The Framework for secondary mathematics: overview and learning objectives

Overview of strands

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1 Mathematical processes and applications		**3 Algebra**	
	1.1 Representing		3.1 Equations, formulae, expressions and identities
	1.2 Analysing – use mathematical reasoning		3.2 Sequences, functions and graphs
	1.3 Analysing – use appropriate mathematical procedures	**4 Geometry and measures**	
	1.4 Interpreting and evaluating		4.1 Geometrical reasoning
	1.5 Communicating and reflecting		4.2 Transformations and coordinates
2 Number			4.3 Construction and loci
	2.1 Place value, ordering and rounding		4.4 Measures and mensuration
	2.2 Integers, powers and roots	**5 Statistics**	
	2.3 Fractions, decimals, percentages, ratio and proportion		5.1 Specifying a problem, planning and collecting data
	2.4 Number operations		5.2 Processing and representing data
	2.5 Mental calculation methods		5.3 Interpreting and discussing results
	2.6 Written calculation methods		5.4 Probability
	2.7 Calculator methods		
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Learning objectives

1 Mathematical processes and applications

Solve problems, explore and investigate in a range of contexts

Increase the **challenge** and build **progression** across the key stage, and for groups of pupils by:

- increasing the **complexity** of the application, e.g. non-routine, multi-step problems, extended enquiries
- reducing the **familiarity** of the context, e.g. new contexts in mathematics, contexts drawn from other subjects, other aspects of pupils' lives
- increasing the **technical demand** of the mathematics required, e.g. more advanced concepts, more difficult procedures
- increasing the degree of **independence** and autonomy in problem-solving and investigation

1.1 Representing

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
identify the necessary information to understand or simplify a context or problem; represent problems, making correct use of symbols, words, diagrams, tables and graphs; use appropriate procedures and tools, including ICT	identify the mathematical features of a context or problem; try out and compare mathematical representations; select appropriate procedures and tools, including ICT	break down substantial tasks to make them more manageable; represent problems and synthesise information in algebraic, geometrical or graphical form; move from one form to another to gain a different perspective on the problem	compare and evaluate representations; explain the features selected and justify the choice of representation in relation to the context	choose and combine representations from a range of perspectives; introduce and use a range of mathematical techniques, the most efficient for analysis and most effective for communication	systematically model contexts or problems through precise and consistent use of symbols and representations, and sustain this throughout the work

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1.2 Analysing – use mathematical reasoning

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
classify and visualise properties and patterns; generalise in simple cases by working logically; draw simple conclusions and explain reasoning; understand the significance of a counter-example; take account of feedback and learn from mistakes	visualise and manipulate dynamic images; conjecture and generalise; move between the general and the particular to test the logic of an argument; identify exceptional cases or counter-examples; make connections with related contexts	use connections with related contexts to improve the analysis of a situation or problem; pose questions and make convincing arguments to justify generalisations or solutions; recognise the impact of constraints or assumptions	identify a range of strategies and appreciate that more than one approach may be necessary; explore the effects of varying values and look for invariance and covariance in models and representations; examine and refine arguments, conclusions and generalisations; produce simple proofs	make progress by exploring mathematical tasks, developing and following alternative approaches; examine and extend generalisations; support assumptions by clear argument and follow through a sustained chain of reasoning, including proof	present rigorous and sustained arguments; reason inductively, deduce and prove; explain and justify assumptions and constraints

1.3 Analysing – use appropriate mathematical procedures

Within the appropriate range and content:

make accurate mathematical diagrams, graphs and constructions on paper and on screen; calculate accurately, selecting mental methods or calculating devices as appropriate; manipulate numbers, algebraic expressions and equations, and apply routine algorithms; use accurate notation, including correct syntax when using ICT; record methods, solutions and conclusions; estimate, approximate and check working

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1.4 Interpreting and evaluating

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
interpret information from a mathematical representation or context; relate findings to the original context; check the accuracy of the solution; explain and justify methods and conclusions; compare and evaluate approaches	use logical argument to interpret the mathematics in a given context or to establish the truth of a statement; give accurate solutions appropriate to the context or problem; evaluate the efficiency of alternative strategies and approaches	justify the mathematical features drawn from a context and the choice of approach; generate fuller solutions by presenting a concise, reasoned argument using symbols, diagrams, graphs and related explanations	make sense of, and judge the value of, own findings and those presented by others; judge the strength of empirical evidence and distinguish between evidence and proof; justify generalisations, arguments or solutions	show insight into the mathematical connections in the context or problem; critically examine strategies adopted and arguments presented; consider the assumptions in the model and recognise limitations in the accuracy of results and conclusions	justify and explain solutions to problems involving an unfamiliar context or a number of features or variables; comment constructively on reasoning, logic, process, results and conclusions

1.5 Communicating and reflecting

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
communicate own findings effectively, orally and in writing, and discuss and compare approaches and results with others; recognise equivalent approaches	refine own findings and approaches on the basis of discussions with others; recognise efficiency in an approach; relate the current problem and structure to previous situations	review and refine own findings and approaches on the basis of discussions with others; look for and reflect on other approaches and build on previous experience of similar situations and outcomes	use a range of forms to communicate findings effectively to different audiences; review findings and look for equivalence to different problems with similar structure	routinely review and refine findings and approaches; identify how other contexts were different from, or similar to, the current situation and explain how and why the same or different strategies were used	use mathematical language and symbols effectively in presenting convincing conclusions or findings; critically reflect on own lines of enquiry when exploring; search for and appreciate more elegant forms of communicating approaches and solutions; consider the efficiency of alternative lines of enquiry or procedures

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2 Number

2.1 Place value, ordering and rounding

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
understand and use decimal notation and place value; multiply and divide integers and decimals by 10, 100, 1000, and explain the effect	read and write positive integer powers of 10; multiply and divide integers and decimals by 0.1 and 0.01	extend knowledge of integer powers of 10; recognise the equivalence of 0.1, $\frac{1}{10}$ and 10^{-1}; multiply and divide by any integer power of 10	convert between ordinary and standard index form representations, using significant figures as appropriate; justify the representation used and choice of accuracy in relation to the problem and audience for the solution	engage in mathematical tasks where using numbers in standard form is essential to the calculations involved; critically examine the effect of numerical representations on the accuracy of the solution, e.g. understand how errors can be compounded in calculations	communicate the solution to a problem, explaining the limitations of accuracy, using upper and lower bounds
compare and order decimals in different contexts; know that when comparing measurements the units must be the same	order decimals				
round positive whole numbers to the nearest 10, 100 or 1000, and decimals to the nearest whole number or one decimal place	round positive numbers to any given power of 10; round decimals to the nearest whole number or to one or two decimal places	use rounding to make estimates and to give solutions to problems to an appropriate degree of accuracy			

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2.2 Integers, powers and roots

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
understand negative numbers as positions on a number line; order, add and subtract integers in context	add, subtract, multiply and divide integers		examine and extend the index laws to establish the meaning of negative, fractional and zero powers, including use of surd notation	examine and extend the index laws to establish the meaning of inverse operations in relation to indices, i.e. the inverse operation of raising a positive number to power n is raising the result of this operation to power $\frac{1}{n}$	solve a problem using rational and irrational numbers, including surds
recognise and use multiples, factors, primes (less than 100), common factors, highest common factors and lowest common multiples in simple cases; use simple tests of divisibility	use multiples, factors, common factors, highest common factors, lowest common multiples and primes; find the prime factor decomposition of a number, e.g. $8000 = 2^6 \times 5^3$	use the prime factor decomposition of a number			
recognise the first few triangular numbers; recognise the squares of numbers to at least 12×12 and the corresponding roots	use squares, positive and negative square roots, cubes and cube roots, and index notation for small positive integer powers	use ICT to estimate square roots and cube roots			
		use index notation for integer powers; know and use the index laws for multiplication and division of positive integer powers			

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2.3 Fractions, decimals, percentages, ratio and proportion

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
express a smaller whole number as a fraction of a larger one; simplify fractions by cancelling all common factors and identify equivalent fractions; convert terminating decimals to fractions, e.g. $0.23 = \frac{23}{100}$; use diagrams to compare two or more simple fractions	recognise that a recurring decimal is a fraction; use division to convert a fraction to a decimal; order fractions by writing them with a common denominator or by converting them to decimals	understand the equivalence of simple algebraic fractions; know that a recurring decimal is an exact fraction	explain the patterns found in recurring decimals; justify why decimals recur or terminate by considering factors of the denominator	explore the historical and cultural roots of the number system and use algebra to justify and prove some of its features, e.g. that all recurring decimals can be expressed as a fraction	show insight into the infinite density of the number line; make sense of the proof that $\sqrt{2}$ is irrational
add and subtract simple fractions and those with common denominators; calculate simple fractions of quantities and measurements (whole-number answers); multiply a fraction by an integer	add and subtract fractions by writing them with a common denominator; calculate fractions of quantities (fraction answers); multiply and divide an integer by a fraction	use efficient methods to add, subtract, multiply and divide fractions, interpreting division as a multiplicative inverse; cancel common factors before multiplying or dividing	understand and apply efficient methods to add, subtract, multiply and divide fractions, interpreting reciprocals as multiplicative inverses		

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2.3 Fractions, decimals, percentages, ratio and proportion (continued)

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
understand percentage as the 'number of parts per 100'; calculate simple percentages and use percentages to compare simple proportions	interpret percentage as the operator 'so many hundredths of' and express one given number as a percentage of another; calculate percentages and find the outcome of a given percentage increase or decrease	recognise when fractions or percentages are needed to compare proportions; solve problems involving percentage changes			
recognise the equivalence of percentages, fractions and decimals	use the equivalence of fractions, decimals and percentages to compare proportions				
understand the relationship between ratio and proportion; use direct proportion in simple contexts; use ratio notation, simplify ratios and divide a quantity into two parts in a given ratio; solve simple problems involving ratio and proportion using informal strategies	apply understanding of the relationship between ratio and proportion; simplify ratios, including those expressed in different units, recognising links with fraction notation; divide a quantity into two or more parts in a given ratio; use the unitary method to solve simple problems involving ratio and direct proportion	use proportional reasoning to solve problems, choosing the correct numbers to take as 100%, or as a whole; compare two ratios; interpret and use ratio in a range of contexts	identify when a problem in number, algebra, geometry or statistics involves proportionality; use multiplicative methods fluently in the solution, including inverse calculations, e.g. with percentages	model real contexts where quantities vary in direct proportion, including repeated proportional change, e.g. growth/decay; use algebraic methods where appropriate and consider limitations of the model (*as in 2.4*)	understand and use direct and inverse proportion; solve problems involving inverse proportion (including $y \propto 1/x^2$) (*as in 2.4*)

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2.4 Number operations

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
understand and use the rules of arithmetic and inverse operations in the context of positive integers and decimals	understand and use the rules of arithmetic and inverse operations in the context of integers and fractions	understand the effects of multiplying and dividing by numbers between 0 and 1; consolidate use of the rules of arithmetic and inverse operations	recognise and use reciprocals as a multiplicative inverse in contexts such as enlargement; explore the behaviour of the reciprocal function ($y = 1/x$) for large and small values of x	model real contexts where quantities vary in direct proportion, including repeated proportional change, e.g. growth/decay; use algebraic methods where appropriate and consider limitations of the model (*as in 2.3*)	understand and use direct and inverse proportion; solve problems involving inverse proportion (including $y \propto 1/x^2$) (*as in 2.3*)
use the order of operations, including brackets	use the order of operations, including brackets, with more complex calculations	understand the order of precedence of operations, including powers			

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2.5 Mental calculation methods

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
recall number facts, including positive integer complements to 100 and multiplication facts to 10 × 10, and quickly derive associated division facts	recall equivalent fractions, decimals and percentages; use known facts to derive unknown facts, including products involving numbers such as 0.7 and 6, and 0.03 and 8		select mental or written strategies and calculating devices appropriate to the stage of the problem; calculate accurately with reciprocals, powers, trigonometrical functions and numbers in standard form (*as in 2.6 and 2.7*)	select and justify an appropriate and efficient combination of methods of calculation, i.e. mental, written, ICT or calculator to solve problems (*as in 2.6*)	appreciate when results of calculations can be more elegantly and exactly communicated using surds and π, rationalising a denominator where appropriate, e.g. a trigonometrical solution (*as in 2.6*)
strengthen and extend mental methods of calculation to include decimals, fractions and percentages, accompanied where appropriate by suitable jottings; solve simple problems mentally	strengthen and extend mental methods of calculation, working with decimals, fractions, percentages, squares and square roots, cubes and cube roots; solve problems mentally	use known facts to derive unknown facts; extend mental methods of calculation, working with decimals, fractions, percentages, factors, powers and roots; solve problems mentally			
make and justify estimates and approximations of calculations	make and justify estimates and approximations of calculations	make and justify estimates and approximations of calculations	examine and refine estimates and approximations of calculations involving rounding		

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2.6 Written calculation methods

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
use efficient written methods to add and subtract whole numbers and decimals with up to two places	use efficient written methods to add and subtract integers and decimals of any size, including numbers with differing numbers of decimal places		select mental or written strategies and calculating devices appropriate to the stage of the problem; calculate accurately with reciprocals, powers, trigonometrical functions and numbers in standard form (*as in 2.5 and 2.7*)	select and justify an appropriate and efficient combination of methods of calculation, i.e. mental, written, ICT or calculator to solve problems (*as in 2.5*)	appreciate when results of calculations can be more elegantly and exactly communicated using surds and π, rationalising a denominator where appropriate, e.g. a trigonometrical solution (*as in 2.5*)
multiply and divide three-digit by two-digit whole numbers; extend to multiplying and dividing decimals with one or two places by single-digit whole numbers	use efficient written methods for multiplication and division of integers and decimals, including by decimals such as 0.6 or 0.06; understand where to position the decimal point by considering equivalent calculations	use efficient written methods to add and subtract integers and decimals of any size; multiply by decimals; divide by decimals by transforming to division by an integer			

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2.7 Calculator methods

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
carry out calculations with more than one step using brackets and the memory; use the square root and sign change keys	carry out more difficult calculations effectively and efficiently using the function keys for sign change, powers, roots and fractions; use brackets and the memory	use a calculator efficiently and appropriately to perform complex calculations with numbers of any size, knowing not to round during intermediate steps of a calculation; use the constant, π and sign change keys; use the function keys for powers, roots and fractions; use brackets and the memory	select mental or written strategies and calculating devices appropriate to the stage of the problem; calculate accurately with reciprocals, powers, trigonometrical functions and numbers in standard form (*as in 2.5 and 2.6*)	critically examine alternative methods, compare strategies for: ● calculating (including calculating devices) ● checking recognise the limitations of some approaches (*as in 2.8*)	reflect on a solution to a problem commenting constructively on the choice of calculating strategies (*as in 2.8*)
enter numbers and interpret the display in different contexts (decimals, percentages, money, metric measures)	enter numbers and interpret the display in different contexts (extend to negative numbers, fractions, time)				

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2.8 Checking results

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
check results by considering whether they are of the right order of magnitude and by working problems backwards	select from a range of checking methods, including estimating in context and using inverse operations	check results using appropriate methods	identify a range of checking strategies and appreciate that more than one way may be necessary in the context of the problem	critically examine alternative methods, compare strategies for: • calculating (including calculating devices) • checking recognise the limitations of some approaches (*as in 2.7*)	reflect on a solution to a problem commenting constructively on the choice of checking strategies (*as in 2.7*)

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3 Algebra

3.1 Equations, formulae, expressions and identities

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
use letter symbols to represent unknown numbers or variables; know the meanings of the words *term*, *expression* and *equation*	recognise that letter symbols play different roles in equations, formulae and functions; know the meanings of the words *formula* and *function*	distinguish the different roles played by letter symbols in equations, identities, formulae and functions	present convincing algebraic arguments to justify generalisations or solutions; relate arguments to the structure of the context or problem; produce simple proofs	examine and refine algebraic arguments presented to explain geometrical and numerical properties; choose and combine representations to present a convincing proof	use symbols and representations consistently to present a formal proof, e.g. deriving the formula for solving quadratic equations
understand that algebraic operations follow the rules of arithmetic	understand that algebraic operations, including the use of brackets, follow the rules of arithmetic; use index notation for small positive integer powers	use index notation for integer powers and simple instances of the index laws	use algebraic representation to synthesise known rules of arithmetic, including the commutative and distributive laws; justify these generalisations, e.g. using spatial representations; use algebraic argument to generalise the index laws for multiplication and division to include zero, negative and fractional powers	appreciate the generality of the forms $a + b = c$ and $ab = c$, where each term can itself be an expression; use this insight into structure to develop fluency in transforming more complex equations	

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3.1 Equations, formulae, expressions and identities (continued)

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
simplify linear algebraic expressions by collecting like terms; multiply a single term over a bracket (integer coefficients)	simplify or transform linear expressions by collecting like terms; multiply a single term over a bracket	simplify or transform algebraic expressions by taking out single-term common factors; add simple algebraic fractions	develop fluency in transforming linear expressions; expand the product of two linear expressions of the form $x \pm n$ and factorise simple quadratic expressions; establish identities such as the difference of two squares; compare and evaluate different representations of the same context; identify equivalent expressions and confirm by transformation	expand and factorise quadratic expressions; simplify or transform algebraic fractions, e.g. by factorising and cancelling common factors; compare and evaluate different representations of the same context; identify equivalent expressions and confirm this by transformation	

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3.1 Equations, formulae, expressions and identities (continued)

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
construct and solve simple linear equations with integer coefficients (unknown on one side only) using an appropriate method (e.g. inverse operations)	construct and solve linear equations with integer coefficients (unknown on either or both sides, without and with brackets) using appropriate methods (e.g. inverse operations, transforming both sides in same way)	construct and solve linear equations with integer coefficients (with and without brackets, negative signs anywhere in the equation, positive or negative solution)	construct linear equations and simple linear inequalities (one variable) to represent real-life situations or mathematical problems; solve linear equations and inequalities, representing the solution in the context of the problem	construct simple quadratic equations to represent real-life situations or mathematical problems and solve them using factorisation, graphical or trial and improvement methods; justify the number of solutions using algebraic or graphical arguments and select appropriate solutions, interpreting their accuracy	represent real-life situations or mathematical problems involving: • more complex quadratic equations, choosing an appropriate method of solution including completing the square and use of the formula • direct or inverse proportion, including $y \propto x^2$, $y \propto 1/x^2$ relate algebraic solutions to graphical representation of the functions
	use graphs and set up equations to solve simple problems involving direct proportion	use algebraic methods to solve problems involving direct proportion; relate algebraic solutions to graphs of the equations; use ICT as appropriate			

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3.1 Equations, formulae, expressions and identities (continued)

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
		use systematic trial and improvement methods and ICT tools to find approximate solutions to equations such as $x^2 + x = 20$	*(See objective above for progression)*	*(See objective above for progression)*	*(See objective above for progression)*
		explore ways of constructing models of real-life situations by drawing graphs and constructing algebraic equations and inequalities			
			construct a pair of simultaneous linear equations to represent real-life situations or mathematical problems; examine and compare algebraic methods of solution; use graphical representation to explain why the intersection of two lines gives the common solution and why some cases have no common solution and others have an infinite number	select and justify optimum methods for solving a pair of simultaneous linear equations in a variety of contexts; construct several linear inequalities in one and two variables to represent real-life situations or mathematical problems; solve the inequalities graphically, identifying and interpreting the solution set in the context of the problem	solve more complex pairs of simultaneous equations generated from real-life contexts or geometrical investigations, including pairs where one is linear and the other is quadratic or of the form $x^2 + y^2 = r^2$

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3.1 Equations, formulae, expressions and identities (continued)

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
use simple formulae from mathematics and other subjects; substitute positive integers into linear expressions and formulae and, in simple cases, derive a formula	use formulae from mathematics and other subjects; substitute integers into simple formulae, including examples that lead to an equation to solve; substitute positive integers into expressions involving small powers, e.g. $3x^2 + 4$ or $2x^3$; derive simple formulae	use formulae from mathematics and other subjects; substitute numbers into expressions and formulae; derive a formula and, in simple cases, change its subject	derive formulae, e.g. in the context of mensuration; interpret a range of formulae drawn from real-life contexts and other subjects, relating the variables to the context and describing their behaviour; solve problems by manipulating formulae	derive and use formulae that involve more variables or more complex algebraic expressions; manipulate formulae in order to reach a solution, show insight into the mathematical connections, e.g. using the context and the formulae to explain the proportional effect of varying values	

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3.2 Sequences, functions and graphs

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
describe integer sequences; generate terms of a simple sequence, given a rule (e.g. finding a term from the previous term, finding a term given its position in the sequence)	generate terms of a linear sequence using term-to-term and position-to-term rules, on paper and using a spreadsheet or graphics calculator	generate terms of a sequence using term-to-term and position-to-term rules, on paper and using ICT	develop, compare and evaluate algebraic and spatial representations of situations that generate sequences; interpret, deduce and justify generalisations for the *n*th term of linear and quadratic sequences, including the properties of square and triangular numbers		
generate sequences from patterns or practical contexts and describe the general term in simple cases	use linear expressions to describe the *n*th term of a simple arithmetic sequence, justifying its form by referring to the activity or practical context from which it was generated	generate sequences from practical contexts and write and justify an expression to describe the *n*th term of an arithmetic sequence			

3.2 Sequences, functions and graphs (continued)

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
express simple functions in words, then using symbols; represent them in mappings	express simple functions algebraically and represent them in mappings or on a spreadsheet	find the inverse of a linear function	compare graphical, algebraic and geometrical representations, including mapping diagrams, to explain the effect of: • rotating the line $y = mx + c$ through 90° about any point • reflecting the line $y = mx + c$ in the line $y = x$ derive properties of perpendicular lines and of the inverse function		
generate coordinate pairs that satisfy a simple linear rule; plot the graphs of simple linear functions, where y is given explicitly in terms of x, on paper and using ICT; recognise straight-line graphs parallel to the x-axis or y-axis	generate points in all four quadrants and plot the graphs of linear functions, where y is given explicitly in terms of x, on paper and using ICT; recognise that equations of the form $y = mx + c$ correspond to straight-line graphs	generate points and plot graphs of linear functions, where y is given implicitly in terms of x (e.g. $ay + bx = 0$, $y + bx + c = 0$), on paper and using ICT; find the gradient of lines given by equations of the form y = $mx + c$, given values for m and c	explore graphs of functions of the form $y = x^n$ (n an integer) and recognise their characteristic shapes; vary the values of a, b and c in functions such as $y = ax^2 + c$, $y = ax^3 + c$, $y = (x + b)^2$ using a graph plotter to explain how this transforms the graph	explore connections between the form of the equation and the resulting graphs of quadratic and cubic functions such as: • $y = (x + 2)(x – 5)$ • $y = (x – 2)(x^2 + 7x + 12)$ • $y = x^2 – 2x + 1$ • $y = x^3 + 3$ include features such as roots of the equation, intercepts and turning points	explore graphs of exponential and trigonometrical functions and recognise their characteristic shapes; apply to the graph $y = f(x)$ the transformations $y = f(x) + a$, $y = af(x)$, $y = f(x + a)$, $y = f(ax)$ for linear, quadratic, sine and cosine functions; use a graph plotter to explain the effect of transformations on the graph and generalise to other functions

3.2 Sequences, functions and graphs (continued)

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
plot and interpret the graphs of simple linear functions arising from real-life situations, e.g. conversion graphs	construct linear functions arising from real-life problems and plot their corresponding graphs; discuss and interpret graphs arising from real situations, e.g. distance–time graphs	construct functions arising from real-life problems and plot their corresponding graphs; interpret graphs arising from real situations, e.g. time series graphs	sketch and interpret graphs that model real-life situations, including those generated from other subjects such as science; use mathematical argument to justify features of their shapes	apply knowledge of mathematical functions to problems involving: ● optimisation, using numerical, algebraic and graphical, techniques, including maxima and minima ● using ICT to fit a curve to data from a real context such as a science experiment ● repeated proportional change, e.g. compound interest	set up a mathematical model of a real-life context or problem, identifying the variables and their functional relationship; use graphs and sketches to explain the behaviour of the variables and to explain or justify the effect of assumptions in the model
		use ICT to explore the graphical representation of algebraic equations and to interpret how properties of the graph are related to features of the equation, e.g. parallel and perpendicular lines			

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3.2 Sequences, functions and graphs (continued)

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
		interpret the meaning of various points and sections of straight-line graphs, including intercepts and intersections, e.g. solving simultaneous linear equations			

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4 Geometry and measures

4.1 Geometrical reasoning

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
use correctly the vocabulary, notation and labelling conventions for lines, angles and shapes		distinguish between conventions, definitions and derived properties	examine and refine arguments in solutions to geometrical problems, distinguishing between practical demonstration and proof; produce simple proofs	examine and create chains of deductive reasoning in solutions to more complex geometrical problems	present rigorous and sustained arguments in the solution of geometrical problems; construct formal geometrical proofs
identify parallel and perpendicular lines; know the sum of angles at a point, on a straight line and in a triangle; recognise vertically opposite angles	identify alternate angles and corresponding angles; understand a proof that: ● the angle sum of a triangle is 180° and of a quadrilateral is 360° ● the exterior angle of a triangle is equal to the sum of the two interior opposite angles	explain how to find, calculate and use: ● the sums of the interior and exterior angles of quadrilaterals, pentagons and hexagons ● the interior and exterior angles of regular polygons	use dynamic images to demonstrate invariant relationships between radii, chords and tangents in circles; develop arguments to explain and justify simple circle properties and theorems	examine and create proofs of the circle theorems; use circle theorems to solve problems	examine the points and lines used to create standard constructions and use the conditions of congruence to present a proof that the standard constructions are exact
		know the definition of a circle and the names of its parts; explain why inscribed regular polygons can be constructed by equal divisions of a circle			

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4.1 Geometrical reasoning (continued)

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
identify and use angle, side and symmetry properties of triangles and quadrilaterals; explore geometrical problems involving these properties, explaining reasoning orally, using step-by-step deduction supported by diagrams	solve geometrical problems using side and angle properties of equilateral, isosceles and right-angled triangles and special quadrilaterals, explaining reasoning with diagrams and text; classify quadrilaterals by their geometrical properties	solve problems using properties of angles, of parallel and intersecting lines and of triangles and other polygons, justifying inferences and explaining reasoning with diagrams and text	solve geometrical problems using properties of lines, angles, polygons and circles; justify arguments and solutions using deductive reasoning	formalise existing knowledge of lines, angles and polygons by: ● using the congruence conditions (SSS, SAS, RHS, ASA) to deduce familiar properties of triangles and quadrilaterals, e.g. an isosceles triangle has two equal angles ● explaining why standard constructions work, e.g. observing that lines joining points where compass arcs meet are sides of a rhombus	(*see objective above for progression*)
	know that if two 2-D shapes are congruent, corresponding sides and angles are equal	understand congruence and explore similarity	draw inferences about properties of similar 2-D shapes and use proportional reasoning to solve geometrical and trigonometrical problems		
		investigate Pythagoras' theorem, using a variety of media, through its historic and cultural roots, including 'picture' proofs	visualise and manipulate dynamic images and use scale drawing to investigate areas of squares on sides of right-angled and non right-angled triangles, relating findings to Pythagoras' theorem; use Pythagoras' theorem to solve problems in 2-D and simple 3-D cases	engage with and explain the stages of a variety of proofs of Pythagoras' theorem; use Pythagoras' theorem to solve more complex 3-D problems	present and justify a formal proof of Pythagoras' theorem

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4.1 Geometrical reasoning (continued)

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
use 2-D representations to visualise 3-D shapes and deduce some of their properties	visualise 3-D shapes from their nets; use geometrical properties of cuboids and shapes made from cuboids; use simple plans and elevations	visualise and use 2-D representations of 3-D objects; analyse 3-D shapes through 2-D projections, including plans and elevations	visualise and describe properties of points, lines and planes in 3-D space, including cross sections created by slicing a 3-D shape		
			visualise and manipulate images to establish trigonometrical relationships by: • generating triangles using a rotating unit radius (circle, centre the origin) • identifying the properties of similar triangles formed by enlargements of the circle use trigonometrical relationships to solve simple problems in 2-D, including bearings	derive the formula $\frac{1}{2}ab\sin C$ for the area of a triangle; use trigonometrical relationships to solve more complex 2-D problems and problems in 3-D, such as the angle between a line and a plane	draw, sketch and compare the graphs of trigonometrical functions and transformations of these graphs; prove the sine and cosine rules and use them to solve 2-D and 3-D problems in a range of contexts

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4.2 Transformations and coordinates

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
understand and use the language and notation associated with reflections, translations and rotations			use precise language and notation to describe and generalise the results of combining transformations of 2-D shapes on paper and using ICT, including: • rotations about any point • reflections in any line • translations using vector notation • a transformation and its inverse generate and analyse patterns, e.g. Islamic designs	explain and demonstrate graphically the effects of combining translations, using vector notation, including: • the rule for addition of vectors • scalar multiplication of a vector (repeated addition)	explain and demonstrate graphically the effects of combining translations, using vector notation, including: • the difference of two vectors • the resultant of two vectors • the commutative and associative properties of vector addition solve simple geometrical problems in 2-D using vectors
recognise and visualise the symmetries of a 2-D shape	identify all the symmetries of 2-D shapes	identify reflection symmetry in 3-D shapes			
transform 2-D shapes by: • reflecting in given mirror lines • rotating about a given point • translating	transform 2-D shapes by rotation, reflection and translation, on paper and using ICT	recognise that translations, rotations and reflections preserve length and angle, and map objects on to congruent images			
explore these transformations and symmetries using ICT	try out mathematical representations of simple combinations of these transformations	explore and compare mathematical representations of combinations of translations, rotations and reflections of 2-D shapes, on paper and using ICT			
		devise instructions for a computer to generate and transform shapes			

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4.2 Transformations and coordinates (continued)

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
	understand and use the language and notation associated with enlargement; enlarge 2-D shapes, given a centre of enlargement and a positive integer scale factor; explore enlargement using ICT	enlarge 2-D shapes, given a centre of enlargement and a positive integer scale factor, on paper and using ICT; identify the scale factor of an enlargement as the ratio of the lengths of any two corresponding line segments; recognise that enlargements preserve angle but not length, and understand the implications of enlargement for perimeter	enlarge 2-D shapes using positive, fractional and negative scale factors, on paper and using ICT; use reciprocals as a multiplicative inverse in the context of enlargement; recognise the similarity of resulting shapes and explain the effect of enlargement on perimeter	enlarge 3-D shapes; identify and explain the effects of enlargement on areas and volumes of similar shapes and solids; relate this understanding to practical contexts, e.g. in biology	
	make scale drawings	use and interpret maps and scale drawings in the context of mathematics and other subjects	apply the properties of similar triangles and Pythagoras' theorem to solving problems presented on a 2-D coordinate grid; use a 3-D coordinate grid to represent simple shapes		
use conventions and notation for 2-D coordinates in all four quadrants; find coordinates of points determined by geometrical information	find the midpoint of the line segment AB, given the coordinates of points A and B	use the coordinate grid to solve problems involving translations, rotations, reflections and enlargements			

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4.3 Construction and loci

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
use a ruler and protractor to: • measure and draw lines to the nearest millimetre and angles, including reflex angles, to the nearest degree • construct a triangle, given two sides and the included angle (SAS) or two angles and the included side (ASA)	use straight edge and compasses to construct: • the midpoint and perpendicular bisector of a line segment • the bisector of an angle • the perpendicular from a point to a line • the perpendicular from a point on a line • a triangle, given three sides (SSS)	use straight edge and compasses to construct triangles, given right angle, hypotenuse and side (RHS)	use properties of 2-D and 3-D shapes to make accurate constructions on paper and using ICT; including constructing triangles from combinations of side and angle facts, reviewing and generalising findings to identify which of these conditions define unique constructions		
use ICT to explore constructions	use ICT to explore these constructions	use ICT to explore constructions of triangles and other 2-D shapes			
use ruler and protractor to construct simple nets of 3-D shapes, e.g. cuboid, regular tetrahedron, square-based pyramid, triangular prism	find simple loci, both by reasoning and by using ICT, to produce shapes and paths, e.g. an equilateral triangle	find the locus of a point that moves according to a simple rule, both by reasoning and by using ICT	visualise and describe the locus of a point that moves according to a more complex rule; explain the path using accurate geometrical vocabulary and notation and use a variety of media, including dynamic geometry software, sketches and graphs	create a chain of reasoning to deduce the equation of a circle by applying Pythagoras' theorem to the locus of a point	

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4.4 Measures and mensuration

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
choose and use units of measurement to measure, estimate, calculate and solve problems in everyday contexts; convert one metric unit to another, e.g. grams to kilograms; read and interpret scales on a range of measuring instruments	choose and use units of measurement to measure, estimate, calculate and solve problems in a range of contexts; know rough metric equivalents of imperial measures in common use, such as miles, pounds (lb) and pints	solve problems involving measurements in a variety of contexts; convert between area measures (e.g. mm^2 to cm^2, cm^2 to m^2, and vice versa) and between volume measures (e.g. mm^3 to cm^3, cm^3 to m^3, and vice versa)	interpret and use compound measures, including from other subjects and real life; solve problems involving rates; convert between compound measures, choosing units most suited to the solution	make connections between the continuity of the number line and continuous measures; critically examine the measurements used in a problem and their effect on the accuracy of the solution, e.g. understand how errors can be compounded	communicate the solution to a problem involving measurement, explaining the limitations of accuracy using upper and lower bounds
distinguish between and estimate the size of acute, obtuse and reflex angles	use bearings to specify direction				
		Interpret and explore combining measures into rates of change in everyday contexts (e.g. km per hour, pence per metre); use compound measures to compare in real-life contexts (e.g. travel graphs and value for money), using ICT as appropriate.			

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4.4 Measures and mensuration (continued)

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
know and use the formula for the area of a rectangle; calculate the perimeter and area of shapes made from rectangles	derive and use formulae for the area of a triangle, parallelogram and trapezium; calculate areas of compound shapes	know and use the formulae for the circumference and area of a circle	present a concise reasoned argument to derive formulae for: • lengths of circular arcs • areas of sectors of a circle • surface area of a cylinder • volume of a cylinder solve problems involving the use of these formulae	present a concise reasoned argument when deriving formulae for the surface areas of pyramids and cones; explore connections between: • formulae for the volume of a pyramid and the related cuboid • formulae for the surface area and volume of a sphere and the circumscribed and inscribed cubes solve problems involving the use of these formulae	solve problems involving more complex shapes and solids, including segments of circles and frustums of cones
calculate the surface area of cubes and cuboids	know and use the formula for the volume of a cuboid; calculate volumes and surface areas of cuboids and shapes made from cuboids	calculate the surface area and volume of right prisms			

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5 Statistics

5.1 Specifying a problem, planning and collecting

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
suggest possible answers, given a question that can be addressed by statistical methods	discuss a problem that can be addressed by statistical methods and identify related questions to explore	suggest a problem to explore using statistical methods, frame questions and raise conjectures	independently devise a suitable plan for a more complex statistical project, selecting suitable hypotheses to address the problem	evaluate possible difficulties with planned approaches; adjust the project plan accordingly, including reconsidering hypotheses	
decide which data would be relevant to an enquiry and possible sources	decide which data to collect to answer a question and the degree of accuracy needed; identify possible sources; consider appropriate sample size	discuss how different sets of data relate to the problem; identify possible primary or secondary sources; determine the sample size and most appropriate degree of accuracy	justify the sampling method selected, identify possible sources of bias and plan how to minimise it	identify practical problems such as non-response or missing data and refine approaches to minimise their impact on the validity of the results	

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5.1 Specifying a problem, planning and collecting (continued)

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
plan how to collect and organise small sets of data from surveys and experiments: ● design data collection sheets or questionnaires to use in a simple survey ● construct frequency tables for gathering discrete data, grouped where appropriate in equal class intervals	plan how to collect the data; construct frequency tables with equal class intervals for gathering continuous data and two-way tables for recording discrete data	design a survey or experiment to capture the necessary data from one or more sources; design, trial and if necessary refine data collection sheets; construct tables for gathering large discrete and continuous sets of raw data, choosing suitable class intervals; design and use two-way tables	decide on the best methods for testing the hypotheses; select, justify and use the data-gathering technique most appropriate to the context, deciding between a range of sources: primary (observation, controlled experiment, data logging) and secondary (spreadsheet data, printed tables, lists)	select, justify and use the data-gathering technique appropriate to complex and unfamiliar problems, identifying potential barriers and limitations; identify what extra information may be required to pursue a further line of enquiry	select and critically evaluate a sampling scheme and a method to investigate a population, including random and stratified sampling; explain the effect on reliability and validity
collect small sets of data from surveys and experiments, as planned	collect data using a suitable method (e.g. observation, controlled experiment, data logging using ICT)	gather data from specified secondary sources, including printed tables and lists, and ICT-based sources, including the internet			

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5.2 Processing and representing data

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
calculate statistics for small sets of discrete data: ● find the mode, median and range, and the modal class for grouped data ● calculate the mean, including from a simple frequency table, using a calculator for a larger number of items	calculate statistics for sets of discrete and continuous data, including with a calculator and spreadsheet; recognise when it is appropriate to use the range, mean, median and mode and, for grouped data, the modal class	calculate statistics and select those most appropriate to the problem or which address the questions posed	use an appropriate range of statistical methods to explore and summarise large data sets, justifying the choices made; include grouping data, estimating and finding the mean, median, quartiles and interquartile range	process data drawn from problems involving seasonality and trends in a time series; choose and combine statistical methods to analyse the problem, including moving averages	

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5.2 Processing and representing data (continued)

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
construct, on paper and using ICT, graphs and diagrams to represent data, including: • bar-line graphs • frequency diagrams for grouped discrete data • simple pie charts	construct graphical representations, on paper and using ICT, and identify which are most useful in the context of the problem, including: • pie charts for categorical data • bar charts and frequency diagrams for discrete and continuous data • simple line graphs for time series • simple scatter graphs • stem-and-leaf diagrams	select, construct and modify, on paper and using ICT, suitable graphical representations to progress an enquiry and identify key features present in the data. Include: • line graphs for time series • scatter graphs to develop further understanding of correlation	construct on paper and using ICT suitable graphical representations, including: • histograms for grouped continuous data with equal class intervals • cumulative frequency tables and diagrams • box plots • scatter graphs and lines of best fit (by eye) justify their suitability with reference to the context of the problem and the audience	choose and combine suitable graphical representations to progress an unfamiliar or non-routine enquiry, including histograms with equal or unequal class intervals	use precise and consistent graphical representation to progress an unfamiliar and non-routine enquiry
		work through the entire handling data cycle to explore relationships within bi-variate data, including applications to global citizenship, e.g. how fair is our society?			

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5.3 Interpreting and discussing results

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
interpret diagrams and graphs (including pie charts) and draw simple conclusions based on the shape of graphs and simple statistics for a single distribution	interpret tables, graphs and diagrams for discrete and continuous data, relating summary statistics and findings to the questions being explored	interpret graphs and diagrams and make inferences to support or cast doubt on initial conjectures; have a basic understanding of correlation	find patterns and exceptions and explain anomalies; including interpretation of social statistics and evaluation of the strength of association within bi-variate data (correlation, lines of best fit)	interpret and compare distributions, including cumulative frequency diagrams; make and discuss inferences, using the shape of the distributions and measures of average and spread, including median and quartiles	explain and justify assumptions and constraints; include interpretation and comparison of histograms with unequal class intervals
compare two simple distributions using the range and one of the mode, median or mean	compare two distributions using the range and one or more of the mode, median and mean	compare two or more distributions and make inferences, using the shape of the distributions and appropriate statistics			
write a short report of a statistical enquiry, including appropriate diagrams, graphs and charts, using ICT as appropriate; justify the choice of presentation	write about and discuss the results of a statistical enquiry using ICT as appropriate; justify the methods used	review interpretations and results of a statistical enquiry on the basis of discussions; communicate these interpretations and results using selected tables, graphs and diagrams	evaluate the results of a statistical enquiry; review and justify or refine the choice of statistical representations and relate summarised data to the questions being explored	critically examine strategies adopted and arguments presented, relating them to the original hypotheses; recognise the limitations of any assumptions and the effects that varying assumptions could have on conclusions drawn from data analysis	use statistical analysis effectively in presenting convincing conclusions; critically reflect on own lines of enquiry; search for and appreciate more elegant forms of communicating conclusions

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5.4 Probability

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
use vocabulary and ideas of probability, drawing on experience	interpret the results of an experiment using the language of probability; appreciate that random processes are unpredictable	interpret results involving uncertainty and prediction	identify when the events in a problem are mutually exclusive or independent; use and interpret tree diagrams to represent outcomes of combined events and to inform the calculation of their probabilities; decide when to add and when to multiply probabilities	interpret the effect on probability of contexts involving selection with and without replacement; choose and combine representations to communicate probabilities as part of a solution to a problem	recognise when and how to work with probabilities associated with independent and mutually exclusive events when interpreting data
understand and use the probability scale from 0 to 1; find and justify probabilities based on equally likely outcomes in simple contexts; identify all the possible mutually exclusive outcomes of a single event	know that if the probability of an event occurring is p then the probability of it not occurring is $1 - p$; use diagrams and tables to record in a systematic way all possible mutually exclusive outcomes for single events and for two successive events	identify all the mutually exclusive outcomes of an experiment; know that the sum of probabilities of all mutually exclusive outcomes is 1 and use this when solving problems			

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5.4 Probability (continued)

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
estimate probabilities by collecting data from a simple experiment and recording it in a frequency table; compare experimental and theoretical probabilities in simple contexts	compare estimated experimental probabilities with theoretical probabilities, recognising that: • if an experiment is repeated the outcome may, and usually will, be different • increasing the number of times an experiment is repeated generally leads to better estimates of probability	compare experimental and theoretical probabilities in a range of contexts; appreciate the difference between mathematical explanation and experimental evidence	use relative frequency as an estimate of probability, including simulation using ICT to generate larger samples; discuss its reliability based on sample size and use to interpret and compare outcomes of experiments	explore a relevant and purposeful problem involving uncertainty; estimate risk by modelling real events through simulation; justify decisions based on experimental probability and comment on the effect of assumptions and sample size on the reliability of conclusions	

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