we know how far we have rotated, how can we work out how far we need to go? Explain, without using numbers, as an example. If angle CXB is *x* what expression will give the angle AXB?	On a straight line, if one angle is *x* the other is 180° – *x* (known as the supplement of *x*).
Diagram B on screen.	Here are two lines labelled AB and CD which intersect at the point X.	
Rotate either line, using the control rays. Rotate slowly a few times to show different angles.	The lines can rotate like this. Can you see four angles formed at the point of intersection? What do you notice about how they change as the lines rotate?	
Point to various pairs of adjacent angles.	The angles are in pairs. Some are next to each other. We call these adjacent angles.	
Point to a pair of adjacent angles and clearly indicate on the diagram the straight line with which they are associated. Drag a pair of coloured markers to identify a pair. Replace markers.	What do we know about these pairs of adjacent angles?	Same given: Pairs of adjacent angles on a straight line add to 180°.
Point to the two pairs of vertically opposite angles. Drag similarly coloured markers to identify a pair.	Some pairs of angles are opposite each other. We call these vertically opposite angles (maybe not the most obvious term, it comes from *vertex*, meaning corner, and the angles are at facing corners).	
Mark one pair of vertically opposite angles with letters *p* and *q* and another pair with letters *a* and *b*.	What can we say about the comparative size of these angles? (Encourage use of 'acute' and 'obtuse' in describing the angles.) Can we be rigorous in our argument?	Want to prove: Vertically opposite angles are equal. Want to prove: $p = q$ and $a = b$
Note: Some pupils may argue this through using a dynamic approach. 'As you rotate it, this angle gets bigger and the vertically opposite one gets bigger by the same amount,' (easily demonstrated). This is good and will help some pupils gain an understanding of the angle relationships. With some persuasion you may also get the following argument.		

Point out: • the pair of angles on one side of line AB • the pair of angles on one side of CD • the angle common to both lines.	We know that each of these pairs of angles adds up to 180°. The vertically opposite angles each pair up with the common angle to make 180°. This must mean that the vertically opposite angles are the same size.	Using algebra: $a + p = 180°$ So $a = 180° - p$ $p + b = 180°$ so $b = 180° - p$ Therefore $a = b$	(Adjacent angles on a straight line add to 180°.)
Perhaps show other pairs of angles that could be used.	Will this argument work for all intersecting lines? Remember when we rotated them: did it matter where we stopped?	Proved: Vertically opposite angles are equal.	

Build-up 2: To establish the definition of corresponding angles

This build-up uses dynamic geometry with diagram C.

Show	**Say**	**Note on paper or board as the layers are built up**
Diagram C on screen. Pupils might use mini-whiteboards to sketch their own constructions.	The blue line CD intersects the red line AB at X. Similarly the green line EF intersects the red line AB at Y.	
Place coloured markers on a pair of corresponding angles, e.g. angle AXD and angle AYF.	Can you see the left-hand top angle at each of the points of intersection? We say that these are in the same position as each other, relative to the intersecting lines.	
Pupils point to or mark pairs of angles.	Which other angles are in the same position as each other, relative to the intersecting lines?	
Use coloured markers to identify pairs of corresponding angles at the points of intersection.	We have been finding pairs of corresponding angles. Use the coloured markers to state pairs of corresponding angles.	By pairs of corresponding angles we mean angles in the same position as each other, relative to the intersecting lines.
Remove all except one pair of coloured markers.	Do you think that pairs of corresponding angles are the same size?	
Slide Y along the transversal (line AB) until it coincides with X. Point to the pair of corresponding angles.	We can see that these corresponding angles are not the same size.	
	Are the other pairs of corresponding angles equal? How would we have to change the diagram to make the corresponding angles equal?	

Build-up 3: To establish that pairs of corresponding angles on parallel lines are equal

This build-up uses dynamic geometry with diagram C.

Show	**Say**	**Note on paper or board as the layers are built up**
Diagram C, with the line MN separate from the rest of the construction (not used in this construction). Move CD and EF towards the centre of AB, but not overlapping and clearly not parallel. Pupils may sketch on mini-whiteboards.	Do you remember our discussion about corresponding angles? Here, we have two line segments, CD and EF, which both cross AB. Can you spot any corresponding angles?	Use the circle markers to indicate the corresponding angles identified.
	Notice that the corresponding angles are not equal in this case. What would we have to do to make them equal?	Follow pupils' suggestions, moving the control rays to make CD and EF parallel, and corresponding angles in the lower diagram equal.
Rotate the blue ray so that it lies on top of the green ray. (This should result in EF and CD being parallel.)	When the corresponding angles are equal, the lines are parallel (as you could have guessed from the position of the control rays).	
Rotate EF and CD apart again. Move first one control ray to a different position, then the other one to the same new orientation. Slide EF and CD together to confirm that the segments are parallel and corresponding angles are equal.	There is nothing special about the angle we chose here. If we change the angle of one of the lines – and then the other, in the same way – then the lines will still be parallel, and the corresponding angles will still be equal.	Establish: Pairs of corresponding angles on parallel lines are equal.
Mark a pair of equal corresponding angles with coloured markers.	To recap: we know that these angles are corresponding, because they are in the same positions relative to line segment AB. Since we know that CD is parallel to EF, we also know that the angles are equal.	
Point out that the same diagram (with CD and EF separate and not overlapping) contains eight angles. Set the challenge given (see right) and then select pupils to give and explain their answers. Formalise the reasoning if necessary – see examples below.	Keeping CD and EF parallel, can you identify pairs of angles that are equal? Convince yourself, and then explain your reasoning to a partner.	We are using coloured markers to show equal angles. Explain the standard conventions of single and double arcs, or letter symbols (also available on the diagrams).
Draw attention to one point of intersection first. Mark equal angles with coloured markers or letters.	Here we have two pairs of vertically opposite angles. We need to say why we are marking angles with the same colour or with the same letter. We will put the reason in brackets.	One pair marked with pink markers or letters *p*, the other pair marked with blue, or letters *q*. (Vertically opposite angles are equal.)
Now look at the second point of intersection. Mark or label equal angles.	We can use the same pair of symbols or letters to mark up this second point of intersection. Here we have pairs of angles that correspond to those already marked.	Again, one pair marked with pink markers, the other pair marked with blue, or use the same letters. (Corresponding angles on parallel lines are equal.)

Build-up 4: To establish the definition of alternate angles

This build-up uses dynamic geometry with diagram C.

Show	**Say**	**Note on paper or board as the construction proceeds**
Diagram C on screen; segment MN is not needed for this construction. Rotate the control rays so that CD is parallel to EF. If pupils are not working at their own machines, ask them to make a pencil sketch of the diagram or use mini-whiteboards. Now ask them to identify in the main diagram the parallel lines and one pair of corresponding angles.	We know that pairs of corresponding angles on parallel lines are equal. I am giving you that fact.	Given: Pairs of corresponding angles on parallel lines are equal.
Pupils point to and describe in full the position of the pair of corresponding angles. Pupils indicate (but do not mark) other pairs of corresponding angles.	Do you remember our discussion about corresponding angles? We gave them their name rather than having to make a lengthy description of their position.	DXY and FYB marked equal. (Corresponding angles on parallel lines are equal.)
Mark with coloured markers or letters angles CXY and XYF.	Now look at this pair of angles. How could we describe their relative positions?	Angle CXY is alternate to angle XYF.
	They are on different or alternate sides of the transversal. Do you remember this is a pair of alternate angles?	
	Can we say anything about the comparative size of the alternate angles we have marked? They look equal – but we must be precise in our reasoning…	Want to prove: Pairs of alternate angles on parallel lines are equal. Want to prove: $\angle$CXY is equal to $\angle$XYF.
Point at angles DXY and CXY. Label them *p* and *q*.	What do we know about this pair of angles?	Given: Angles on straight line add to 180°. $\angle CXY + \angle DXY = 180°$ $p + q = 180°$

Build-up 5: To prove that pairs of alternate angles on parallel lines are equal

This build-up uses dynamic geometry with diagram C.

Show	Say	Note on paper or board as the construction proceeds
Diagram C on screen; segment MN is not needed for this construction. Rotate the control rays so that CD is parallel to EF. If pupils are not working at their own machines, ask them to make a pencil sketch of the diagram or use mini-whiteboards. Now ask them to identify the parallel lines in the main diagram, and one pair of corresponding angles.	We know that pairs of corresponding angles on parallel lines are equal. I am giving you that fact.	Given: Pairs of corresponding angles on parallel lines are equal.
Pupils point to and describe in full the position of the pair of corresponding angles. Pupils indicate (but do not mark) other pairs of corresponding angles.	Do you remember our discussion about corresponding angles? We gave them their name rather than have to make a lengthy description of their position.	∠DXY and ∠FYB marked equal. (Corresponding angles on parallel lines are equal.)
Mark with coloured markers or letters angles CXY and XYF.	Now look at this pair of angles. How could we describe their relative positions?	∠CXY is alternate to ∠XYF.
	They are on different or alternate sides of the transversal. Do you remember that this is a pair of alternate angles?	
	Can we say anything about the comparative size of the alternate angles we have marked? They look equal – but we must be precise in our reasoning…	Want to prove: Pairs of alternate angles on parallel lines are equal. Want to prove: ∠CXY is equal to ∠XYF.
Point at angles DXY and CXY. Label them *p* and *q*.	What do we know about this pair of angles?	Given: Angles on straight line add to 180°. ∠CXY + ∠DXY = 180° $p + q = 180°$
	What other pairs of angles must add up to 180°?	∠XYF + ∠FYB = 180° (Adjacent angles on a straight line add to 180º.) (Could also be labelled *q* and *p* as ∠FYZ corresponds to ∠DXY.)
	What can we deduce from these two statements? Angle DXB is equal to angle FYB, so angle CXB must be equal to angle AYF.	∠CXY = ∠XYF

Ask pupils to talk through a similar chain of reasoning for other pairs of alternate angles. Encourage them to label angles with the same letter when they can prove them to be equal.		
Pupils could sketch different sets of parallel lines and transversals.	Remember that we could slide parallel lines apart or together and change the angle of the transversal. Would our argument still be true?	Proved: Pairs of alternate angles on parallel lines are equal.

Build-up 6: To prove that the angles in a triangle add to 180°

This build-up uses dynamic geometry with diagram C.

Show	**Say**	**Note on paper or board as the layers are built up**
Diagram C with line segment MN positioned so that both CD and EF intersect MN. Arrange CD with a positive gradient and EF with a negative gradient, but not forming a triangle. New intersection points V and W will appear on the line MN.	These line segments, AB and MN, are parallel. If I change the angle of AB – like this (use red control vector) – MN moves so that it remains parallel. Or I could move AB and MN apart – but they remain parallel.	AB is parallel to MN.
Mark acute alternate angles VXB and MVX with coloured blue markers or letters *p*.	We know that pairs of alternate angles on parallel lines are equal. I am giving you that fact.	Given: Pairs of alternate angles on parallel lines are equal.
Move the parallel lines apart and relabel the pair of equal alternate angles (always choosing the pair of acute angles).	Do you remember the discussion about alternate angles? We could pull the parallel lines further apart or squash them closer and still have a pair of equal alternate angles.	
Now mark the acute alternate angles NWY and XYW with coloured pink markers or letters *q*.	We can do the same for this line; however we move EF, these angles will remain equal.	
Slide CD or EF so that a triangle is formed. (Note there are two possible ways of doing this: either W and V or X and Y will coincide.) Replace the markers or labels showing pairs of equal alternate angles, as they will not move on the screen.	Can you see how a triangle is formed?	
Point to the three interior angles. Mark remaining angle with a brown marker or letter *r*.	We want to know what we can say about the sum of the angles inside the triangle using the facts we have just built up.	Want to prove: Angles in a triangle add to 180°. Want to prove: The pink, blue and brown angles (or *p*, *q* and *r*) add to 180°.
Point to the three marked angles on the straight line. Make sure that pupils are clear which straight line is being used.	There is one more fact here that we have been given on a previous occasion. What is it?	Given: Angles on a straight line add to 180°.
	So, we can show that the three interior angles of the triangle add to make a straight line; and we already know that the angles on a straight line have a total of 180°.	Proved: The angles in a triangle add to 180°. $p + q + r = 180°$
Change the orientation of the line segments, and invite pupils to work through the chain of reasoning to show that the interior angles of a triangle add to 180°.		

Build-up 7: To prove that the angles in a triangle add to 180°

This build-up uses dynamic geometry with diagram C.

Show	Say	Note on paper or board as the layers are built up
Diagram C with line segment MN positioned so that both CD and EF intersect MN. Arrange CD with a positive gradient and EF with a negative gradient, but not forming a triangle. New intersection points V and W will appear on MN.	We have already looked at one proof that the angles of a triangle add up to 180°. Now we are going to look at an alternative proof. Once again, we start with a pair of parallel lines, and two traversals.	AB is parallel to MN.
Mark acute alternate angles VXB and MVX with coloured blue markers or the letters *p*.	As before, we start with a pair of alternate angles on parallel lines that we know are equal. I am giving you that fact.	Given: Pairs of alternate angles on parallel lines are equal.
Move the parallel lines apart and relabel the pair of equal alternate angles (always choosing the pair of acute angles).	Remember that we could pull the parallel lines further apart or squash them closer and still have a pair of equal alternate angles.	
Now mark the acute corresponding angles FWV and WYX with coloured pink markers or letters *q*.	Previously, we labelled a pair of equal alternate angles on this line segment. This time we will label a pair of equal corresponding angles.	Given: Pairs of corresponding angles on parallel lines are equal.
Slide CD or EF so that a triangle is formed. (Note there are two possible ways of doing this: either W and V or X and Y will coincide.) Replace the markers or labels showing pairs of equal corresponding angles.	Now, just like last time, we will slide this segment over to form a triangle. Let's mark the corresponding angles on EF.	
Point to the three interior angles. Mark the remaining angle with a brown marker or letter *r*.	We want to know what we can say about the sum of the angles inside the triangle using the facts we have just built up.	Want to prove: The angles in a triangle add to 180°. Want to prove: The pink, blue and brown marked angles add to 180°.
Point to the three marked angles on the straight line FWE (or FVE). Make sure that pupils are clear which straight line is being used.	Just like last time, there is one more fact here that we have been given on a previous occasion. What is it?	Given: Angles on a straight line add to 180°. $p + q + r = 180°$
	So, we can show that the three interior angles of the triangle add to make a straight line; and we already know that the angles on a straight line have a total of 180°.	Proved: The angles in a triangle add to 180°.
Change the orientation of the line segments, and invite pupils to work through the chain of reasoning to show that the interior angles of a triangle add to 180°.		