Pupils should be taught to:

Find the nth term, justifying its form by referring to the context in which it was generated

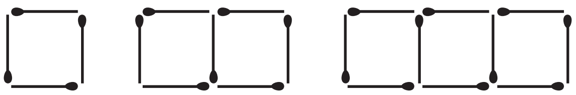
As outcomes, Year 7 pupils should, for example:

Generate sequences from simple practical contexts.

- Find the first few terms of the sequence.
- Describe how it continues by reference to the context.
- Begin to describe the general term, first using words, then symbols; justify the generalisation by referring to the context.

For example:

- Growing matchstick squares

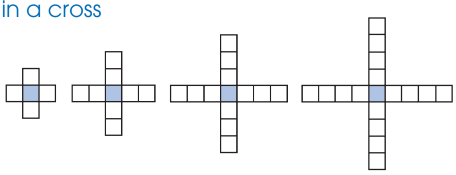


Number of squares	1	2	3	4	…
Number of matchsticks	4	7	10	13	…

Justify the pattern by explaining that the first square needs 4 matches, then 3 matches for each additional square, or you need 3 matches for every square plus an extra one for the first square.

In the nth arrangement there are 3n +1 matches.
Possible justification:
Every square needs three matches, plus one more for the first square, which needs four. For n squares, the number of matches needed is 3n, plus one for the first square.

- Squares in a cross



Size of cross	1	2	3	4	…
Number of squares	5	9	13	17	…

Justify the pattern by explaining that the first cross needs 5 squares, then 4 extra for the next cross, one on each 'arm', or start with the middle square, add 4 squares to make the first pattern, 4 more to make the second, and so on.

In the nth cross there are 4n + 1 squares.

Possible justification:
The crosses have four 'arms', each increasing by one square in the next arrangement. So the nth cross has 4n squares in the arms, plus one in the centre.

- Making different amounts using 1p, 2p and 3p stamps

Amount	1p	2p	3p	4p	5p	6p	…
No. of ways	1	2	3	4	5	7	…

From experience of practical examples, begin to appreciate that some sequences do not continue in an 'obvious' way and simply 'spotting a pattern' may lead to incorrect results.

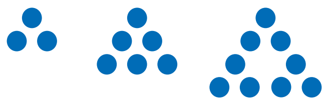
As outcomes, Year 8 pupils should, for example:

Generate sequences from practical contexts.

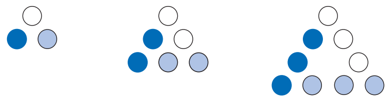
- Find the first few terms of the sequence; describe how it continues using a term-to-term rule.
- Describe the general (nth) term, and justify the generalisation by referring to the context.
- When appropriate, compare different ways of arriving at the generalisation.

For example:

- Growing triangles



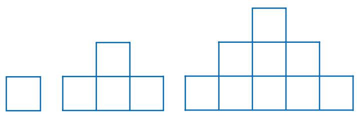
This generates the sequence: 3, 6, 9…
Possible explanations:
We add three each time because we add one more dot to each side of the triangle to make the next triangle.
It's the 3 times table because we get…



3 lots of 1   3 lots of 2   3 lots of 3   etc.

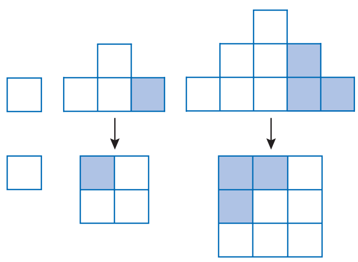
The general (nth) term is 3 × n or 3n.
Possible justification:
This follows because the 10th term would be '3 lots of 10'.

- 'Pyramid' of squares



This generates the sequence: 1, 4, 9, …
Possible explanation:
The next 'pyramid' has another layer, each time increasing by the next odd number 3, 5, 7, …

The general (nth) term is n × n or n^2.
Possible justification:
The pattern gives square numbers. Each 'pyramid' can be rearranged into a square pattern, as here:



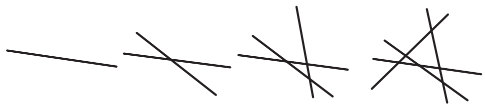
As outcomes, Year 9 pupils should, for example:

Generate sequences from practical contexts.

- Find the first few terms of the sequence; describe how it continues using a term-to-term rule.
- Use algebraic expressions to describe the nth term, justifying them by referring to the context.
- When appropriate, compare different ways of arriving at the generalisation.

For example:

- Maximum crossings for a given number of lines

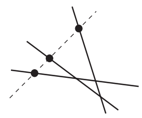


Number of lines	1	2	3	4	…
Maximum crossings	0	1	3	6	…

Predict how the sequence might continue and test for several more terms.

Discuss and follow an explanation, such as:

A new line must cross all existing lines.

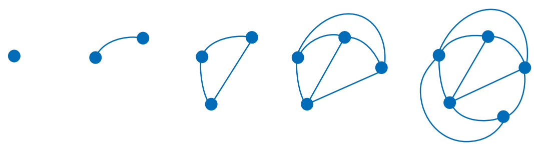


So when a new line is added, the number of extra crossings will equal the existing number of lines, e.g. when there is one line, an extra line adds one crossing, when there are two lines, an extra line adds two crossings, and so on.

No. of lines	1	2	3	4	5	…
Max. crossings	0	1	3	6	10	…
Increase		1	2	3	4	…

- Joining points to every other point

Joins may be curved or straight. Keeping to the rule that lines are not allowed to cross, what is the maximum number of joins that can be made?



No. of points	1	2	3	4	5	…
Maximum joins	0	1	3	6	9	…

Predict how the sequence might continue, try to draw it, discuss and provide an explanation.

Pupils should be taught to:

Find the nth term, justifying its form by referring to the context in which it was generated (continued)

As outcomes, Year 7 pupils should, for example:

Begin to find a simple rule for the nth term of some simple sequences.

For example, express in words the nth term of counting sequences such as:

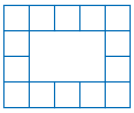
• 6, 12, 18, 24, 30, …	nth term is six times n
• 6, 11, 16, 21, 26, …	nth term is five times n plus one
• 9, 19, 29, 39, 49, …	nth term is ten times n minus one
• 40, 30, 20, 10, 0, …	nth term is fifty minus ten times n

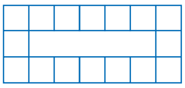
Link to generating sequences using term-to-term and position-to-term definitions (pages 148–51).

As outcomes, Year 8 pupils should, for example:

- Paving stones

  Investigate the number of square concrete slabs that will surround rectangular ponds of different sizes. Try some examples:





2 by 3 pond1 by 5 pond
needs 14 slabs needs 16 slabs

Collect a range of different sizes and count the slabs. Deduce that, for an l by w rectangle, you need $2l + 2w + 4$ slabs.
Possible justification:
You always need one in each corner, so it's twice the length plus twice the width, plus the four in the corners.

Other ways of counting could lead to equivalent forms, such as $2(l + 2) + 2w$ or $2(l + 1) + 2(w + 1)$.

Confirm that these formulae give consistent values for the first few terms.

Check predicted terms for correctness.

Link to simplifying and transforming algebraic expressions (pages 116–19).

Begin to use linear expressions to describe the nth term of an arithmetic sequence, justifying its form by referring to the activity from which it was generated.

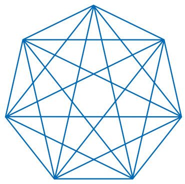
Develop an expression for the nth term of sequences such as:

- 7, 12, 17, 22, 27, … $2 + 5n$
- 100, 115, 130, 145, 160, … $15n + 85$
- 2.5, 4.5, 6.5, 8.5, 10.5, … $(4n + 1)/2$
- $^{-}12$, $^{-}7$, $^{-}2$, 3, 8, … $5n - 17$
- 4, $^{-}2$, $^{-}8$, $^{-}14$, $^{-}20$, … $10 - 6n$

As outcomes, Year 9 pupils should, for example:

- Diagonals of polygons

  How many diagonals does a polygon have altogether (crossings allowed)?



No. of sides	1	2	3	4	5	6	7	…
No. of diagonals	–	–	0	2	5	9	14	…

Explain that, starting with 3 sides, the terms increase by 2, 3, 4, 5, …

Follow an explanation to derive a formula for the nth term, such as:
Any vertex can be joined to all the other vertices, except the two adjacent ones.
In an n-sided polygon this gives $n - 3$ diagonals, and for n vertices a total of $n(n - 3)$ diagonals. But each diagonal can be drawn from either end, so this formula counts each one twice.
So the number of diagonals in an n-sided polygon is $\frac{1}{2}n(n - 3)$.

Confirm that this formula gives consistent results for the first few terms.

Know that for sequences generated from an activity or context:
- Predicted values of terms need to be checked for correctness.
- A term-to-term or position-to-term rule needs to be justified by a mathematical explanation derived from consideration of the context.

Use linear expressions to describe the nth term of an arithmetic sequence, justifying its form by referring to the activity or context from which it was generated.

Find the nth term of any linear (arithmetic) sequence. For example:

- Find the nth term of 21, 27, 33, 39, 45, …

  The difference between successive terms is 6, so the nth term is of the form $T(n) = 6n + b$.
  $T(1) = 21$, so $6 + b = 21$, leading to $b = 15$.
  $T(n) = 6n + 15$
  Check by testing a few terms.

- Find the nth term of these sequences:
  54, 62, 70, 78, 86, …
  68, 61, 54, 47, 40, …
  2.3, 2.5, 2.7, 2.9, 3.1, …
  $^{-}5$, $^{-}14$, $^{-}23$, $^{-}32$, $^{-}41$, …