Geometrical reasoning study units: Problems

Set 2: Problems requiring pupils to construct appropriate diagrams of their own

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Introduction

This is the second of three sections that present a series of geometrical reasoning problems for classroom use. The problems presented here all require pupils to construct an appropriate diagram of their own. (The problems in the previous section present problems were all based on a diagram presented to the pupils.)

Requiring pupils to draw their own diagram increases the level of challenge. Experience with the visualisation exercises described in the study units will be helpful; if pupils learn to form a mental image of the situation presented in the problem, it will be much easier to produce an appropriate diagram on paper.

Pupils will need support in building the skills required to tackle this kind of problem. For this reason, the questions include alternate wording, which presents a more structured approach to drawing the required diagram and solving the problem. You can think about the alternate wording as providing some cognitive ‘scaffolding’ for pupils learning to devise their own diagrams.

Problems for which a diagram must be drawn (with alternative wordings)

1 Prove that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices.

- Alternative wording
  Draw ∠ABC. Extend side AB beyond vertex B to point D.
  Prove that ∠CBD = ∠ACB + ∠CAB.

2 Prove that any straight line drawn parallel to the non-equal side of an isosceles triangle makes equal angles with the other sides.

- Alternative wording
  Draw an isosceles triangle ABC with AB = AC. Draw any straight line through the triangle parallel to the side BC.
  Prove that the straight line makes equal angles with AC and AB.

3 From any point on the bisector of an angle, a straight line is drawn parallel to one arm of the angle. Prove that the triangle thus formed is isosceles.

- Alternative wording
  Draw angle ABC and a line BD bisecting that angle. Draw a straight line DE so that DE is parallel to AB and meets BC at F.
  Prove that ∆BDF is isosceles.

4 From X, a point on the side BC of an isosceles triangle ABC (where AB = AC), a straight line is drawn at right angles to BC, cutting AB at Y and CA extended at Z. Prove that the triangle AYZ is isosceles.

- Alternative wording
  Draw an isosceles triangle ABC with AB = AC. Extend side CA beyond vertex A. Choose a point X on BC closer to B than C. Draw a straight line at X at right angles to BC, intersecting AB at Y and the extended side CA at Z.
  Prove that ∆AYZ is isosceles.

5 If the straight line that bisects an exterior angle of a triangle is parallel to the opposite side, prove that the triangle is isosceles.

- Alternative wording
  Draw a triangle ABC, extend the side AB beyond B and mark the exterior angle at B. Draw a straight line BD which bisects the exterior angle at B. You are given that the line BD is parallel to AC.
  Prove that ∆ABC is isosceles.

6 AB and CD are two straight lines intersecting at D, and the adjacent angles so formed are bisected. If through any point Y on DC a straight line XYZ is drawn parallel to AB and meeting the bisectors at X and Z, prove that XY is equal to YZ.

- Alternative wording
  Draw a line AB and mark a point D part way along AB. Draw any line DC. Draw a line DE so that it bisects .ADC. Draw a line DF so that it bisects .BDC. Draw a line parallel to AB and intersecting DE at X, DC at Y, DF at Z.
  Prove that XY = YZ.

7 If a straight line meets two parallel straight lines and the two interior angles on the same side are bisected, prove that the bisectors meet at right angles.

- Alternative wording
  Labelling lines from left to right, draw a line AB and draw a second line CD parallel to AB. Draw a transversal intersecting AB at E and CD at F. Draw a line which bisects ∠AEF and another line which bisects ∠CFE.
  Prove that these two lines intersect at right angles.

8 Prove that the angle formed by the intersection of the bisectors of two adjacent angles of a quadrilateral is equal to half the sum of the remaining two angles.

- Alternative wording
  Draw a quadrilateral ABCD (label the vertices in order). Draw the line AE so that ∠DAB is bisected. Draw the line DF so that ∠CDA is bisected. Label the point G where AE and DF intersect.
  Prove that ∠DGA = $\frac{1}{2}$(∠ABC + ∠BCD).

9 A is a vertex of an isosceles triangle ABC, in which AB = AC. BA is extended to D, so that AD is equal to BA. If DC is drawn, prove that ∠BCD is a right angle.

- Alternative wording
  Draw an isosceles triangle ABC with AB = BC. Extend side BA beyond vertex A to a point D so that BA = AD. Join D to C.
  Prove that ∠BCD = 90°.