

NUMBERS AND THE NUMBER SYSTEM

Pupils should be taught to:

Recognise squares and cubes, and the corresponding roots; use index notation and simple instances of the index laws

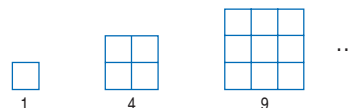
As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
property, consecutive, classify...
square number, squared, square root... triangular number...
the notation 6^2 as six squared and the square root sign $\sqrt{\quad}$.

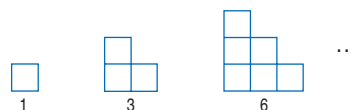
Use **index notation** to write squares such as 2^2 , 3^2 , 4^2 , ...

Recognise:

- **squares** of numbers 1 to 12 and the corresponding **roots**;

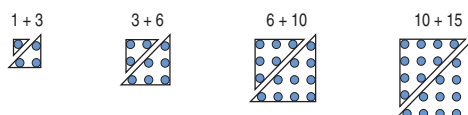


- **triangular numbers**: 1, 3, 6, 10, 15, ...



Work out the values of squares such as 15^2 , 21^2 .

Investigate the relationship between square numbers and triangular numbers, using interlocking cubes or pegboard.



Link to generating sequences from practical contexts (pages 146–7).

Square roots

Find **square roots** of multiples of 100 and 10 000 by factorising. For example, find:

- $\sqrt{900} = \sqrt{(9 \times 100)}$
 $= \sqrt{9} \times \sqrt{100} = 3 \times 10 = 30$
- $\sqrt{160000} = \sqrt{(16 \times 100 \times 100)}$
 $= \sqrt{16} \times \sqrt{100} \times \sqrt{100} = 4 \times 10 \times 10 = 400$

Use a **calculator**, including the square root key, to find square roots, rounding as appropriate.

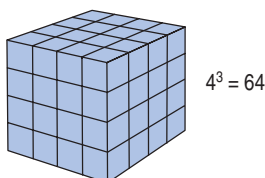
Recognise that squaring and finding the square root are the inverse of each other.

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to: cube number, cubed, cube root... power... the notation 6^3 as six cubed, and 6^4 as six to the power 4... and the cube root sign $\sqrt[3]{\quad}$.

Use index notation to write cubes and small positive integer powers of 10.

Know **cubes** of 1, 2, 3, 4, 5 and 10 and the corresponding **roots**.



Work out the values of cubes, such as 6^3 , $^{-}9^3$, $(0.1)^3$.

Know that $100 = 10 \times 10 = 10^2$, and that successive powers of 10 (10 , 10^2 , 10^3 , ...) underpin decimal (base 10) notation. For example:

- 1 thousand is 10^3 ;
- 10 thousand is 10^4 ;
- 1 million is 10^6 ;
- 1 billion is 10^9 (one thousand millions).

Link to place value (pages 36–7), prime factor decomposition of a number and tree diagrams (pages 52–3), and generating sequences from practical contexts (pages 146–7).

Squares, cubes and square roots

Know that a positive integer has two square roots, one positive and one negative; by convention the square root sign $\sqrt{\quad}$ denotes the positive square root.

Find square roots by factorising, for example:
 $\sqrt{196} = \sqrt{(4 \times 49)} = 2 \times 7 = 14$

Find an upper and lower bound for a square root by comparing with the roots of two consecutive square numbers:

$$\sqrt{4} < \sqrt{7} < \sqrt{9} \quad \text{so} \quad 2 < \sqrt{7} < 3$$

Use a **calculator** to find cubes, squares and estimate square roots, including using the square root key. For example:

- Find the square root of 12.

$3^2 = 9$ (3 to the power 2)
 $4^2 = 16$
 so the square root of 12 lies between 3 and 4.

3^2	9
4^2	16
3.5^2	12.25
3.4^2	

Try 3.5, and so on.

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to: index, indices, index notation, index law...

Use index notation for small integer powers. For example:

- $19^3 = 6859$ $6^5 = 7776$ $14^4 = 38416$

Know that $x^0 = 1$, for all values of x .

Know that:

$$10^{-1} = \frac{1}{10} = 0.1 \qquad 10^{-2} = \frac{1}{100} = 0.01$$

Know how to use the x^y key on a **calculator** to calculate powers.

Recognise applications of indices in biology, where cells and organisms grow by doubling, giving rise to the powers of 2.

Link to writing numbers in standard form (pages 38–9).

Square roots and cube roots

Know that:

- $\sqrt{a} + \sqrt{b} \neq \sqrt{(a + b)}$

Know that:

- there are two square roots of a positive integer, one positive and one negative, written as $\pm\sqrt{\quad}$;
- the cube root of a positive number is positive and the cube root of a negative number is negative.

Use **ICT** to estimate square roots or cube roots to the required number of decimal places. For example:

- Estimate the solution of $x^2 = 70$.

The positive value of x lies between 8 and 9, since $8^2 = 64$ and $9^2 = 81$.

Try numbers from 8.1 to 8.9 to find a first approximation lying between 8.3 and 8.4. Next try numbers from 8.30 to 8.40.

Link to using trial and improvement and ICT to find approximate solutions to equations (pages 132–5).

NUMBERS AND THE NUMBER SYSTEM

Pupils should be taught to:

Recognise squares and cubes, and the corresponding roots; use index notation and simple instances of the index laws (continued)

As outcomes, Year 7 pupils should, for example:

Investigate problems such as:

- Without using a calculator, find a number that when multiplied by itself gives 2304.
- Describe the pattern formed by the last digits of square numbers. Do any numbers not appear as the last digits? Could 413 be a square number? Or 517?
- Can every square number up to 12×12 be expressed as the sum of two prime numbers?
- Some triangular numbers are equal to the sum of two other triangular numbers. Find some examples.

See Y456 examples (pages 20–1).

As outcomes, Year 8 pupils should, for example:

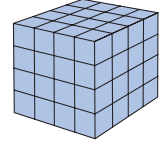
Investigate problems such as:

- Using a **calculator**, find two consecutive numbers with a product of 7482.
- Some numbers are equal to the sum of two squares: for example, $34 = 3^2 + 5^2$. Which numbers less than 100 are equal to the sum of two squares? Which can be expressed as the sum of two squares in at least two different ways?
- What are the 20 whole numbers up to 30 that can be written as the difference of two squares?
- Find the smallest number that can be expressed as the sum of two cubes in two different ways.
- What are the three smallest numbers that are both triangular and square?

As outcomes, Year 9 pupils should, for example:

Investigate problems such as:

- Estimate the cube root of 20.
- The outside of a cube made from smaller cubes is painted blue. How many small cubes have 0, 1, 2 or 3 faces painted blue? Investigate.
- Three integers, each less than 100, fit the equation $a^2 + b^2 = c^2$. What could the integers be?



Link to Pythagoras' theorem (pages 186–9); graphs of quadratic and cubic functions (pages 170–1).

Use simple instances of the index laws and start to multiply and divide numbers in index form.

Recognise that:

- indices are added when multiplying, e.g.

$$4^3 \times 4^2 = (4 \times 4 \times 4) \times (4 \times 4)$$

$$= 4 \times 4 \times 4 \times 4 \times 4$$

$$= 4^5 = 4^{(3+2)}$$
- indices are subtracted when dividing, e.g.

$$4^5 \div 4^2 = (4 \times 4 \times 4 \times 4 \times 4) \div (4 \times 4)$$

$$= 4 \times 4 \times 4$$

$$= 4^3 = 4^{(5-2)}$$
- $4^2 \div 4^5 = 4^{(2-5)} = 4^{-3}$
- $7^5 \div 7^5 = 7^0 = 1$

Generalise to algebra. Apply simple instances of the index laws (small integral powers), as in:

- $n^2 \times n^3 = n^{2+3} = n^5$
- $p^3 \div p^2 = p^{3-2} = p$

Know and use the general forms of the index laws for multiplication and division of integer powers.

$$p^a \times p^b = p^{a+b}, \quad p^a \div p^b = p^{a-b}, \quad (p^a)^b = p^{ab}$$

Begin to extend understanding of index notation to negative and fractional powers; recognise that the index laws can be applied to these as well.

2^{-4}	2^{-3}	2^{-2}	2^{-1}	2^0	2^1	2^2	2^3	2^4
$\frac{1}{2^4} = \frac{1}{16}$	$\frac{1}{2^3} = \frac{1}{8}$	$\frac{1}{2^2} = \frac{1}{4}$	$\frac{1}{2^1} = \frac{1}{2}$	1	2	4	8	16

Know the notation $5^{1/2} = \sqrt{5}$ and $5^{1/3} = \sqrt[3]{5}$.

Extend to simple surds (unresolved roots):

- $\sqrt{3} \times \sqrt{3} = 3$
- $\sqrt{3} \times \sqrt{3} \times \sqrt{3} = 3\sqrt{3}$
- $\sqrt{32} = \sqrt{(2 \times 16)} = \sqrt{2} \times \sqrt{16} = 4\sqrt{2}$
- Can a square have an exact area of 32 cm^2 ? If so, what is its exact perimeter?