Mathematics

Programme of study for key stage 3 and attainment targets

(This is an extract from The National Curriculum 2007)

www.qca.org.uk/curriculum

Curriculum aims

Learning and undertaking activities in mathematics contribute to achievement of the curriculum aims for all young people to become:

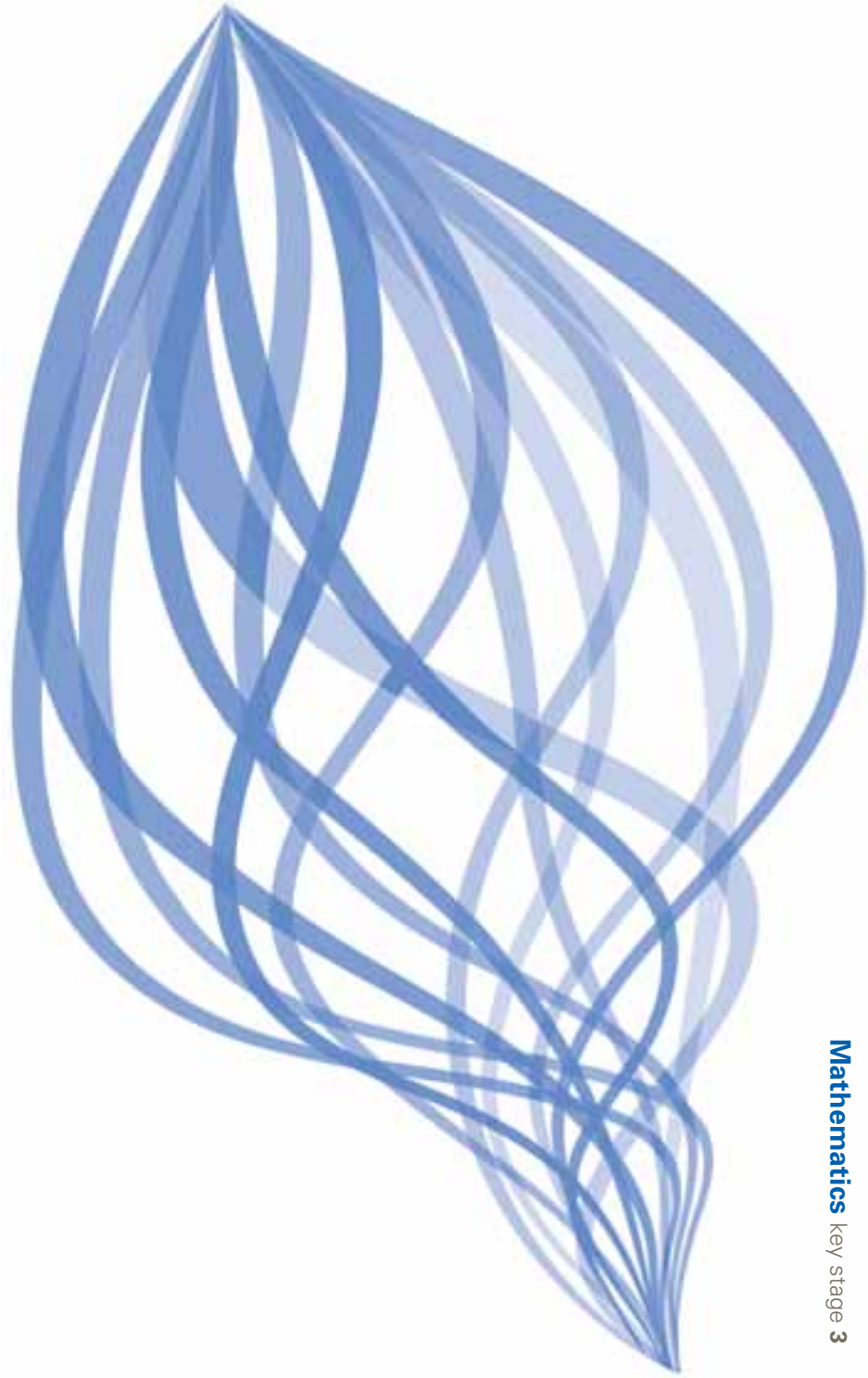
- successful learners who enjoy learning, make progress and achieve
- confident individuals who are able to live safe, healthy and fulfilling lives
- responsible citizens who make a positive contribution to society.

The importance of mathematics

Mathematical thinking is important for all members of a modern society as a habit of mind for its use in the workplace, business and finance; and for personal decision-making. Mathematics is fundamental to national prosperity in providing tools for understanding science, engineering, technology and economics. It is essential in public decision-making and for participation in the knowledge economy.

Mathematics equips pupils with uniquely powerful ways to describe, analyse and change the world. It can stimulate moments of pleasure and wonder for all pupils when they solve a problem for the first time, discover a more elegant solution, or notice hidden connections. Pupils who are functional in mathematics and financially capable are able to think independently in applied and abstract ways, and can reason, solve problems and assess risk.

Mathematics is a creative discipline. The language of mathematics is international. The subject transcends cultural boundaries and its importance is universally recognised. Mathematics has developed over time as a means of solving problems and also for its own sake.



1 Key concepts

There are a number of key concepts that underpin the study of mathematics. Pupils need to understand these concepts in order to deepen and broaden their knowledge, skills and understanding.

1.1 Competence

a Applying suitable mathematics accurately within the classroom and beyond.
b Communicating mathematics effectively.
c Selecting appropriate mathematical tools and methods, including ICT.

1.2 Creativity

a Combining understanding, experiences, imagination and reasoning to construct new knowledge.
b Using existing mathematical knowledge to create solutions to unfamiliar problems.
c Posing questions and developing convincing arguments.

EXPLANATORY NOTES

Study of mathematics: This is concerned with the learning processes for mathematics.

Applying suitable mathematics: This requires fluency and confidence in a range of mathematical techniques and processes that can be applied in a widening range of familiar and unfamiliar contexts, including managing money, assessing risk, problem-solving and decision-making.

Communicating mathematics: Pupils should be familiar with and confident about mathematical notation and conventions and be able to select the most appropriate way to communicate mathematics, both orally and in writing. They should also be able to understand and interpret mathematics presented in a range of forms.

Mathematical tools: Pupils should be familiar with a range of resources and tools, including graphic calculators, dynamic geometry and spreadsheets, which can be used to work on mathematics.

Mathematical methods: At the heart of mathematics are the concepts of equivalence, proportional thinking, algebraic structure, relationships, axiomatic systems, symbolic representation, proof, operations and their inverses.

Posing questions: This involves pupils adopting a questioning approach to mathematical activity, asking questions such as 'How true?' and 'What if…?'

1.3 Applications and implications of mathematics

a Knowing that mathematics is a rigorous, coherent discipline.
b Understanding that mathematics is used as a tool in a wide range of contexts.
c Recognising the rich historical and cultural roots of mathematics.
d Engaging in mathematics as an interesting and worthwhile activity.

1.4 Critical understanding

a Knowing that mathematics is essentially abstract and can be used to model, interpret or represent situations.
b Recognising the limitations and scope of a model or representation.

Mathematics equips pupils with uniquely powerful ways to describe, analyse and change the world

EXPLANATORY NOTES

Mathematics is used as a tool: This includes using mathematics as a tool for making financial decisions in personal life and for solving problems in fields such as building, plumbing, engineering and geography. Current applications of mathematics in everyday life include internet security, weather forecasting, modelling changes in society and the environment, and managing risk (eg insurance, investments and pensions). Mathematics can be used as a way of perceiving the world, for example the symmetry in architecture and nature and the geometry of clothing.

Historical and cultural roots of mathematics: Mathematics has a rich and fascinating history and has been developed across the world to solve problems and for its own sake. Pupils should learn about problems from the past that led to the development of particular areas of mathematics, appreciate that pure mathematical findings sometimes precede practical applications, and understand that mathematics continues to develop and evolve.

Limitations and scope: Mathematics equips pupils with the tools to model and understand the world around them. This enables them to engage with complex issues, such as those involving financial capability or environmental dilemmas. For example, mathematical skills are needed to compare different methods of borrowing and paying back money, but the final decision may include other dimensions, such as comparing the merits of using a credit card that promotes a particular charity with one offering the lowest overall cost. The mathematical model or representation may have properties that are not relevant to the situation.

2 Key processes

These are the essential skills and processes in mathematics that pupils need to learn to make progress.

2.1 Representing

Pupils should be able to:

a identify the mathematical aspects of a situation or problem
b choose between representations
c simplify the situation or problem in order to represent it mathematically, using appropriate variables, symbols, diagrams and models
d select mathematical information, methods and tools to use.

2.2 Analysing

Use mathematical reasoning

Pupils should be able to:

a make connections within mathematics
b use knowledge of related problems
c visualise and work with dynamic images
d identify and classify patterns
e make and begin to justify conjectures and generalisations, considering special cases and counter-examples
f explore the effects of varying values and look for invariance and covariance
g take account of feedback and learn from mistakes
h work logically towards results and solutions, recognising the impact of constraints and assumptions
i appreciate that there are a number of different techniques that can be used to analyse a situation
j reason inductively and deduce.

EXPLANATORY NOTES

Processes in mathematics: The key processes in this section are clearly related to the different stages of problem-solving and the handling data cycle.

Representing: Representing a situation places it into the mathematical form that will enable it to be worked on. Pupils should begin to explore mathematical situations, identify the major mathematical features of a problem, try things out and experiment, and create representations that contain the major features of the situation.

Select mathematical information, methods and tools: This involves using systematic methods to explore a situation, beginning to identify ways in which it is possible to break a problem down into more manageable tasks, and identifying and using existing mathematical knowledge that might be needed. In statistical investigations it includes planning to minimise sources of bias when conducting experiments and surveys, and using a variety of methods for collecting primary and secondary data. ICT tools can be used for mathematical applications, including iteration and algorithms.

Make connections: For example, realising that an equation, a table of values and a line on a graph can all represent the same thing, or understanding that an intersection between two lines on a graph can represent the solution to a problem.

Generalisations: Pupils should recognise the range of factors that affect a generalisation.

Varying values: This involves changing values to explore a situation, including the use of ICT (eg to explore statistical situations with underlying random or systematic variation).

Different techniques: For example, working backwards and looking at simpler cases.

Analyse a situation: This includes using mathematical reasoning to explain and justify inferences when analysing data.

Reason inductively: This involves using particular examples to suggest a general statement.

Deduce: This involves using reasoned arguments to derive or draw a conclusion from something already known.

Use appropriate mathematical procedures

Pupils should be able to:

k make accurate mathematical diagrams, graphs and constructions on paper and on screen
l calculate accurately, selecting mental methods or calculating devices as appropriate
m manipulate numbers, algebraic expressions and equations and apply routine algorithms
n use accurate notation, including correct syntax when using ICT
o record methods, solutions and conclusions
p estimate, approximate and check working.

EXPLANATORY NOTES

Calculating devices as appropriate: For example, when calculation without a calculator will take an inappropriate amount of time.

Record methods: This includes representing the results of analyses in various ways (eg tables, diagrams and symbolic representation).

2.3 Interpreting and evaluating

Pupils should be able to:

a form convincing arguments based on findings and make general statements
b consider the assumptions made and the appropriateness and accuracy of results and conclusions
c be aware of the strength of empirical evidence and appreciate the difference between evidence and proof
d look at data to find patterns and exceptions
e relate findings to the original context, identifying whether they support or refute conjectures
f engage with someone else's mathematical reasoning in the context of a problem or particular situation
g consider the effectiveness of alternative strategies.

2.4 Communicating and reflecting

Pupils should be able to:

a communicate findings effectively
b engage in mathematical discussion of results
c consider the elegance and efficiency of alternative solutions
d look for equivalence in relation to both the different approaches to the problem and different problems with similar structures
e make connections between the current situation and outcomes, and situations and outcomes they have already encountered.

EXPLANATORY NOTES

Interpreting: This includes interpreting data and involves looking at the results of an analysis and deciding how the results relate to the original problem.

Evidence: This includes evidence gathered when using ICT to explore cases.

Patterns and exceptions: Pupils should recognise that random processes are unpredictable.

Someone else's mathematical reasoning: Pupils should interpret information presented by the media and through advertising.

Communicating and reflecting: Pupils should communicate findings to others and reflect on different approaches.

Alternative solutions: These include solutions using ICT.

3 Range and content

This section outlines the breadth of the subject on which teachers should draw when teaching the key concepts and key processes.

The study of mathematics should enable pupils to apply their knowledge, skills and understanding to relevant real-world situations.

The study of mathematics should include:

3.1 Number and algebra

a rational numbers, their properties and their different representations
b rules of arithmetic applied to calculations and manipulations with rational numbers
c applications of ratio and proportion
d accuracy and rounding
e algebra as generalised arithmetic
f linear equations, formulae, expressions and identities
g analytical, graphical and numerical methods for solving equations
h polynomial graphs, sequences and functions

The subject transcends cultural boundaries and its importance is universally recognised

EXPLANATORY NOTES

Rules of arithmetic: This includes knowledge of operations and inverse operations and how calculators use precedence. Pupils should understand that not all calculators use algebraic logic and may give different answers for calculations such as $1 + 2 \times 3$.

Calculations and manipulations with rational numbers: This includes using mental and written methods to make sense of everyday situations such as temperature, altitude, financial statements and transactions.

Ratio and proportion: This includes percentages and applying concepts of ratio and proportion to contexts such as value for money, scales, plans and maps, cooking and statistical information (eg 9 out of 10 people prefer…).

Accuracy and rounding: This is particularly important when using calculators and computers.

Linear equations: This includes setting up equations, including inequalities and simultaneous equations. Pupils should be able to recognise equations with no solutions or an infinite number of solutions.

Polynomial graphs: This includes gradient properties of parallel and perpendicular lines.

Sequences and functions: This includes a range of sequences and functions based on simple rules and relationships.

3.2 Geometry and measures

a properties of 2D and 3D shapes
b constructions, loci and bearings
c Pythagoras' theorem
d transformations
e similarity, including the use of scale
f points, lines and shapes in 2D coordinate systems
g units, compound measures and conversions
h perimeters, areas, surface areas and volumes

3.3 Statistics

a the handling data cycle
b presentation and analysis of grouped and ungrouped data, including time series and lines of best fit
c measures of central tendency and spread
d experimental and theoretical probabilities, including those based on equally likely outcomes.

EXPLANATORY NOTES

2D and 3D shapes: These include circles and shapes made from cuboids.

Constructions, loci and bearings: This includes constructing mathematical figures using both straight edge and compasses, and ICT.

Scale: This includes making sense of plans, diagrams and construction kits.

Compound measures: This includes making sense of information involving compound measures, for example fuel consumption, speed and acceleration.

Surface areas and volumes: This includes 3D shapes based on prisms.

The handling data cycle: This is closely linked to the mathematical key processes and consists of:

- specifying the problem and planning (representing)
- collecting data (representing and analysing)
- processing and presenting the data (analysing)
- interpreting and discussing the results (interpreting and evaluating).

Presentation and analysis: This includes the use of ICT.

Spread: For example, the range and inter-quartile range.

Probabilities: This includes applying ideas of probability and risk to gambling, safety issues, and simulations using ICT to represent a probability experiment, such as rolling two dice and adding the scores.

4 Curriculum opportunities

During the key stage pupils should be offered the following opportunities that are integral to their learning and enhance their engagement with the concepts, processes and content of the subject.

The curriculum should provide opportunities for pupils to:

a develop confidence in an increasing range of methods and techniques
b work on sequences of tasks that involve using the same mathematics in increasingly difficult or unfamiliar contexts, or increasingly demanding mathematics in similar contexts
c work on open and closed tasks in a variety of real and abstract contexts that allow them to select the mathematics to use
d work on problems that arise in other subjects and in contexts beyond the school
e work on tasks that bring together different aspects of concepts, processes and mathematical content
f work collaboratively as well as independently in a range of contexts
g become familiar with a range of resources, including ICT, so that they can select appropriately.

EXPLANATORY NOTES

Other subjects: For example, representing and analysing data in geography, using formulas and relationships in science, understanding number structure and currency exchange in modern foreign languages, measuring and making accurate constructions in design and technology, and managing money in economic wellbeing and financial capability.

Contexts beyond the school: For example, conducting a survey into consumer habits, planning a holiday budget, designing a product, and measuring for home improvements. Mathematical skills contribute to financial capability and to other aspects of preparation for adult life.

Work collaboratively: This includes talking about mathematics, evaluating their own and others' work and responding constructively, problem-solving in pairs or small groups and presenting ideas to a wider group.

Become familiar with a range of resources: This includes using practical resources and ICT, such as spreadsheets, dynamic geometry, graphing software and calculators, to develop mathematical ideas.

Attainment targets

Attainment target 1: Mathematical processes and applications

Level 4

Pupils develop their own strategies for solving problems and use these strategies both in working within mathematics and in applying mathematics to practical contexts. When solving problems, with or without a calculator, they check their results are reasonable by considering the context or the size of the numbers. They look for patterns and relationships, presenting information and results in a clear and organised way. They search for a solution by trying out ideas of their own.

Level 5

In order to explore mathematical situations, carry out tasks or tackle problems, pupils identify the mathematical aspects and obtain necessary information. They calculate accurately, using ICT where appropriate. They check their working and results, considering whether these are sensible. They show understanding of situations by describing them mathematically using symbols, words and diagrams. They draw simple conclusions of their own and explain their reasoning.

Level 7

Starting from problems or contexts that have been presented to them, pupils explore the effects of varying values and look for invariance in models and representations, working with and without ICT. They progressively refine or extend the mathematics used, giving reasons for their choice of mathematical presentation and explaining features they have selected. They justify their generalisations, arguments or solutions, looking for equivalence to different problems with similar structures. They appreciate the difference between mathematical explanation and experimental evidence.

Level 8

Pupils develop and follow alternative approaches. They compare and evaluate representations of a situation, introducing and using a range of mathematical techniques. They reflect on their own lines of enquiry when exploring mathematical tasks. They communicate mathematical or statistical meaning to different audiences through precise and consistent use of symbols that is sustained throughout the work. They examine generalisations or solutions reached in an activity and make further progress in the activity as a result. They comment constructively on the reasoning and logic, the process employed and the results obtained.

Level 6

Pupils carry out substantial tasks and solve quite complex problems by independently and systematically breaking them down into smaller, more manageable tasks. They interpret, discuss and synthesise information presented in a variety of mathematical forms, relating findings to the original context. Their written and spoken language explains and informs their use of diagrams. They begin to give mathematical justifications, making connections between the current situation and situations they have encountered before.

Exceptional performance

Pupils critically examine the strategies adopted when investigating within mathematics itself or when using mathematics to analyse tasks. They explain why different strategies were used, considering the elegance and efficiency of alternative lines of enquiry or procedures. They apply the mathematics they know in a wide range of familiar and unfamiliar contexts. They use mathematical language and symbols effectively in presenting a convincing, reasoned argument. Their reports include mathematical justifications, distinguishing between evidence and proof and explaining their solutions to problems involving a number of features or variables.

Attainment target 2: Number and algebra

Level 4

Pupils use their understanding of place value to multiply and divide whole numbers by 10 or 100. When solving number problems, they use a range of mental methods of computation with the four operations, including mental recall of multiplication facts up to 10 × 10 and quick derivation of corresponding division facts. They use efficient written methods of addition and subtraction and of short multiplication and division. They recognise approximate proportions of a whole and use simple fractions and percentages to describe these. They begin to use simple formulae expressed in words.

Level 5

Pupils use their understanding of place value to multiply and divide whole numbers and decimals. They order, add and subtract negative numbers in context. They use all four operations with decimals to two places. They solve simple problems involving ratio and direct proportion. They calculate fractional or percentage parts of quantities and measurements, using a calculator where appropriate. They construct, express in symbolic form and use simple formulae involving one or two operations. They use brackets appropriately. They use and interpret coordinates in all four quadrants.

Attainment target 3: Geometry and measures

Level 4

Pupils make 3D mathematical models by linking given faces or edges, and draw common 2D shapes in different orientations on grids. They reflect simple shapes in a mirror line. They choose and use appropriate units and tools, interpreting, with appropriate accuracy, numbers on a range of measuring instruments. They find perimeters of simple shapes and find areas by counting squares.

Level 5

When constructing models and drawing or using shapes, pupils measure and draw angles to the nearest degree and use language associated with angles. They know the angle sum of a triangle and that of angles at a point. They identify all the symmetries of 2D shapes. They convert one metric unit to another. They make sensible estimates of a range of measures in relation to everyday situations. They understand and use the formula for the area of a rectangle.

Attainment target 4: Handling data

Level 4

Pupils collect discrete data and record them using a frequency table. They understand and use the mode and range to describe sets of data. They group data in equal class intervals where appropriate, represent collected data in frequency diagrams and interpret such diagrams. They construct and interpret simple line graphs.

Level 5

Pupils understand and use the mean of discrete data. They compare two simple distributions using the range and one of the mode, median or mean. They interpret graphs and diagrams, including pie charts, and draw conclusions. They understand and use the probability scale from 0 to 1. They find and justify probabilities and approximations to these by selecting and using methods based on equally likely outcomes and experimental evidence, as appropriate. They understand that different outcomes may result from repeating an experiment.

Level 6

Pupils order and approximate decimals when solving numerical problems and equations, using trial and improvement methods. They evaluate one number as a fraction or percentage of another. They understand and use the equivalences between fractions, decimals and percentages, and calculate using ratios in appropriate situations. They add and subtract fractions by writing them with a common denominator. They find and describe in words the rule for the next term or *n*th term of a sequence where the rule is linear. They formulate and solve linear equations with whole-number coefficients. They represent mappings expressed algebraically, and use Cartesian coordinates for graphical representation interpreting general features.

Level 6

Pupils recognise and use common 2D representations of 3D objects. They know and use the properties of quadrilaterals. They solve problems using angle and symmetry, properties of polygons and angle properties of intersecting and parallel lines, and explain these properties. They devise instructions for a computer to generate and transform shapes and paths. They understand and use appropriate formulae for finding circumferences and areas of circles, areas of plane rectilinear figures and volumes of cuboids when solving problems.

Level 6

Pupils collect and record continuous data, choosing appropriate equal class intervals over a sensible range to create frequency tables. They construct and interpret frequency diagrams. They construct pie charts. They draw conclusions from scatter diagrams, and have a basic understanding of correlation. When dealing with a combination of two experiments, they identify all the outcomes. When solving problems, they use their knowledge that the total probability of all the mutually exclusive outcomes of an experiment is 1.

Level 7

When making estimates, pupils round to one significant figure and multiply and divide mentally. They understand the effects of multiplying and dividing by numbers between 0 and 1. They solve numerical problems involving multiplication and division with numbers of any size, using a calculator efficiently and appropriately. They understand and use proportional changes, calculating the result of any proportional change using only multiplicative methods. They find and describe in symbols the next term or *n*th term of a sequence where the rule is quadratic. They use algebraic and graphical methods to solve simultaneous linear equations in two variables.

Level 7

Pupils understand and apply Pythagoras' theorem when solving problems in two dimensions. They calculate lengths, areas and volumes in plane shapes and right prisms. They enlarge shapes by a fractional scale factor, and appreciate the similarity of the resulting shapes. They determine the locus of an object moving according to a rule. They appreciate the imprecision of measurement and recognise that a measurement given to the nearest whole number may be inaccurate by up to one half in either direction. They understand and use compound measures, such as speed.

Level 7

Pupils specify hypotheses and test them by designing and using appropriate methods that take account of variability or bias. They determine the modal class and estimate the mean, median and range of sets of grouped data, selecting the statistic most appropriate to their line of enquiry. They use measures of average and range, with associated frequency polygons, as appropriate, to compare distributions and make inferences. They understand relative frequency as an estimate of probability and use this to compare outcomes of experiments.

Attainment target 2: Number and algebra

Level 8

Pupils solve problems that involve calculating with powers, roots and numbers expressed in standard form. They choose to use fractions or percentages to solve problems involving repeated proportional changes or the calculation of the original quantity given the result of a proportional change. They evaluate algebraic formulae or calculate one variable, given the others, substituting fractions, decimals and negative numbers. They manipulate algebraic formulae, equations and expressions, finding common factors and multiplying two linear expressions. They solve inequalities in two variables. They sketch and interpret graphs of linear, quadratic, cubic and reciprocal functions, and graphs that model real situations.

Exceptional performance

Pupils understand and use rational and irrational numbers. They determine the bounds of intervals. They understand and use direct and inverse proportion. In simplifying algebraic expressions, they use rules of indices for negative and fractional values. In finding formulae that approximately connect data, they express general laws in symbolic form. They solve simultaneous equations in two variables where one equation is linear and the other is quadratic. They solve problems using intersections and gradients of graphs.

Attainment target 3: Geometry and measures

Level 8

Pupils understand and use congruence and mathematical similarity. They use sine, cosine and tangent in right-angled triangles when solving problems in two dimensions.

Exceptional performance

Pupils sketch the graphs of sine, cosine and tangent functions for any angle, and generate and interpret graphs based on these functions. They use sine, cosine and tangent of angles of any size, and Pythagoras' theorem when solving problems in two and three dimensions. They construct formal geometric proofs. They calculate lengths of circular arcs and areas of sectors, and calculate the surface area of cylinders and volumes of cones and spheres. They appreciate the continuous nature of scales that are used to make measurements.

Attainment target 4: Handling data

Level 8

Pupils interpret and construct cumulative frequency tables and diagrams. They estimate the median and interquartile range and use these to compare distributions and make inferences. They understand how to calculate the probability of a compound event and use this in solving problems.

Exceptional performance

Pupils interpret and construct histograms. They understand how different methods of sampling and different sample sizes may affect the reliability of conclusions drawn. They select and justify a sample and method to investigate a population. They recognise when and how to work with probabilities associated with independent, mutually exclusive events.