Geometrical reasoning study units: Problems

Set 1: Problems based on diagrams presented to pupils

Contents

Introduction

This is the first of three sections that present a series of geometrical reasoning problems for classroom use. The problems presented here all start from a given diagram. (The following sections present problems in which pupils are expected to construct their own diagrams, and problems that require a more extended or open-ended approach.)

Throughout these sections, the key idea is to help pupils generate solutions to geometrical problems in a more rigorous and better-articulated way. The activities involving visualisations and build-ups in the study units are designed to support this process of formalising pupils' geometrical reasoning, and the same approaches should be used here. Obtaining the correct answers to the problems is important but not sufficient; pupils need to structure, justify and communicate the reasoning leading to the solutions.

The materials consist of:

- Allowed assumptions
- Problems with diagrams
- Appendix: A solution to question 1.

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Allowed assumptions

The following is a suggested way of organising the work of pairs of pupils. The instructions below should be presented on the board or provided on paper to each pair of pupils.

In the following problems you are expected to explain each step of your reasoning.

You are allowed to take the following facts as given.

1 Angles on a straight line add to 180°.

2 Vertically opposite angles are equal.

3 Pairs of corresponding angles on parallel lines are equal.

4 Pairs of alternate angles on parallel lines are equal.

5 The interior angles of a triangle add to 180°.

6 The interior angles of an n-sided polygon add to $(n - 2) \times 180°$.

7 Angles opposite the equal sides of an isosceles triangle are equal.

8 In a triangle with two equal angles, the sides opposite the equal angles are equal and the triangle is isosceles.

Nothing else must be assumed!

Work in pairs.

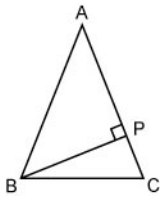
- **Solve the problem** on your own if you can. You may need to draw a diagram and label some points and angles.
- **Compare notes**: take it in turns to explain your thinking to each other – be ready to act as a 'doubter' and question your partner so that they make everything clear.
- **Agree on a solution**, talk through the steps and carefully write out the proof, making sure that a reason is given for each step. Read and, if necessary, redraft your explanation.
- **Present the alternative**, if the two of you have taken different steps to reach the solution.

Problems with diagrams

1 In the isosceles triangle shown, AB = AC.

From B, a line BP has been drawn to meet the opposite side AC at right angles.

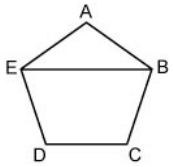
Prove that $\angle PBC = \frac{1}{2}\angle CAB$.



2 The diagram shows a regular pentagon ABCDE.

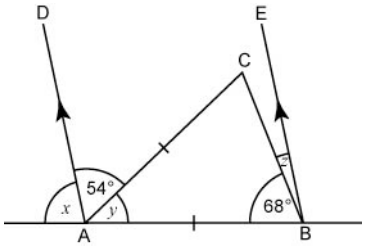
Calculate $\angle ABE$, explaining each step of your reasoning.

Suppose you started with a regular hexagon? Or a regular n-gon?



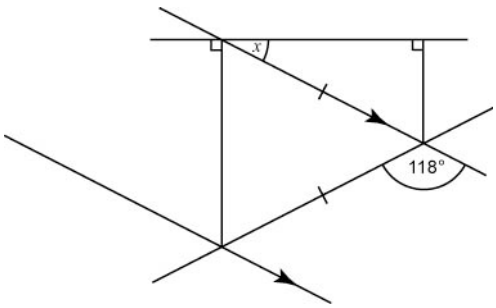
3 In the diagram, ABC is an isosceles triangle with AB = AC, and AD is parallel to BE.

Find the values of x, y and z.



4 Find the value of x in the diagram.

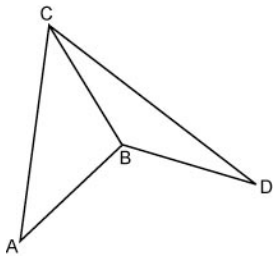
(Label the vertices and angles, to help you explain the steps in your reasoning.)



5 ABC is an isosceles triangle with AB = BC.

CBD is an isosceles triangle with BC = BD.

Prove that $\angle ABD = 2\angle ACD$.

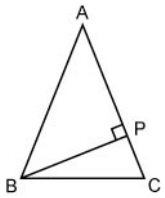


Appendix: A solution to question 1

1 In the isosceles triangle shown, AB = AC.

From B, a line BP has been drawn to meet the opposite side AC at right angles.

Prove that ∠PBC = $\frac{1}{2}$∠CAB.



Allowed assumptions

1	Angles on a straight line add to 180°.
2	Vertically opposite angles are equal.
3	Pairs of corresponding angles on parallel lines are equal.
4	Pairs of alternate angles on parallel lines are equal.
5	The interior angles of a triangle add to 180°.
6	The interior angles of an n-sided polygon add to (n – 2) × 180°.
7	Angles opposite the equal sides of an isosceles triangle are equal.
8	In a triangle with two equal angles, the sides opposite the equal angles are equal and the triangle is isosceles.

Solution

Say…		**Using assumption**
We want to show that ∠PBC = $\frac{1}{2}$∠CAB		
(i)	We know that ∠ABC = ∠BCA	8
(ii)	We know that ∠CAB + ∠ABC + ∠BCA = 180°	5
(iii)	Substituting (i) into (ii): ∠CAB + 2∠ABC = 180°	
(iv)	Dividing (iii) by 2: $\frac{1}{2}$∠CAB + ∠ABC = 90°	
(v)	∠PCB = ∠ACB ⇒ ∠PCB = ∠ABC	8
(vi)	Substituting (v) into (iv): $\frac{1}{2}$∠CAB + ∠PCB = 90°	
(vii)	∠PBC + ∠PCB + 90° = 180° ⇒ ∠PBC + ∠PCB = 90°	5
(viii)	Comparing (vi) and (vii), ∠PBC = $\frac{1}{2}$∠CAB	