The Framework for secondary mathematics: overview and learning objectives

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Learning objectives

1 Mathematical processes and applications

Solve problems, explore and investigate in a range of contexts

Increase the **challenge** and build **progression** across the key stage, and for groups of pupils by:

- increasing the **complexity** of the application, e.g. non-routine, multi-step problems, extended enquiries
- reducing the **familiarity** of the context, e.g. new contexts in mathematics, contexts drawn from other subjects, other aspects of pupils' lives
- increasing the **technical demand** of the mathematics required, e.g. more advanced concepts, more difficult procedures
- increasing the degree of **independence** and autonomy in problem-solving and investigation

1.1 Representing

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• identify the necessary information to understand or simplify a context or problem; represent problems, making correct use of symbols, words, diagrams, tables and graphs; use appropriate procedures and tools, including ICT	• identify the mathematical features of a context or problem; try out and compare mathematical representations; select appropriate procedures and tools, including ICT	• break down substantial tasks to make them more manageable; represent problems and synthesise information in algebraic, geometrical or graphical form; move from one form to another to gain a different perspective on the problem	• compare and evaluate representations; explain the features selected and justify the choice of representation in relation to the context	• choose and combine representations from a range of perspectives; introduce and use a range of mathematical techniques, the most efficient for analysis and most effective for communication	• systematically model contexts or problems through precise and consistent use of symbols and representations, and sustain this throughout the work

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1.2 Analysing – use mathematical reasoning

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• classify and visualise properties and patterns; generalise in simple cases by working logically; draw simple conclusions and explain reasoning; understand the significance of a counter-example; take account of feedback and learn from mistakes	• visualise and manipulate dynamic images; conjecture and generalise; move between the general and the particular to test the logic of an argument; identify exceptional cases or counter-examples; make connections with related contexts	• use connections with related contexts to improve the analysis of a situation or problem; pose questions and make convincing arguments to justify generalisations or solutions; recognise the impact of constraints or assumptions	• identify a range of strategies and appreciate that more than one approach may be necessary; explore the effects of varying values and look for invariance and covariance in models and representations; examine and refine arguments, conclusions and generalisations; produce simple proofs	• make progress by exploring mathematical tasks, developing and following alternative approaches; examine and extend generalisations; support assumptions by clear argument and follow through a sustained chain of reasoning, including proof	• present rigorous and sustained arguments; reason inductively, deduce and prove; explain and justify assumptions and constraints

1.3 Analysing – use appropriate mathematical procedures

Within the appropriate range and content:

make accurate mathematical diagrams, graphs and constructions on paper and on screen; calculate accurately, selecting mental methods or calculating devices as appropriate; manipulate numbers, algebraic expressions and equations, and apply routine algorithms; use accurate notation, including correct syntax when using ICT; record methods, solutions and conclusions; estimate, approximate and check working

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1.4 Interpreting and evaluating

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• interpret information from a mathematical representation or context; relate findings to the original context; check the accuracy of the solution; explain and justify methods and conclusions; compare and evaluate approaches	• use logical argument to interpret the mathematics in a given context or to establish the truth of a statement; give accurate solutions appropriate to the context or problem; evaluate the efficiency of alternative strategies and approaches	• justify the mathematical features drawn from a context and the choice of approach; generate fuller solutions by presenting a concise, reasoned argument using symbols, diagrams, graphs and related explanations	• make sense of, and judge the value of, own findings and those presented by others; judge the strength of empirical evidence and distinguish between evidence and proof; justify generalisations, arguments or solutions	• show insight into the mathematical connections in the context or problem; critically examine strategies adopted and arguments presented; consider the assumptions in the model and recognise limitations in the accuracy of results and conclusions	• justify and explain solutions to problems involving an unfamiliar context or a number of features or variables; comment constructively on reasoning, logic, process, results and conclusions

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1.5 Communicating and reflecting

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• communicate own findings effectively, orally and in writing, and discuss and compare approaches and results with others; recognise equivalent approaches	• refine own findings and approaches on the basis of discussions with others; recognise efficiency in an approach; relate the current problem and structure to previous situations	• review and refine own findings and approaches on the basis of discussions with others; look for and reflect on other approaches and build on previous experience of similar situations and outcomes	• use a range of forms to communicate findings effectively to different audiences; review findings and look for equivalence to different problems with similar structure	• routinely review and refine findings and approaches; identify how other contexts were different from, or similar to, the current situation and explain how and why the same or different strategies were used	• use mathematical language and symbols effectively in presenting convincing conclusions or findings; critically reflect on own lines of enquiry when exploring; search for and appreciate more elegant forms of communicating approaches and solutions; consider the efficiency of alternative lines of enquiry or procedures

2 Number

2.1 Place value, ordering and rounding

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• understand and use decimal notation and place value; multiply and divide integers and decimals by 10, 100, 1000, and explain the effect • compare and order decimals in different contexts; know that when comparing measurements the units must be the same	• read and write positive integer powers of 10; multiply and divide integers and decimals by 0.1, 0.01 • order decimals	• extend knowledge of integer powers of 10; recognise the equivalence of 0.1, $\frac{1}{10}$ and 10^{-1}; multiply and divide by any integer power of 10	• express numbers in standard index form, both in conventional notation and on a calculator display • convert between ordinary and standard index form representations	• use standard index form to make sensible estimates for calculations involving multiplication and/or division	
• round positive whole numbers to the nearest 10, 100 or 1000, and decimals to the nearest whole number or one decimal place	• round positive numbers to any given power of 10; round decimals to the nearest whole number or to one or two decimal places	• use rounding to make estimates and to give solutions to problems to an appropriate degree of accuracy	• round to a given number of significant figures; use significant figures to approximate answers when multiplying or dividing large numbers	• understand how errors can be compounded in calculations	• understand upper and lower bounds

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2.2 Integers, powers and roots

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• understand negative numbers as positions on a number line; order, add and subtract integers in context • recognise and use multiples, factors, primes (less than 100), common factors, highest common factors and lowest common multiples in simple cases; use simple tests of divisibility • recognise the first few triangular numbers; recognise the squares of numbers to at least 12×12 and the corresponding roots	• add, subtract, multiply and divide integers • use multiples, factors, common factors, highest common factors, lowest common multiples and primes; find the prime factor decomposition of a number, e.g. $8000 = 2^6 \times 5^3$ • use squares, positive and negative square roots, cubes and cube roots, and index notation for small positive integer powers	• use the prime factor decomposition of a number • use ICT to estimate square roots and cube roots • use index notation for integer powers; know and use the index laws for multiplication and division of positive integer powers	• use index notation with negative and fractional powers, recognising that the index laws can be applied to these as well • know that $n^{\frac{1}{2}} = \sqrt{n}$ and $n^{\frac{1}{3}} = \sqrt[3]{n}$ for any positive number n	• use inverse operations, understanding that the inverse operation of raising a positive number to power n is raising the result of this operation to power $\frac{1}{n}$	• understand and use rational and irrational numbers

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2.3 Fractions, decimals, percentages, ratio and proportion

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• express a smaller whole number as a fraction of a larger one; simplify fractions by cancelling all common factors and identify equivalent fractions; convert terminating decimals to fractions, e.g. $0.23 = \frac{23}{100}$; use diagrams to compare two or more simple fractions	• recognise that a recurring decimal is a fraction; use division to convert a fraction to a decimal; order fractions by writing them with a common denominator or by converting them to decimals	• understand the equivalence of simple algebraic fractions; know that a recurring decimal is an exact fraction	• distinguish between fractions with denominators that have only prime factors 2 or 5 (terminating decimals), and other fractions (recurring decimals)	• use an algebraic method to convert a recurring decimal to a fraction	
• add and subtract simple fractions and those with common denominators; calculate simple fractions of quantities and measurements (whole-number answers); multiply a fraction by an integer	• add and subtract fractions by writing them with a common denominator; calculate fractions of quantities (fraction answers); multiply and divide an integer by a fraction	• use efficient methods to add, subtract, multiply and divide fractions, interpreting division as a multiplicative inverse; cancel common factors before multiplying or dividing	• understand and apply efficient methods to add, subtract, multiply and divide fractions, interpreting division as a multiplicative inverse		

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• understand percentage as the 'number of parts per 100'; calculate simple percentages and use percentages to compare simple proportions • recognise the equivalence of percentages, fractions and decimals • understand the relationship between ratio and proportion; use direct proportion in simple contexts; use ratio notation, simplify ratios and divide a quantity into two parts in a given ratio; solve simple problems involving ratio and proportion using informal strategies	• interpret percentage as the operator 'so many hundredths of' and express one given number as a percentage of another; calculate percentages and find the outcome of a given percentage increase or decrease • use the equivalence of fractions, decimals and percentages to compare proportions • apply understanding of the relationship between ratio and proportion; simplify ratios, including those expressed in different units, recognising links with fraction notation; divide a quantity into two or more parts in a given ratio; use the unitary method to solve simple problems involving ratio and direct proportion	• recognise when fractions or percentages are needed to compare proportions; solve problems involving percentage changes • use proportional reasoning to solve problems, choosing the correct numbers to take as 100%, or as a whole; compare two ratios; interpret and use ratio in a range of contexts	• understand and use proportionality and calculate the result of any proportional change using multiplicative methods • calculate an original amount when given the transformed amount after a percentage change; use calculators for reverse percentage calculations by doing an appropriate division	• calculate an unknown quantity from quantities that vary in direct proportion using algebraic methods where appropriate	• understand and use direct and inverse proportion; solve problems involving inverse proportion (including inverse squares) using algebraic methods

2.4 Number operations

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• understand and use the rules of arithmetic and inverse operations in the context of positive integers and decimals • use the order of operations, including brackets	• understand and use the rules of arithmetic and inverse operations in the context of integers and fractions • use the order of operations, including brackets, with more complex calculations	• understand the effects of multiplying and dividing by numbers between 0 and 1; consolidate use of the rules of arithmetic and inverse operations • understand the order of precedence of operations, including powers	• recognise and use reciprocals; understand 'reciprocal' as a multiplicative inverse; know that any number multiplied by its reciprocal is 1, and that zero has no reciprocal because division by zero is not defined	• use a multiplier raised to a power to represent and solve problems involving repeated proportional change, e.g. compound interest	

2.5 Mental calculation methods

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• recall number facts, including positive integer complements to 100 and multiplication facts to 10×10, and quickly derive associated division facts	• recall equivalent fractions, decimals and percentages; use known facts to derive unknown facts, including products involving numbers such as 0.7 and 6, and 0.03 and 8	• use known facts to derive unknown facts; extend mental methods of calculation, working with decimals, fractions, percentages, factors, powers and roots; solve problems mentally			• use surds and π in exact calculations, without a calculator; rationalise a denominator such as $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
• strengthen and extend mental methods of calculation to include decimals, fractions and percentages, accompanied where appropriate by suitable jottings; solve simple problems mentally	• strengthen and extend mental methods of calculation, working with decimals, fractions, percentages, squares and square roots, cubes and cube roots; solve problems mentally				
• make and justify estimates and approximations of calculations	• make and justify estimates and approximations of calculations	• make and justify estimates and approximations of calculations	• make and justify estimates and approximations of calculations by rounding numbers to one significant figure and multiplying or dividing mentally		

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2.6 Written calculation methods

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• use efficient written methods to add and subtract whole numbers and decimals with up to two places • multiply and divide three-digit by two-digit whole numbers; extend to multiplying and dividing decimals with one or two places by single-digit whole numbers	• use efficient written methods to add and subtract integers and decimals of any size, including numbers with differing numbers of decimal places • use efficient written methods for multiplication and division of integers and decimals, including by decimals such as 0.6 or 0.06; understand where to position the decimal point by considering equivalent calculations	• use efficient written methods to add and subtract integers and decimals of any size; multiply by decimals; divide by decimals by transforming to division by an integer			

2.7 Calculator methods

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• carry out calculations with more than one step using brackets and the memory; use the square root and sign change keys • enter numbers and interpret the display in different contexts (decimals, percentages, money, metric measures)	• carry out more difficult calculations effectively and efficiently using the function keys for sign change, powers, roots and fractions; use brackets and the memory • enter numbers and interpret the display in different contexts (extend to negative numbers, fractions, time)	• use a calculator efficiently and appropriately to perform complex calculations with numbers of any size, knowing not to round during intermediate steps of a calculation; use the constant, π and sign change keys; use the function keys for powers, roots and fractions; use brackets and the memory	• use an extended range of function keys, including the reciprocal and trigonometric functions • use standard index form, expressed in conventional notation and on a calculator display; know how to enter numbers in standard index form	• use calculators to explore exponential growth and decay, using a multiplier and the power key • calculate with standard index form, using a calculator as appropriate	• use calculators, or written methods, to calculate the upper and lower bounds of calculations in a range of contexts, particularly when working with measurements

2.8 Checking results

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• check results by considering whether they are of the right order of magnitude and by working problems backwards	• select from a range of checking methods, including estimating in context and using inverse operations	• check results using appropriate methods	• check results using appropriate methods	• check results using appropriate methods	• check results using appropriate methods

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3 Algebra

3.1 Equations, formulae, expressions and identities

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• use letter symbols to represent unknown numbers or variables; know the meanings of the words *term, expression* and *equation* • understand that algebraic operations follow the rules of arithmetic	• recognise that letter symbols play different roles in equations, formulae and functions; know the meanings of the words *formula* and *function* • understand that algebraic operations, including the use of brackets, follow the rules of arithmetic; use index notation for small positive integer powers	• distinguish the different roles played by letter symbols in equations, identities, formulae and function • use index notation for integer powers and simple instances of the index laws	• know and use the index laws in generalised form for multiplication and division of integer powers • square a linear expression; expand the product of two linear expressions of the form $x \pm n$ and simplify the corresponding quadratic expression; establish identities such as $a^2 - b^2 = (a + b)(a - b)$	• factorise quadratic expressions, including the difference of two squares, e.g. $x^2 - 9 = (x + 3)(x - 3)$ cancel common factors in rational expressions, e.g. $\frac{2(x+1)^2}{(x+1)}$ • simplify simple algebraic fractions to produce linear expressions; use factorisation to simplify compound algebraic fractions	

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• simplify linear algebraic expressions by collecting like terms; multiply a single term over a bracket (integer coefficients) • construct and solve simple linear equations with integer coefficients (unknown on one side only) using an appropriate method (e.g. inverse operations)	• simplify or transform linear expressions by collecting like terms; multiply a single term over a bracket • construct and solve linear equations with integer coefficients (unknown on either or both sides, without and with brackets) using appropriate methods (e.g. inverse operations, transforming both sides in same way) • use graphs and set up equations to solve simple problems involving direct proportion	• simplify or transform algebraic expressions by taking out single-term common factors; add simple algebraic fractions • construct and solve linear equations with integer coefficients (with and without brackets, negative signs anywhere in the equation, positive or negative solution) • use systematic trial and improvement methods and ICT tools to find approximate solutions to equations such as $x^2 + x = 20$	• solve linear equations in one unknown with integer and fractional coefficients; solve linear equations that require prior simplification of brackets, including those with negative signs anywhere in the equation • solve linear inequalities in one variable; represent the solution set on a number line • solve a pair of simultaneous linear equations by eliminating one variable; link a graph of an equation or a pair of equations to the algebraic solution; consider cases that have no solution or an infinite number of solutions	• solve equations involving algebraic fractions with compound expressions as the numerators and/or denominators • solve linear inequalities in one and two variables; find and represent the solution set • explore 'optimum' methods of solving simultaneous equations in different forms • solve quadratic equations by factorisation	• solve quadratic equations by factorisation, completing the square and using the quadratic formula, including those in which the coefficient of the quadratic term is greater than 1 • solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns, where one is linear in each unknown and the other is linear in one unknown and quadratic in the other or of the form $x^2 + y^2 = r^2$

		• use algebraic methods to solve problems involving direct proportion; relate algebraic solutions to graphs of the equations; use ICT as appropriate **Y9 extension objective** • *explore ways of constructing models of real-life situations by drawing graphs and constructing algebraic equations and inequalities*			
• use simple formulae from mathematics and other subjects; substitute positive integers into linear expressions and formulae and, in simple cases, derive a formula	• use formulae from mathematics and other subjects; substitute integers into simple formulae, including examples that lead to an equation to solve; substitute positive integers into expressions involving small powers, e.g. $3x^2 + 4$ or $2x^3$; derive simple formulae	• use formulae from mathematics and other subjects; substitute numbers into expressions and formulae; derive a formula and, in simple cases, change its subject	• derive and use more complex formulae; change the subject of a formula, including cases where a power of the subject appears in the question or solution, e.g. find r given that $A = \pi r^2$	• derive and use more complex formulae; change the subject of a formula, including cases where the subject occurs twice	• derive relationships between different formulae that produce equal or related results

3.2 Sequences, functions and graphs

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• describe integer sequences; generate terms of a simple sequence, given a rule (e.g. finding a term from the previous term, finding a term given its position in the sequence) • generate sequences from patterns or practical contexts and describe the general term in simple cases	• generate terms of a linear sequence using term-to-term and position-to-term rules, on paper and using a spreadsheet or graphics calculator • use linear expressions to describe the nth term of a simple arithmetic sequence, justifying its form by referring to the activity or practical context from which it was generated	• generate terms of a sequence using term-to-term and position-to-term rules, on paper and using ICT • generate sequences from practical contexts and write and justify an expression to describe the nth term of an arithmetic sequence	• find the next term and the nth term of quadratic sequences and explore their properties; deduce properties of the sequences of triangular and square numbers from spatial patterns		

• express simple functions in words, then using symbols; represent them in mappings	• express simple functions algebraically and represent them in mappings or on a spreadsheet	• find the inverse of a linear function • generate points and plot graphs of linear functions, where y is given implicitly in terms of x (e.g. $ay + bx = 0$, $y + bx + c = 0$), on paper and using ICT; find the gradient of lines given by equations of the form $y = mx + c$ given values for m and c	• plot the graph of the inverse of a linear function • understand that equations in the form $y = mx + c$ represent a straight line and that m is the gradient and c is the value of the y-intercept; investigate the gradients of parallel lines and lines perpendicular to these lines	• identify the equations of straight-line graphs that are parallel; find the gradient and equation of a straight-line graph that is perpendicular to a given line • plot graphs of more complex quadratic and cubic functions; estimate values at specific points, including maxima and minima	• know and understand that the intersection points of the graphs of a linear and quadratic function are the approximate solutions to the corresponding simultaneous equations • construct the graphs of simple loci, including the circle $x^2 + y^2 = r^2$; find graphically the intersection points of a given straight line with this circle and know this represents the solution to the corresponding two simultaneous equations

• generate coordinate pairs that satisfy a simple linear rule; plot the graphs of simple linear functions, where y is given explicitly in terms of x, on paper and using ICT; recognise straight-line graphs parallel to the x-axis or y-axis • plot and interpret the graphs of simple linear functions arising from real-life situations, e.g. conversion graphs	• generate points in all four quadrants and plot the graphs of linear functions, where y is given explicitly in terms of x, on paper and using ICT; recognise that equations of the form $y = mx + c$ correspond to straight-line graphs • construct linear functions arising from real-life problems and plot their corresponding graphs; discuss and interpret graphs arising from real situations, e.g. distance–time graphs	• construct functions arising from real-life problems and plot their corresponding graphs; interpret graphs arising from real situations, e.g. time series graphs	• explore simple properties of quadratic functions; plot graphs of simple quadratic and cubic functions, e.g. $y = x^2$, $y = 3x^2 + 4$, $y = x^3$ • understand that the point of intersection of two different lines in the same two variables that simultaneously describe a real situation is the solution to the simultaneous equations represented by the lines	• find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function • identify and sketch graphs of linear and simple quadratic and cubic functions; understand the effect on the graph of addition of (or multiplication by) a constant	• plot and recognise the characteristic shapes of graphs of simple cubic functions (e.g. $y = x^3$), reciprocal functions (e.g. $y = \frac{1}{x}$, $x \neq 0$), exponential functions ($y = k^x$ for integer values of x and simple positive values of k) and trigonometric functions, on paper and using ICT • apply to the graph $y = f(x)$ the transformations $y = f(x) + a$, $y = f(ax)$, $y = f(x + a)$ and $y = af(x)$ for linear, quadratic, sine and cosine functions

		Y9 extension objectives ● *Use ICT to explore the graphical representation of algebraic equations and interpret how properties of the graph are related to features of the equation, e.g. parallel and perpendicular lines* ● *interpret the meaning of various points and sections of straight-line graphs, including intercepts and intersection, e.g. solving simultaneous linear equations*			

4 Geometry and measures

4.1 Geometrical reasoning

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• use correctly the vocabulary, notation and labelling conventions for lines, angles and shapes • identify parallel and perpendicular lines; know the sum of angles at a point, on a straight line and in a triangle; recognise vertically opposite angles	• identify alternate angles and corresponding angles; understand a proof that: – the angle sum of a triangle is 180° and of a quadrilateral is 360° – the exterior angle of a triangle is equal to the sum of the two interior opposite angles	• distinguish between conventions, definitions and derived properties • explain how to find, calculate and use: – the sums of the interior and exterior angles of quadrilaterals, pentagons and hexagons – the interior and exterior angles of regular polygons	• distinguish between practical demonstration and proof in a geometrical context • solve multi-step problems using properties of angles, of parallel lines, and of triangles and other polygons, justifying inferences and explaining reasoning with diagrams and text	• show step-by-step deduction in solving more complex geometrical problems	• understand the necessary and sufficient conditions under which generalisations, inferences and solutions to geometrical problems remain valid

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• identify and use angle, side and symmetry properties of triangles and quadrilaterals; explore geometrical problems involving these properties, explaining reasoning orally, using step-by-step deduction supported by diagrams	• solve geometrical problems using side and angle properties of equilateral, isosceles and right-angled triangles and special quadrilaterals, explaining reasoning with diagrams and text; classify quadrilaterals by their geometrical properties	• know the definition of a circle and the names of its parts; explain why inscribed regular polygons can be constructed by equal divisions of a circle • solve problems using properties of angles, of parallel and intersecting lines, and of triangles and other polygons, justifying inferences and explaining reasoning with diagrams and text	• know that the tangent at any point on a circle is perpendicular to the radius at that point; explain why the perpendicular from the centre to the chord bisects the chord	• prove and use the facts that: – the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference – the angle subtended at the circumference by a semicircle is a right angle – angles in the same segment are equal – opposite angles in a cyclic quadrilateral sum to 180°	• prove and use the alternate segment theorem

	• know that if two 2-D shapes are congruent, corresponding sides and angles are equal	• understand congruence and explore similarity	• know that if two 2-D shapes are similar, corresponding angles are equal and corresponding sides are in the same ratio; understand from this that any two circles and any two squares are mathematically similar while in general any two rectangles are not		• prove the congruence of triangles and verify standard ruler and compass constructions using formal arguments
		Y9 extension objective • *investigate Pythagoras' theorem, using a variety of media, through its historical and cultural roots including 'picture' proofs*			

• use 2-D representations to visualise 3-D shapes and deduce some of their properties	• visualise 3-D shapes from their nets; use geometric properties of cuboids and shapes made from cuboids; use simple plans and elevations	• visualise and use 2-D representations of 3-D objects; analyse 3-D shapes through 2-D projections, including plans and elevations	• understand and apply Pythagoras' theorem when solving problems in 2-D and simple problems in 3-D • understand and use trigonometric relationships in right-angled triangles, and use these to solve problems, including those involving bearings	• understand and use Pythagoras' theorem to solve 3-D problems • use trigonometric relationships in right-angled triangles to solve 3-D problems, including finding the angles between a line and a plane	• calculate the area of a triangle using the formula $\frac{1}{2}ab\sin C$ • draw, sketch and describe the graphs of trigonometric functions for angles of any size, including transformations involving scalings in either or both of the x and y directions • use the sine and cosine rules to solve 2-D and 3-D problems

4.2 Transformations and coordinates

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• understand and use the language and notation associated with reflections, translations and rotations	• identify all the symmetries of 2-D shapes	• identify reflection symmetry in 3-D shapes			
• recognise and visualise the symmetries of a 2-D shape • transform 2-D shapes by: – reflecting in given mirror lines – rotating about a given point – translating	• transform 2-D shapes by rotation, reflection and translation, on paper and using ICT	• recognise that translations, rotations and reflections preserve length and angle, and map objects on to congruent images	• transform 2-D shapes by combinations of translations, rotations and reflections, on paper and using ICT; use congruence to show that translations, rotations and reflections preserve length and angle		

• explore these transformations and symmetries using ICT	• try out mathematical representations of simple combinations of these transformations	• devise instructions for a computer to generate and transform shapes	• use any point as the centre of rotation; measure the angle of rotation, using fractions of a turn or degrees; understand that translations are specified by a vector		
		• explore and compare mathematical representations of combinations of translations, rotations and reflections of 2-D shapes, on paper and using ICT			

	• understand and use the language and notation associated with enlargement; enlarge 2-D shapes, given a centre of enlargement and a positive integer scale factor; explore enlargement using ICT	• enlarge 2-D shapes, given a centre of enlargement and a positive integer scale factor, on paper and using ICT; identify the scale factor of an enlargement as the ratio of the lengths of any two corresponding line segments; recognise that enlargements preserve angle but not length, and understand the implications of enlargement for perimeter	• enlarge 2-D shapes using positive, fractional and negative scale factors, on paper and using ICT; recognise the similarity of the resulting shapes; understand and use the effects of enlargement on perimeter	• understand and use the effects of enlargement on areas and volumes of shapes and solids	
	• make scale drawings	• use and interpret maps and scale drawings in the context of mathematics and other subjects			

• use conventions and notation for 2-D coordinates in all four quadrants; find coordinates of points determined by geometric information	• find the midpoint of the line segment AB, given the coordinates of points A and B	• use the coordinate grid to solve problems involving translations, rotations, reflections and enlargements	• find the points that divide a line in a given ratio, using the properties of similar triangles; calculate the length of AB, given the coordinates of points A and B	• understand and use vector notation to describe transformation of 2-D shapes by combinations of translations; calculate and represent graphically the sum of two vectors	• calculate and represent graphically the sum of two vectors, the difference of two vectors and a scalar multiple of a vector; calculate the resultant of two vectors • understand and use the commutative and associative properties of vector addition • solve simple geometrical problems in 2-D using vectors

4.3 Construction and loci

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• use a ruler and protractor to: – measure and draw lines to the nearest millimetre and angles, including reflex angles, to the nearest degree – construct a triangle, given two sides and the included angle (SAS) or two angles and the included side (ASA) • use ICT to explore constructions • use ruler and protractor to construct simple nets of 3-D shapes, e.g. cuboid, regular tetrahedron, square-based pyramid, triangular prism	• use straight edge and compasses to construct: – the midpoint and perpendicular bisector of a line segment – the bisector of an angle – the perpendicular from a point to a line – the perpendicular from a point on a line – a triangle, given three sides (SSS) • use ICT to explore these constructions • find simple loci, both by reasoning and by using ICT, to produce shapes and paths, e.g. an equilateral triangle	• use straight edge and compasses to construct triangles, given right angle, hypotenuse and side (RHS) • use ICT to explore constructions of triangles and other 2-D shapes • find the locus of a point that moves according to a simple rule, both by reasoning and by using ICT	• understand from experience of constructing them that triangles given SSS, SAS, ASA or RHS are unique, but that triangles given SSA or AAA are not • find the locus of a point that moves according to a more complex rule, both by reasoning and by using ICT		

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4.4 Measures and mensuration

Year 7	Year 8	Year 9	Year10	Year 11	Extension
• choose and use units of measurement to measure, estimate, calculate and solve problems in everyday contexts; convert one metric unit to another, e.g. grams to kilograms; read and interpret scales on a range of measuring instruments • distinguish between and estimate the size of acute, obtuse and reflex angles	• choose and use units of measurement to measure, estimate, calculate and solve problems in a range of contexts; know rough metric equivalents of imperial measures in common use, such as miles, pounds (lb) and pints • use bearings to specify direction	• solve problems involving measurements in a variety of contexts; convert between area measures (e.g. mm^2 to cm^2, cm^2 to m^2, and vice versa) and between volume measures (e.g. mm^3 to cm^3, cm^3 to m^3, and vice versa)	• understand and use measures of speed (and other compound measures such as density or pressure); solve problems involving constant or average rates of change	• apply knowledge that measurements given to the nearest whole unit may be inaccurate by up to one half of the unit in either direction and use this to understand how errors can be compounded in calculations	• recognise limitations in the accuracy of measurements and judge the proportional effect on solutions
		Y9 extension objective • *interpret and explore combining measures into rates of change in everyday contexts (e.g. km per hour, pence per metre); use compound measures to compare in real-life contexts (e.g. travel graphs and value for money), using ICT as appropriate*			

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• know and use the formula for the area of a rectangle; calculate the perimeter and area of shapes made from rectangles • calculate the surface area of cubes and cuboids	• derive and use formulae for the area of a triangle, parallelogram and trapezium; calculate areas of compound shapes • know and use the formula for the volume of a cuboid; calculate volumes and surface areas of cuboids and shapes made from cuboids	• know and use the formulae for the circumference and area of a circle • calculate the surface area and volume of right prisms	• solve problems involving lengths of circular arcs and areas of sectors • solve problems involving surface areas and volumes of cylinders	• solve problems involving surface areas and volumes of cylinders, pyramids, cones and spheres • understand and use the formulae for the length of a circular arc and area and perimeter of a sector • consider the dimensions of a formula and begin to recognise the difference between formulae for perimeter, area and volume in simple contexts	• solve problems involving more complex shapes and solids, including segments of circles and frustums of cones • understand the difference between formulae for perimeter, area and volume by considering dimensions

5 Statistics

5.1 Specifying a problem, planning and collecting data

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• suggest possible answers, given a question that can be addressed by statistical methods • decide which data would be relevant to an enquiry and possible sources	• discuss a problem that can be addressed by statistical methods and identify related questions to explore • decide which data to collect to answer a question, and the degree of accuracy needed; identify possible sources; consider appropriate sample size	• suggest a problem to explore using statistical methods, frame questions and raise conjectures • discuss how different sets of data relate to the problem; identify possible primary or secondary sources; determine the sample size and most appropriate degree of accuracy	• independently devise a suitable plan for a substantial statistical project and justify the decisions made • identify possible sources of bias and plan how to minimise it • break a task down into an appropriate series of key statements (hypotheses), and decide upon the best methods for testing these	• consider possible difficulties with planned approaches, including practical problems; adjust the project plan accordingly • deal with practical problems such as non-response or missing data • identify what extra information may be required to pursue a further line of enquiry	• select and justify a sampling scheme and a method to investigate a population, including random and stratified sampling • understand how different methods of sampling and different sample sizes may affect the reliability of conclusions drawn

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• plan how to collect and organise small sets of data from surveys and experiments: – design data collection sheets or questionnaires to use in a simple survey – construct frequency tables for gathering discrete data, grouped where appropriate in equal class intervals • collect small sets of data from surveys and experiments, as planned	• plan how to collect the data; construct frequency tables with equal class intervals for gathering continuous data and two-way tables for recording discrete data • collect data using a suitable method (e.g. observation, controlled experiment, data logging using ICT)	• design a survey or experiment to capture the necessary data from one or more sources; design, trial and if necessary refine data collection sheets; construct tables for gathering large discrete and continuous sets of raw data, choosing suitable class intervals; design and use two-way tables • gather data from specified secondary sources, including printed tables and lists, and ICT-based sources, including the internet	• gather data from primary and secondary sources, using ICT and other methods, including data from observation, controlled experiment, data logging, printed tables and lists		

5.2 Processing and representing data

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• calculate statistics for small sets of discrete data: – find the mode, median and range, and the modal class for grouped data – calculate the mean, including from a simple frequency table, using a calculator for a larger number of items	• calculate statistics for sets of discrete and continuous data, including with a calculator and spreadsheet; recognise when it is appropriate to use the range, mean, median and mode and, for grouped data, the modal class	• calculate statistics and select those most appropriate to the problem or which address the questions posed	• use an appropriate range of statistical methods to explore and summarise data; including estimating and finding the mean, median, quartiles and interquartile range for large data sets (by calculation or using a cumulative frequency diagram)	• use an appropriate range of statistical methods to explore and summarise data; including calculating an appropriate moving average for a time series • use a moving average to identify seasonality and trends in time series data, using them to make predictions	

• construct, on paper and using ICT, graphs and diagrams to represent data, including: – bar-line graphs – frequency diagrams for grouped discrete data – simple pie charts	• construct graphical representations, on paper and using ICT, and identify which are most useful in the context of the problem. Include: – pie charts for categorical data – bar charts and frequency diagrams for discrete and continuous data – simple line graphs for time series – simple scatter graphs – stem-and-leaf diagrams	• select, construct and modify, on paper and using ICT, suitable graphical representations to progress an enquiry and identify key features present in the data. Include: – line graphs for time series – scatter graphs to develop further understanding of correlation	• select, construct and modify, on paper and using ICT, suitable graphical representation to progress an enquiry and identify key features present in the data. Include: – cumulative frequency tables and diagrams – box plots – scatter graphs and lines of best fit (by eye)	• select, construct and modify, on paper and using ICT, suitable graphical representation to progress an enquiry, including histograms for grouped continuous data with equal class intervals	• construct histograms, including those with unequal class intervals
		Y9 extension objective • *work through the entire handling data cycle to explore relationships within bivariate data, including applications to global citizenship, e.g. how fair is our society?*			

5.3 Interpreting and discussing results

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• interpret diagrams and graphs (including pie charts), and draw simple conclusions based on the shape of graphs and simple statistics for a single distribution • compare two simple distributions using the range and one of the mode, median or mean • write a short report of a statistical enquiry, including appropriate diagrams, graphs and charts, using ICT as appropriate; justify the choice of presentation	• interpret tables, graphs and diagrams for discrete and continuous data, relating summary statistics and findings to the questions being explored • compare two distributions using the range and one or more of the mode, median and mean • write about and discuss the results of a statistical enquiry using ICT as appropriate; justify the methods used	• interpret graphs and diagrams and make inferences to support or cast doubt on initial conjectures; have a basic understanding of correlation • compare two or more distributions and make inferences, using the shape of the distributions and appropriate statistics • review interpretations and results of a statistical enquiry on the basis of discussions; communicate these interpretations and results using selected tables, graphs and diagrams	• analyse data to find patterns and exceptions, and try to explain anomalies; include social statistics such as index numbers, time series and survey data • appreciate that correlation is a measure of the strength of association between two variables; distinguish between positive, negative and zero correlation, using lines of best fit; appreciate that zero correlation does not necessarily imply 'no relationship' but merely 'no linear relationship'	• interpret and use cumulative frequency diagrams to solve problems • recognise the limitations of any assumptions and the effects that varying the assumptions could have on the conclusions drawn from data analysis • compare two or more distributions and make inferences, using the shape of the distributions and measures of average and spread, including median and quartiles	• use, interpret and compare histograms, including those with unequal class intervals

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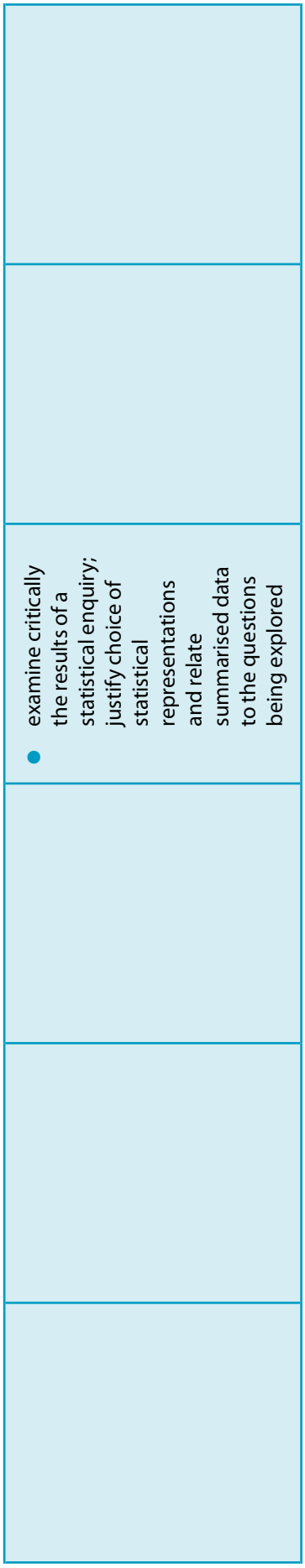

5.4 Probability

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• use vocabulary and ideas of probability, drawing on experience • understand and use the probability scale from 0 to 1; find and justify probabilities based on equally likely outcomes in simple contexts; identify all the possible mutually exclusive outcomes of a single event	• interpret the results of an experiment using the language of probability; appreciate that random processes are unpredictable • know that if the probability of an event occurring is p, then the probability of it not occurring is $1 - p$; use diagrams and tables to record in a systematic way all possible mutually exclusive outcomes for single events and for two successive events	• interpret results involving uncertainty and prediction • identify all the mutually exclusive outcomes of an experiment; know that the sum of probabilities of all mutually exclusive outcomes is 1 and use this when solving problems	• use tree diagrams to represent outcomes of two or more events and to calculate probabilities of combinations of independent events • know when to add or multiply two probabilities: if A and B are mutually exclusive, then the probability of A or B occurring is P(A) + P(B), whereas if A and B are independent events, the probability of A and B occurring is P(A) × P(B)	• use tree diagrams to represent outcomes of compound events, recognising when events are independent and distinguishing between contexts involving selection both with and without replacement	• recognise when and how to work with probabilities associated with independent and mutually exclusive events when interpreting data

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• estimate probabilities by collecting data from a simple experiment and recording it in a frequency table; compare experimental and theoretical probabilities in simple contexts	• compare estimated experimental probabilities with theoretical probabilities, recognising that: – if an experiment is repeated the outcome may, and usually will, be different – increasing the number of times an experiment is repeated generally leads to better estimates of probability	• compare experimental and theoretical probabilities in a range of contexts; appreciate the difference between mathematical explanation and experimental evidence	• understand relative frequency as an estimate of probability and use this to compare outcomes of experiments	• understand that if an experiment is repeated, the outcome may – and usually will – be different, and that increasing the sample size generally leads to better estimates of probability and population parameters	