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# Using a basic calculator

Calculators are powerful tools and, as with all tools, pupils need to learn how to use them properly. You may want to stress to the target pupils that it is not appropriate to use a calculator for calculations that can more quickly and reliably be carried out mentally. It is also important for them to learn how to use a calculator efficiently, including how to use calculator functions such as the square root key or memory.

To help pupils to judge when it is and when it is not sensible to use a calculator, it is useful to look from time to time at a mixed set of calculations presented on an overhead projector transparency and to discuss which of the calculations can be done mentally and which need a calculator.

You may need to control the use of calculators in mathematics lessons by distinguishing the times when pupils **must not** use a calculator, the times when they **must** use one, and the times when they **may choose** whether or not to use one. All three types of sessions have a place in the learning programme.

# Calculator usage

The advantages of using calculators in mathematics lessons have been well rehearsed. For example, a calculator makes a useful teaching aid to help pupils to learn about numbers and the number system. A calculator designed for the overhead projector is particularly effective for generating discussion, in either a whole-class or group setting.

Calculators can also stimulate problem solving in mathematics and give opportunities for developing mathematical concepts that would otherwise be inaccessible to the pupils. A calculator allows them to work with real data, without the need for artificial simplifications. In this way, the calculator can open up a range of real problems that might arise in mathematics or in other subjects, such as science or geography.

Nevertheless, however a calculator is used, it does not do the strategic thinking for the user. It is important, before any step in a calculation is carried out, to decide what operation is appropriate. Equally important is the ability to interpret the display and to ask whether the result makes sense. Whenever pupils use a calculator, they should be encouraged to ask themselves questions such as:

- 'Roughly, what size of answer should I expect?'
- 'What numbers do I need to enter?'
- 'Do I have to add, subtract, multiply or divide these numbers?'
- 'In what order do I key in the numbers and operations?'
- 'How do I interpret the display?'
- 'Does the result make sense?'

The three important roles of the calculator are using it to:

- · work with real data and awkward numbers
- · explore and gain understanding of mathematical ideas and patterns
- promote mental calculation skills.

These uses are discussed in more detail in the following pages.

## **Calculator skills**

Pupils need to be taught the technical skills of using a basic calculator. The target pupils, in particular, will not necessarily have these skills and some teaching time needs to be planned when the skills will be developed and practised. Pupils should know how to:

- clear the display before starting a calculation
- use the [+], [-], [x] and [÷] keys, the [=] key and decimal point to calculate with realistic data
- change an accidental wrong entry by using the [clear entry] key
- recognise a negative number output and use the [sign change] key where appropriate
- key in and interpret money calculations, for example:
   key in £4.35 + £3.85 as 4.35 [+] 3.85 [=], and interpret the result 8.2 as £8.20
   key in £6.30 + 85p as 6.3 [+] 0.85 [=], recognising that '0.' signals no pounds and only pence (alternatively, change money to pence and divide final answer by 100 to convert back to pounds)
- key in and interpret measurements of time, for example key in 4 hours 15 minutes as 4.25 hours (alternatively, change hours to minutes and work in minutes)
- find whole-number remainders after division with a calculator, for example be able to change 1000 minutes to hours and minutes
- interpret decimals, for example, know that a number such as 81.75 lies between 81 and 82
- interpret a rounding error, if it appears (eg interpret 6.9999999 as 7)
- key in fractions, recognise the equivalent decimal form, and use this to compare and order fractions
- read the display of, say, 0.3333333 as 'point three recurring', know that it represents one third, and that 0.6666666 represents two thirds
- use the square and square root keys
- select the correct key sequence to carry out calculations involving more than one step, for example  $8 \times (37 + 58)$
- have a feel for the approximate size of an answer, and check it appropriately, for example: performing the inverse calculation or by clearing and repeating the calculation.

When they progress to using a scientific calculator, pupils will later need to be taught how to:

- use the memory and select the correct key sequence to carry out calculations involving more than one operation including brackets, for example (23 + 41) × (87 + 48)
- use the fraction key
- use the relevant keys to find powers and roots
- use the  $\pi$  key.

# Using a calculator for real data and awkward numbers

Enquiry into problems of genuine interest to pupils may involve calculations that are too awkward to deal with mentally. A mathematics programme in which all the numbers or measurements are simple enough to be dealt with mentally or with pencil and paper creates a limited view of mathematics and fails to give pupils an understanding of how mathematics is used in the real world.

Real contexts require a decision about which mathematical operations are necessary for the task. Examples in which pupils are directed, for example, to add or divide two given numbers offer practice, but not in the decision-making skills needed to solve real problems. When the development of these skills is the main objective of a lesson, it is appropriate to use a calculator. Pupils are then less likely to be distracted by any difficulties they may have in carrying out calculations.

A calculator will not interpret the display that arises from a calculation. If a pupil needs, say, to find how many coaches, each with a capacity of 54 people, will be needed to take 347 pupils on an outing, they may begin by using a calculator to divide 347 by 54. The display 6.4259259 is not, as it stands, a real solution to the problem and has to be interpreted. The pupil needs to know that the number of coaches has to be a whole number, in this case the next largest one: 7. If the pupil also wants to find the number of spare places on the coaches, further work has to be done. Multiplying  $7 \times 54$ , giving 378, the pupil can see that there are 31 spare seats.

The examples given below are related to the national curriculum at levels 3, 4 and 5, and are suitable for the target group of pupils as they enter and move through key stage 3. The examples include cases that require pupils to make decisions as to which operations they need to use, which calculator keys they need to press and in which order to press them. Many of them require pupils to interpret the resulting display on their calculator.

The least and greatest heights of girls in a class are 118.7 cm and 161.2 cm. Find the range of heights of the girls in the class.

This is a realistic data-handling activity where the awkwardness of the numbers warrants the use of the calculator. The activity can be extended to other contexts in mathematics or other subjects where the range of values is of interest.

Collect some information on comparative prices for large and small amounts of any items. Ask, for example:

'Which is the better buy: 567 g of ketchup at 89p or 340 g at 49p?'

As the price of many goods is now given on labels as, for example, a cost per 100 g, comparisons are easier to make without calculation. But tasks such as these give opportunity to explore ideas of ratio.

A model village has houses that were built to a scale of 1 in 17.

One model shop is 47 cm high. What would be its real size?

The height of the village school is 11.85 m. What is the height of the model?

Pupils will need to decide whether they have to multiply or divide by 17 before they can use their calculator to carry out the appropriate operation. This kind of activity can be extended to map scales.

250 kilometres is approximately the same as 155.3 miles.

How many kilometres are there to 60 miles?

Pupils could think how they would use the information to find the number of kilometres in 1 mile (ie by dividing 250 by 155.3), and then multiplying by 60 to find the distance in kilometres.

To change a Celsius temperature to Fahrenheit, multiply by 1.8 and add 32. To change a Fahrenheit temperature to Celsius, subtract 32 and divide by 1.8.

Using these rules, what is the Celsius equivalent of 83°F? What is the Fahrenheit equivalent of 15°C?

The factor 1.8 arises from comparing the 180 degrees between the freezing point and boiling point of water on the Fahrenheit scale with the 100 degrees between the same two temperatures on the Celsius scale.

A machine makes 752 300 drawing pins every day.

These are packed in boxes of 125 pins.

How many boxes can be filled in one day?

While it may be possible for some pupils to carry out a calculation like this mentally, for others the numbers will be too large. In either case, the first task is to decide what sort of calculation to perform – in this case, 752 300 has to be divided by 125.

Petrol costs 78.9 pence a litre.

My car's petrol tank holds 41.5 litres.

How much does it cost to fill my car's petrol tank?

My car can travel about 340 miles on a full tank of petrol.

What is the cost per mile?

The first decision has to be to multiply 78.9 or 0.789 by 41.5 to get the total cost. Then the calculator display has to be interpreted. Knowing the cost for 340 miles, pupils will need to divide by 340 to find the cost per mile, and again interpret the display.

The planet Mars is 0.11 times heavier than Earth.

Jupiter is 317.8 times heavier than Earth.

How many times heavier is Jupiter than Mars?

To find how many times heavier Jupiter is than Mars, pupils will need to find how many times 317.8 is bigger than 0.11. Some pupils tend to assume all comparisons are made by subtraction, so they will need to think carefully about the importance of the word 'times' in the question. They need to find, in effect, how many times 0.11 will divide into 317.8. They will also need to interpret the display.

In 1955, it was decided that there were 31 556 926 seconds in that year. How many minutes were there in the year? How many hours in the year?

Pupils need to take care in entering large numbers. After dividing by 60, they will need to interpret the calculator display to give an answer in minutes. At what stage should the interpretation of the display be made when calculating the number of hours in the year?

A computer printer can print 346 characters per second. How many characters are there on this page? How long will it take to print the page?

Pupils will need to decide how to arrive at the number of characters in a given piece of writing. Then the calculator will help them to arrive at an estimation of how long it will take to print.

Investigate the sequence: 1, 1, 2, 3, 5, 8, 13, 21, ... Each number is the sum of the two previous numbers. Divide each number by the one before it:  $1 \div 1$ ,  $2 \div 1$ ,  $3 \div 2$ ,  $5 \div 3$ , ... What do you notice if you keep going?

Some pupils may be familiar with the Fibonacci sequence; it has many interesting properties. In this case, as pupils carry out the successive divisions, their calculator display will eventually show numbers that always begin 1.618.... In fact, the ratio of adjacent terms in this sequence gets closer and closer to what is known as the Golden Ratio, which could be the beginning of further investigation.

The two examples that follow can be used as extension activities for more able pupils. They are typical of calculator activities that a teacher could devise for pupils working beyond level 5 in mathematics.

The distance from Brussels to Milan is approximately 785 miles. An aeroplane flight took 1 hour 35 minutes. What was the average speed of the aeroplane?

Knowing that the journey time is 95 minutes, pupils will need to divide 785 miles by 95 and then multiply the result by 60 to find the distance travelled per hour.

Six similar tubes of sweets were found to contain 46, 49, 47, 45, 51 and 49 sweets respectively.

What was the mean number of sweets in the tubes?

How many tubes of sweets can be filled from a container that has 5000 sweets?

The mean number of sweets per tube can be calculated in order to get an idea of how many would be filled from the large container. Alternatively, pupils could use the smallest number and the largest number of sweets to see what effect that has on the number of tubes that can be filled.

# Using a calculator to explore and gain understanding of mathematical ideas and patterns

Calculators can help pupils to enhance their understanding of some fundamental mathematical ideas, such as negative numbers or place value. Calculators can also give opportunities to explore mathematical ideas that would otherwise be inaccessible to pupils, but which have intrinsic value. These include divisibility, recurring and terminating decimals and the effect of multiplying by numbers between 0 and 1. These ideas lend themselves readily to classroom discussion during key stage 3.

A calculator used with an overhead projector is invaluable in promoting whole-class discussion, as everyone can the see the display and the effect of entering or changing any number or operation. Tools for use with an interactive whiteboard also usually include display calculators.

#### **Numbers in words**

Each person in the group should have a calculator.

One person reads out the numbers.

The others enter them into their calculator, pressing the + key after each one.

Does everyone get the check number?

Set 1 Set 2

One hundred and fifty-six

Twelve pounds sixty-three pence
Two hundred and seven
Seven hundred and five
Three hundred and twelve

Twelve pounds sixty-three pence
Sighty-nine pence
Six pounds and six pence

Six thousand, one hundred and forty

Ten pounds and ten pence

One thousand and eighty Fifty pence

Check number: 8600 Check number: 31.6 (£31.60)

This activity allows you to check whether all the pupils in the group can translate numbers in words to numbers in figures. You may need to start with a shorter list for pupils at level 3. The activity can be extended to decimals, and to other measurements, including time.

#### Bull's eye

Choose a starting number (eg 17) and a target number (eg 100).

Find what number to multiply the starting number by to get as close as you can to the target number.

This gives pupils useful practice in estimation. A variation in which they may only multiply by whole numbers is a useful introductory activity before a lesson on long division.

#### Largest answer

Use the digits 1, 2, 3, 4 and 5 in any arrangement and one × sign.

You could have  $241 \times 35$ ,  $123 \times 54$  or  $2431 \times 5$  etc.

What is the largest answer you can get?

This activity helps to focus attention on place value. Having to think about getting the largest product encourages pupils to think about where to put each number. Pupils can choose other sets of numbers, for example 5, 6, 7 and 8.

#### **Prime factors**

The number 74 865 can be written as the product of prime numbers. What are they?

Pupils will need to use what they know about number in order to begin this activity. For example, they may decide to divide the number by 5 first.

#### **Matching multiplications**

Give pairs of pupils two sets of numbers, such as sets A and B.

Set A: 7, 15, 23, 34, 47, 53, 137

Set B: 782, 645, 371, 795, 1219, 959, 705, 345, 1802, 1081, 510, 1598

Players take turns.

The first player chooses a number in set B.

The player says which two numbers in set A will multiply together to make the selected number from set B.

The second player checks with a calculator.

If the first player is correct, the numbers in set B are marked with the first player's initials. The winner is the first to mark four numbers in set B.

This game encourages pupils to consider what happens when two numbers are multiplied. For example, if one of the numbers ends in a 5, the product will also end in a 5. If one number ends in a 2, the product must be even. If one number ends in a 6, the product must end in a number that is a units digit in the 6 times table (6, 2, 8, 4 or 0). Such considerations help to improve mental calculation strategies.

#### Place value

Pupils work in pairs. One pupil enters a three-digit number on a calculator, for example 374. The second pupil suggests a change to one of the digits, for example 354, and challenges the first pupil to change the number to 354 with just one subtraction (in this case, – 20). The second pupil then challenges the first to make the new display into, say, 854, with just one addition (in this case, + 500). Each pair could set themselves a target number, such as 888 or 654, at which they will aim.

This game helps to reinforce the idea of place value. Pupils can announce, in advance, what operation they will use, in which case the calculator provides a check.

#### **Matching divisions**

Choose any number from this set: 1, 2, 3, 5, 8, 4, 6.

Choose another from this set: 3, 6, 8, 12, 10, 4, 6, 16, 32, 20, 15.

Divide the first number by the second.

Find some pairs that give the same answer.

Make up some more pairs that will give the same answer.

This activity helps pupils to work with decimal equivalents of fractions.

#### Multiplying by a number close to 1

Choose any two- or three-digit number.

Use the calculator to multiply the number by 1.1.

Try it with different starting numbers just greater than 1.

What can you say about the answers?

Now try multiplying by 0.9.

What happens now?

In this activity the calculator shows the effect of multiplying by numbers just greater than or less than 1. During discussion, pupils can explore the reason why this is so.

#### **Exploring simple sequences**

Use an overhead projector calculator to create sequences.

- Enter 10 1 = and keep pressing the = key.
   Count down with the display. Go past 0 and see what happens.
- Enter 103 10 = and keep pressing the = key.
- Enter 57 5 = or 12 + 5 = and keep pressing the = key.

Ask questions about each sequence.

- 'How does the units digit in the display change?'
- 'Why does this happen?'
- 'What will the next number be?'
- 'What number will appear in the display after ten presses of the = key?'
- 'Is it possible to exactly hit the number 20, 200, ...?'
- 'After how many key presses will you pass 100?'

This activity works well with an overhead projector so that all pupils can see the display. Encourage prediction and reasoning by asking: 'What will come next?'

# Using a calculator to promote mental skills

Opportunities to use a calculator to develop mental calculation skills often take the form of challenges between two pupils, one of whom works mentally while the other attempts to arrive at the solution more quickly with a calculator. This sort of activity encourages the use of mental methods; the challenge is to beat the calculator.

#### Beat the calculator

Pupils work in pairs with a pack of cards showing a set of additions of two whole numbers that are close together such as 35 + 36, 48 + 49, 125 + 126. The cards are placed face down in a pack. Players turn the cards over one by one. They then try to be the first to get the answer, one by mental calculation and the other with a calculator.

Alternative sets of cards can include division facts, or a set of simple multiplications such as  $3 \times 8$ ,  $21 \times 7$ ,  $5 \times 9$ ,  $13 \times 3$ , ...

This is a case where the calculator and mental calculation are compared for speed and accuracy. Games like these encourage pupils to increase their speed of calculation and the number of facts that they can recall rapidly.

#### **Broken keys**

Use only the 1, 0, 5, + and = keys.

Make each of these numbers. Press as few keys as possible.

16, 37, 88, 638

This well-known calculator activity encourages pupils to calculate mentally before they attempt to make the numbers using the calculator. Discuss different approaches.

#### **Counting forwards and backwards**

Set up two overhead projector calculators to act as simultaneous counters. For example, set calculator A to count forward in twos, starting at 0 (0 + 2 = = =), and calculator B to count backwards in threes from 100 (100 - 3 = = =).

Ask questions such as:

- 'What number will appear next on each calculator?'
- 'What happens to the sum of the pair of numbers? What happens to the difference?'
- 'Will the same numbers ever appear on both calculators at the same time? If so, which numbers?'

The role of the calculator in this activity is to give a context for asking a range of numerical questions that provoke mental calculation and stimulate class discussion.

### The 100 complements game

Each player needs some counters of their own colour.

Players take turns. The first player chooses two numbers with a sum of 100 from this array.

50	65	26	30	19	85	10	96
35	39	36	18	31	52	73	46
55	28	37	77	22	12	17	20
51	81	59	1	69	70	9	25
72	90	78	88	48	5	82	39
11	75	64	49	50	63	38	99
80	27	45	61	23	74	41	54
91	4	15	62	95	83	15	89

The second player checks with a calculator. If the first player is correct, the player covers the two numbers with coloured counters.

The winner is the player who covers the most numbers.

This game provides motivation for mental calculation. The calculator acts as a referee; the pupil with a calculator is motivated to check the other player's accuracy.

#### Interpreting the display

Four books cost £6.

What does one book cost?

At a school concert, programmes cost 35p each.

How much money is collected when 140 programmes are sold?

A minibus has 12 seats.

How many minibuses are needed for an outing of 70 pupils and 7 teachers?

Simple questions like these can be used to develop pupils' ability to interpret the calculator display in the context of the original problem. For the first problem, the calculator display will show 1.5, which has to be interpreted as £1.50; in the second problem, the display of 490 has to be interpreted as £4.90. In the third problem, pupils have to round up the answer to the nearest integer.

### **Operations**

Set up two calculators with two different functions. For example, one pupil has a calculator that has been programmed to perform + 2 (2 + =). A second pupil has a calculator programmed to multiply by  $2 (2 \times =)$ . The class suggests a number for the first pupil to key into their calculator. The pupil shows the result to the second pupil who keys it into the other calculator and gives the class the final result. They then have to work out what the calculators have been programmed to do. The class can then predict the outcome for given inputs and the input for given outputs.

The calculator is used to give the motivation for the class to carry out mental calculation. Other functions can be tried.