

NUMBERS AND THE NUMBER SYSTEM

Pupils should be taught to:

Recognise squares and cubes, and the corresponding roots; use index notation and simple instances of the index laws (continued)

As outcomes, Year 7 pupils should, for example:

Investigate problems such as:

- Without using a calculator, find a number that when multiplied by itself gives 2304.
- Describe the pattern formed by the last digits of square numbers. Do any numbers not appear as the last digits? Could 413 be a square number? Or 517?
- Can every square number up to 12×12 be expressed as the sum of two prime numbers?
- Some triangular numbers are equal to the sum of two other triangular numbers. Find some examples.

See Y456 examples (pages 20–1).

As outcomes, Year 8 pupils should, for example:

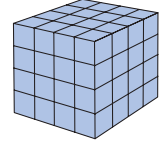
Investigate problems such as:

- Using a **calculator**, find two consecutive numbers with a product of 7482.
- Some numbers are equal to the sum of two squares: for example, $34 = 3^2 + 5^2$. Which numbers less than 100 are equal to the sum of two squares? Which can be expressed as the sum of two squares in at least two different ways?
- What are the 20 whole numbers up to 30 that can be written as the difference of two squares?
- Find the smallest number that can be expressed as the sum of two cubes in two different ways.
- What are the three smallest numbers that are both triangular and square?

As outcomes, Year 9 pupils should, for example:

Investigate problems such as:

- Estimate the cube root of 20.
- The outside of a cube made from smaller cubes is painted blue. How many small cubes have 0, 1, 2 or 3 faces painted blue? Investigate.
- Three integers, each less than 100, fit the equation $a^2 + b^2 = c^2$. What could the integers be?



Link to Pythagoras' theorem (pages 186–9); graphs of quadratic and cubic functions (pages 170–1).

Use simple instances of the index laws and start to multiply and divide numbers in index form.

Recognise that:

- indices are added when multiplying, e.g.
 $4^3 \times 4^2 = (4 \times 4 \times 4) \times (4 \times 4)$
 $= 4 \times 4 \times 4 \times 4 \times 4$
 $= 4^5 = 4^{(3+2)}$
- indices are subtracted when dividing, e.g.
 $4^5 \div 4^2 = (4 \times 4 \times 4 \times 4 \times 4) \div (4 \times 4)$
 $= 4 \times 4 \times 4$
 $= 4^3 = 4^{(5-2)}$
- $4^2 \div 4^5 = 4^{(2-5)} = 4^{-3}$
- $7^5 \div 7^5 = 7^0 = 1$

Generalise to algebra. Apply simple instances of the index laws (small integral powers), as in:

- $n^2 \times n^3 = n^{2+3} = n^5$
- $p^3 \div p^2 = p^{3-2} = p$

Know and use the general forms of the index laws for multiplication and division of integer powers.

$$p^a \times p^b = p^{a+b}, \quad p^a \div p^b = p^{a-b}, \quad (p^a)^b = p^{ab}$$

Begin to extend understanding of index notation to negative and fractional powers; recognise that the index laws can be applied to these as well.

2^{-4}	2^{-3}	2^{-2}	2^{-1}	2^0	2^1	2^2	2^3	2^4
$\frac{1}{2^4} = \frac{1}{16}$	$\frac{1}{2^3} = \frac{1}{8}$	$\frac{1}{2^2} = \frac{1}{4}$	$\frac{1}{2^1} = \frac{1}{2}$	1	2	4	8	16

Know the notation $5^{1/2} = \sqrt{5}$ and $5^{1/3} = \sqrt[3]{5}$.

Extend to simple surds (unresolved roots):

- $\sqrt{3} \times \sqrt{3} = 3$
- $\sqrt{3} \times \sqrt{3} \times \sqrt{3} = 3\sqrt{3}$
- $\sqrt{32} = \sqrt{(2 \times 16)} = \sqrt{2} \times \sqrt{16} = 4\sqrt{2}$
- Can a square have an exact area of 32 cm^2 ? If so, what is its exact perimeter?