Geometrical reasoning study units

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Visualisation scripts

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Visualisation 1: Two intersecting lines

Imagine a line…

- Make it horizontal...
- Imagine that it extends to infinity in both directions.

Imagine a vertical line, crossing (intersecting) the horizontal one and extending to infinity in both directions…

- Can you see four right angles?

Fix your mind on the point of intersection of the two lines… Keeping the horizontal line fixed, **very slowly** rotate the vertical line clockwise about this point.

As you slowly rotate the line, think to yourself:

- What happens to the right angles?

Continue rotating the line very slowly until it is at 45°, then stop…

- How many angles of 45° can you see?…
- What size are the other angles?

Now, keeping the sloping line fixed at 45°, **very slowly** move (translate) the horizontal line upwards, keeping it horizontal.

As you slowly move (translate) the line, what happens to the point of intersection?

- What happens to the angles at this point?…
- Do they change in size?

Now slowly move (translate) the horizontal line downwards, past its original position.

- What happens to the point of intersection?
- What happens to the angles at this point?…
- Do they change in size?

Visualisation 2: Two parallels and a transversal

Imagine two parallel lines of infinite length…

- Make them horizontal, not too far apart.
- Keep them fixed in this position.

Imagine a vertical line that crosses the parallel lines…

Think to yourself:

- How many points of intersection can I see?…
- Can you see eight right angles?
- Count them to make sure.

Now imagine a point on the vertical line, halfway between the parallel lines. Imagine pushing a pin into the vertical line so that it is fixed at this point… Now **very slowly** rotate the vertical line clockwise about the point fixed by the pin.

As you slowly rotate the line, think to yourself:

- What happens to the points where it crosses the parallel lines?
- What happens to the right angles?

Continue rotating **very slowly** until the line is sloping at 45°, then stop…

- How many angles of 45° can you see?...
- What size are the other angles?

Keeping the sloping line (transversal) fixed at 45°, make the parallel lines move **very slowly** apart, keeping them still horizontal and parallel.

As you slowly move (translate) the lines apart, think to yourself:

- What happens to the points of intersection?…
- What happens to the angles at these points – do they change in size?

Now push the parallel lines slowly towards each other…

- What happens to the points of intersection?…
- What happens to the angles at these points – do they change in size?

Pupils find it difficult to see shapes within sets of intersecting lines. They are sometimes reluctant to extend the lines forming a shape and this is often needed for a proof. The following two exercises encourage pupils to practise extending lines forming shapes while maintaining a fixed picture of the original shape, focusing on vertically opposite angles at points of intersection.

Visualisation 3: Extending the sides of a triangle

Imagine a triangle – make sure all the sides and all the angles are different.

Think to yourself:

- Which interior angle is the biggest?…
- Which is the smallest?

Check the vertices of the triangle. Pin these in place – **they must not move**. Fix your mind on one of the sides and allow the line to grow in both directions. Remember, the vertices stay still so the line extends beyond the vertices at each end.

Think to yourself:

- How would I describe what I can see now?

Fix your mind on another of the sides and allow the line to grow in both directions. Remember, the vertices stay still so the line extends beyond the vertices at each end.

Finally do the same to the last side. Allow the line to grow past the two vertices, remembering that these are pinned in position and do not move as the line grows.

Look at the three vertices of the triangle. The growing lines have produced something like an X shape at each vertex. Look at one of the vertices…

- Can you see four angles formed around this X shape?

Shade your triangle so that you can easily see the original shape.

Concentrate on the smallest interior angle of your triangle… Look at the intersection now formed here…

- Can you see another angle at the intersection that will be the same size as this smallest angle?

Visualisation 4: Extending the sides of a parallelogram

Imagine a parallelogram… Think to yourself:

- Roughly how big is each interior angle?…
- Which is the bigger pair of angles?…
- Which is the smaller?

Check the vertices of the parallelogram. Pin these in place – **they must not move**. Fix your mind on one of the sides and allow the line to grow in both directions. Remember, the vertices stay still so the line extends beyond the vertices at each end.

Think to yourself:

- How would I describe what I can see now?

Fix your mind on the side parallel to the line you have extended. Allow this side to grow in both directions. Remember, the vertices stay still so the line extends beyond the vertices at each end.

Do the same for the other pair of sides. Allow the lines to grow past the vertices, remembering that these are pinned in position and do not move as the lines grow.

Look at the vertices of the parallelogram. The growing lines have produced something like an X shape at each vertex. Look at one of the vertices…

- Can you see four angles formed around this X shape?

Shade your parallelogram so that you can easily see the original shape.

Concentrate on the smaller pair of interior angles… Look at one of the intersections now formed here…

- Can you see another angle at this intersection that will be the same size as this smallest angle?

The next two examples take the opposite approach to that of visualisations 3 and 4, starting with sets of lines and visualising the shapes within the intersections.

Visualisation 5: Three intersecting lines

Imagine three very long, straight lines. Let them wander freely, changing direction and position, crossing (intersecting) each other but remaining straight.

Now fix the lines so that they all cross (intersect) at the same point. How many angles can you see at this point? Count them to yourself.

Now arrange the lines so that they cross (intersect) in three different points.

- What shape do they enclose?

Concentrate on the enclosed shape.

- Roughly how big is each interior angle?

Concentrate on **one** of the points of intersection.

- How many angles surround this point?
- Are any of them equal?

Visualisation 6: Two pairs of parallel lines

Imagine a pair of parallel lines, infinitely long. Make them horizontal, not too far apart.

Imagine another pair of parallel lines; arrange them vertically, to cross the first pair at right angles…

- What shape is enclosed by the two pairs of lines?…
- How many right angles can you see? Count them.

Keeping the horizontal lines fixed as before, pull the vertical lines apart.

- Can you enclose a long, thin rectangle?

Now slowly push the vertical lines together… Stop when they enclose a square…

- What shape do you get if you push them even closer together?
- Still keeping the horizontal pair of lines fixed, can you rotate the other pair so that the lines enclose a parallelogram?

Now slowly push the sloping lines together, or pull them apart, until the lines enclose a rhombus.

- How many angles can you see? Count them.
- Are they all the same?

This last example might lead to a discussion about the congruence of shapes.

Visualisation 7: Halving a rectangle

Imagine a rectangle.

Now imagine one of the lines of symmetry of that rectangle. Ask yourself:

- How many times does the line intersect the rectangle?…
- How many right angles can I see?

Imagine pushing a pin right into the centre of the rectangle, so that the line of symmetry is fixed at the centre of the rectangle.

Now, keeping the rectangle in the same place, **very slowly** rotate the line anticlockwise about the point fixed by the pin. Notice this is no longer a line of symmetry…

Stop rotating your line. Imagine cutting the rectangle along the line and pulling the two bits of the rectangle apart.

- What can you say about the two new shapes?

As a follow-up, you may want to consider this alternative to class discussion.

- *Without drawing any diagrams, write down all the things you can now say about the two new shapes.*
- *Compare notes with a friend. Convince them of your ideas, using diagrams if you wish.*
- *If you had started with a rhombus, what would have changed? What would be the same?*