Build-up scripts (acetate overlay versions)

Contents

Build-up 1: To prove that vertically opposite angles are equal
Build-up 2: To establish the definition of corresponding angles
Build-up 3: To establish that pairs of corresponding angles on parallel lines are equal
Build-up 4: To establish the definition of alternate angles
Build-up 5: To prove that pairs of alternate angles on parallel lines are equal
Build-up 6: To prove that the angles in a triangle add to 180°
Build-up 7: To prove that the angles in a triangle add to 180°

Build-up 1: To prove that vertically opposite angles are equal

This build-up uses figures 1 (two copies) and 2.

Show	Say	Note on paper or board as the layers are built up
Fig. 1 on screen. Mark a point on the line and indicate the 180° angle by tracing out the 180° arc with a finger or pointer.	We know that angles on a straight line add to 180°. I am giving you that fact.	Given: Angles on a straight line add to 180°.
Place a new fig. 1 on top of existing fig. 1 so that the lines coincide.	Here is another straight line. I can put it directly on top of the original.	
Rotate top fig. 1 about the point marked. Rotate a few times to show different angles.	I can rotate it like this. We now have a point of intersection. Can you see four angles formed here? Can you see them changing as I rotate the lines?	
Point to various pairs of adjacent angles.	The angles are in pairs. Some are next to each other. We call these adjacent angles.	
Point to a pair of adjacent angles and indicate the position of the straight line by running a finger or pointer along it.	What do we know about these pairs of adjacent angles?	Same given: Pairs of adjacent angles on a straight line add to 180°.
Point to the two pairs of vertically opposite angles.	Some pairs of angles are opposite each other. We call these vertically opposite angles (maybe not the most obvious term, it comes from *vertex*, meaning corner, and the angles are at facing corners).	
Replace with fig. 2 as a freeze-frame. Mark one pair of vertically opposite angles with letters p and q and another pair with letters a and b.	What can we say about the comparative size of these angles? Can we be rigorous in our argument?	Want to prove: Vertically opposite angles are equal. Want to prove: $p = q$ and $a = b$
Note: Some pupils may argue this through using a dynamic approach. ‘As you rotate it, this angle gets bigger and the vertically opposite one gets bigger by the same amount,’ (easily demonstrated). This is good and will help some pupils gain an understanding of the angle relationships. With some persuasion you may also get the following argument.		

Position fig. 1 on top of fig.2, along each straight line as the argument is made. Point out the angle each straight line has in common (in this case angle marked *p*).	We know *a* and *p* make 180°. We know *p* and *b* make 180°. That must mean that *a* and *b* are the same size.	$a + p = 180°$ $p + b = 180°$ $p = q$	(Adjacent angles on a straight line add to 180°.)
Position fig. 1 on top of fig.2, along each straight line as the argument is made. Point out the angle each straight line has in common (in this case angle marked *b*).	Repeat the logic for the other pair (p and q).	$p + b = 180°$ $b + q = 180°$ $p = q$	(Adjacent angles on straight line add to 180°.)
Perhaps use two copies of fig. 1 again.	Will this argument work for all intersecting lines? Remember when we rotated them: did it matter where we stopped?	Proved: Vertically opposite angles are equal.	

Build-up 2: To establish the definition of corresponding angles

This build-up uses figures 2, 3 and 7.

Show	**Say**	**Note on paper or board as the layers are built up**
Fig. 2 on screen.	Here are two intersecting lines.	
Fig. 3 on screen.	Here is a different pair of intersecting lines.	
Slide fig. 2 and fig. 3 so that one line is produced (as shown in fig. 7).	We can slide these so that one of the straight lines is continuous through both points of intersection.	
Trace a finger or pointer along the transversal (the long line in fig. 7).	Can you see the join? This could have started as one straight line crossing the other pair of lines. (We would call this a transversal.)	
Point to or dot the appropriate angles.	Can you see the right-hand top angle at each of the points of intersection? We say that these are in the same position as each other, relative to the intersecting lines.	
Pupils point to or mark pairs of angles.	Which other angles are in the same position as each other, relative to the intersecting lines?	
Fig. 7 on screen. Mark angles at each point of intersection with letters (*a*, *b*, *c*, *d* and *p*, *q*, *r*, *s*).	We have been finding pairs of corresponding angles. Use the letters to state pairs of corresponding angles.	By pairs of corresponding angles we mean angles in the same position as each other, relative to the intersecting lines.
Fig. 2 and fig. 3 replaced on screen with just one pair of corresponding angles marked.	Do you think that pairs of corresponding angles are the same size?	
Translate fig. 3 along the transversal to sit on top of fig. 2. Point to the pair of corresponding angles.	We can see that these corresponding angles are not the same size.	
Fig. 7 on screen with letters still marking all eight angles.	Who can remind me of another pair of corresponding angles and convince me that they are not equal?	

Build-up 3: To establish that pairs of corresponding angles on parallel lines are equal

This build-up uses figures 2 (two copies), 3, 7 and 10.

Note: This process does not prove that corresponding angles on parallel lines are equal. Partway through we effectively have to establish the meaning of parallel lines. It is important nonetheless to convince pupils of their equality, used as a given in our system.

Show	**Say**	**Note on paper or board as the layers are built up**	
Fig. 7 on screen.	Do you remember our discussion about corresponding angles?		
Fig. 2 and fig. 3 on screen.	Do you remember how we generated this final diagram? Under what circumstances would we have a diagram with all pairs of corresponding angles equal?		
Pupils may suggest: Replace fig. 3 by a duplicate fig. 2 on top of the first fig. 2 to show identical angles.	Why do the figures have to be identical?		
Overlay fig. 2 with a second copy. Slide the top fig. 2 so that one line is produced (as shown in fig. 10).	We can slide these apart so that one of the straight lines is continuous through both points of intersection.		
Show the two lines. Indicate the drawing of just one transversal.	Can you see the join? This could have started as a pair of lines crossed by one transversal.		
Indicate different pairs of lines at a variety of different angles, using a finger or a pointer. Slide fig. 2 back on top of itself to show what is special about this case.	Could we draw the pair of lines anyhow? … What is special about the way we draw this pair of lines?		
Slide top fig. 2 back out so that one line is produced (as shown in fig. 10). Mark arrows to indicate parallel lines. Identify the transversal.	The lines as built up here produce a pair of parallel lines.	Given: Parallel lines are established here.	
Slide figures back together and apart again to confirm this. Show, with the same letter, a pair of identical angles.	We say that this is a pair of corresponding angles because they are in the same relative position. Now we also know that they are equal because they are on a pair of parallel lines.	Establish: Pairs of corresponding angles on parallel lines are equal.	
Replace with fig. 10 as a freeze-frame. Ask a few pupils to mark up the diagram and talk through their chain of reasoning for the equality of some pairs of angles.	There are eight angles at these two points of intersection. How many different sizes of angle are there? The simplest way of recording this is to use the same symbol for angles known to be equal.	Convention: When angles are equal we mark them with the same symbol.	
Draw attention to one point of intersection first. Mark equal angles (letters *g* and *h*).	Here we have two pairs of vertically opposite angles. We need to say why we are marking angles with the same letter. We usually put the reason in brackets.	One pair each marked as *g*, the other pair each marked as *h*.	(Vertically opposite angles are equal.)

Now look at the second point of intersection. Mark equal angles (letters *g* and *h*).	We can use the same pair of symbols to mark up this second point of intersection. Here we have pairs of angles which correspond to those marked *g* and *h*.	One pair each marked as *g*, the other pair each marked as *h*.	(Corresponding angles on parallel lines are equal.)

Build-up 4: To establish the definition of alternate angles

This build-up uses figures 4, 6, 7 and 11.

Show	Say	Note on paper or board as the layers are built up
Fig. 4 on screen.	Here is an angle formed by two lines.	
Fig. 6 on screen.	Here is another angle formed by two lines.	
Slide fig. 4 and fig. 6 so that one common line is formed and a zigzag is produced (as shown in fig. 11).	We can slide these together to form a zigzag.	
Indicate the zigzag shape, using a finger or a pointer.	Can you see the join? This could have started as one diagram.	
Point to or dot the appropriate angles.	Can you see the acute angles contained in the zigzag?	
Indicate the transversal, using a finger or a pointer, and indicate the position of the two alternate angles.	We can describe these angles as being formed by the top line and the transversal, and by the bottom line and the transversal. But they are on alternate sides of the transversal. We call these alternate angles.	By pairs of alternate angles we mean angles formed on either side of a transversal crossing two straight lines and contained between the two straight lines.
Fig. 7 on screen.	Can you pick a zigzag out of this diagram? Mark the alternate angles.	
Fig. 4 and fig. 6 replaced on screen with just the pair of alternate angles marked.	Do you think that pairs of alternate angles are the same size?	
Rotate fig. 6 to sit on top of fig. 4. Point to the pair of marked angles.	We can see that these angles are not the same size.	
Fig. 7 on screen. Overlay fig. 4 and fig. 6.	Who can remind me why this is labelled as a pair of alternate angles and convince me that they are not equal?	

Build-up 5: To prove that pairs of alternate angles on parallel lines are equal

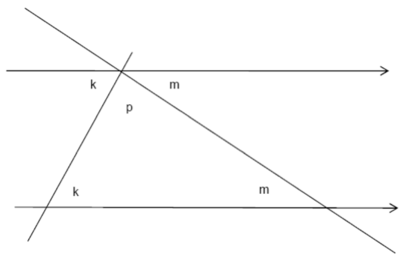
This build-up uses figures 1 and 10.

Note: An alternative proof could be developed using 'vertically opposite angles' in the sequence in place of 'angles on a straight line'.

Show	**Say**	**Note on paper or board as the layers are built up**
Fig. 10 on screen. Pupils mark arrows to show the parallel lines and one pair of corresponding angles, using the same letter.	We know that pairs of corresponding angles on parallel lines are equal. I am giving you that fact.	Given: Pairs of corresponding angles on parallel lines are equal
Pupils point to and describe in full the position of one pair of corresponding angles. Pupils indicate (but do not mark) other pairs of corresponding angles.	Do you remember our discussion about corresponding angles? We gave them their name rather than have to make a lengthy description of their position.	One pair marked as g. (Corresponding angles on parallel lines are equal.)
Mark two alternate angles as x and y (choose angles that are different from the marked corresponding angles).	Now look at this pair of angles. How could we describe their relative positions?	Angle marked x is alternate to angle marked y.
Pupils indicate (but do not mark) another pair of alternate angles.	They are on different or alternate sides of the transversal. Do you remember this is a pair of alternate angles?	
	Can we say anything about the comparative sizes of the alternate angles we have marked? … We must be precise in our reasoning…	Want to prove: Pairs of alternate angles on parallel lines are equal. Want to prove: $x = y$
Position fig. 1 over one of the straight lines formed by two of the angles marked.	What fact am I giving you?	Given: Angles on a straight line add to 180°.
Position fig. 1 in turn on each of the straight lines formed by two of the angles marked.	What does this show you about each pair of angles?	$g + x = 180°$ $g + y = 180°$ (Adjacent angles on a straight line add to 180º.)
	What can we deduce from these two statements? Angle marked g is common to both equations so the values of x and y must be the same.	$x = y$
Remove all overlays. Show a clean version of fig. 10. Ask a few pupils to label the diagram and talk through their chain of reasoning for each pair of alternate angles.		
Pupils could sketch different sets of parallel lines and transversals.	Remember that we could slide parallel lines apart or together and change the angle of the transversal. Would our argument still be true?	Proved: Pairs of alternate angles on parallel lines are equal.

Build-up 6: To prove that the angles in a triangle add to 180°

This build-up uses figures 1, 5, 8 and 9.

Show	**Say**	**Note on paper or board as the layers are built up**	
Fig. 5 on screen. Mark parallel lines with arrows.	We know that pairs of alternate angles on parallel lines are equal. I am giving you that fact.	Given: Pairs of alternate angles on parallel lines are equal.	
Label the pair of equal alternate angles *k* (choose the pair of acute angles). You might want to use your hands to indicate the parallel lines moving closer together or further apart. 	Do you remember the discussion about alternate angles? We could pull the parallel lines further apart or squash them closer and still have a pair of equal alternate angles.	One pair marked as *k*.	(Alternate angles on parallel lines are equal.)
Fig. 8 on top of fig. 5 but not forming triangle. Mark parallel lines with arrows.	This is the same given fact but the transversal has a different slope and is in the opposite direction.		
Label new pair of equal alternate angles on fig. 8, using a different letter (*m*) to first pair on fig. 5 (choose the pair of acute angles).	Let's recap the facts so far… We have two pairs of parallel lines that are drawn at exactly the same distance apart. We have one pair of alternate angles, already marked, that are equal to each other. We now have a second (different) pair to mark that are equal to each other.	One pair marked as *m*.	(Alternate angles on parallel lines are equal.)
Slide fig. 8 so that a triangle is formed. (Note there are two possible ways of doing this.)	Can you see how a triangle is formed?		
Point to the three interior angles. Mark the remaining angle as *p*.	We want to know what we can say about the sum of the angles inside the triangle using the facts we have just built up.	Want to prove: The angles in a triangle add to 180°. Want to prove: $k + m + p = 180°$	
Point to the three marked angles on the straight line. Overlay fig. 1 to make clear which straight line is being used.	There is one more fact here that we have been given on a previous occasion. What is it?	Given: Angles on a straight line add to 180°.	
Point to the three angles on the line (*k*, *p* and *m*). Point to the pairs of alternate angles to convince that the same three	Could someone use the two facts we have been given to prove that the angles in a	$k + m + p =$ 180°	(Angles on a straight line add to 180°.)

angles appear as the sum of the interior angles.	triangle must add up to 180°?	Same angle marked *k* in triangle. Same angle marked *m* in triangle.	(Alternate angles on parallel lines are equal.)
		Proved: The angles in a triangle add to 180°.	
Replace with fig. 9 as a freeze-frame. A few pupils mark up and point to diagram and talk through their chain of reasoning.			

Build-up 7: To prove that the angles in a triangle add to 180°

This build-up uses figures 1, 5, 8 and 9.

Show	**Say**	**Note on paper or board as the layers are built up**	
Fig. 5 on screen. Mark parallel lines with arrows.	We know that pairs of alternate angles on parallel lines are equal. I am giving you that fact.	Given: Pairs of alternate angles on parallel lines are equal.	
Using your hands, indicate pulling the parallel lines apart and then squashing them together. Label equal alternate angles as *x* (choose the acute pair).	Do you remember the discussion about alternate angles? We could pull the parallel lines further apart or squash them closer and still have an equal pair of alternate angles.	One pair marked as *x*.	(Alternate angles on parallel lines are equal.)
Fig. 8 on top of fig. 5 but not forming triangle. Mark parallel lines with arrows.	We know that pairs of corresponding angles on parallel lines are equal. I am giving you that fact.	Given: Pairs of corresponding angles on parallel lines are equal.	
On fig. 8 label equal pairs of corresponding angles as *y* (choose the pair of acute angles on the left of the transversal).	Let's recap the facts so far… We have two pairs of parallel lines that are drawn at exactly the same distance apart. We have one pair of alternate angles, already marked, that are equal to each other. We now have a pair of corresponding angles to mark that are equal to each other.	One pair marked as *y*.	(Corresponding angles on parallel lines are equal.)
Slide fig. 8 so that a triangle is formed.	Can you see how a triangle is formed?		
Point to the three interior angles. Mark the remaining angle as *z*.	We want to know what we can say about the sum of the angles inside the triangle, using the facts we have just built up.	Want to prove: The angles in a triangle add to 180°. Want to prove: $x + y + z = 180°$	
Point to the three marked angles on the straight line. Overlay fig. 1 to make clear which straight line is being used.	There is one more fact here that we have been given on a previous occasion. What is it?	Given: Angles on a straight line add to 180°.	
Point to the three angles on the line (*x*, *y* and *z*).	Could someone use the two facts we have been given to prove that the angles in a triangle must add up to 180°?	$x + y + z = 180°$	(Angles on a straight line add to 180°.)
Point to the pair of alternate angles and the pair of corresponding angles to convince that the same three angles appear as the sum of the interior angles.		Same angle marked *x* in triangle. Same angle marked *y* in triangle.	(Alternate angles on parallel lines are equal) (Corresponding angles on parallel lines are equal)
Replace with fig. 9 as a freeze-frame. A few pupils point to diagram and talk through their chain of reasoning.		Proved: The angles in a triangle add to 180°.	

