7 Estimating and checking

Estimating and approximating

Mathematics is sometimes regarded as a subject in which all answers have to be either right or wrong. But there are many occasions in everyday life when an approximate answer is appropriate. For example, the newspaper headline: '75 000 fans watch Manchester United beat Arsenal' doesn't mean that exactly 75 000 people were there. One of two things may have happened. Either the exact number of people entering the grounds has been counted as they passed through the gate, and the exact number has been rounded to the nearest 1000 or 5000, or someone has estimated the number. Either is good enough for the newspaper headline.

Similarly, the value of π is usually taken as 3.14, because that is sufficiently accurate for the purpose, even though π is an irrational number that cannot be written in a precise form as a decimal.

It is often hard for pupils who have been accustomed to giving right answers to understand the idea that it is not always either possible or necessary to be exact. The idea of 'suitable for the purpose' needs time to develop.

The target pupils will probably be familiar with the idea of estimating in the context of measurement. The length of the classroom, for example, might be estimated in metres. The number of metres will be approximate but will probably be sufficiently accurate for the purpose. The pupils will probably be less familiar with the idea that every measurement is an approximation which depends on the sophistication of the measuring tool and the degree of accuracy required by the task.

Estimating the answer before embarking on a calculation helps to reduce later mistakes. So, before carrying out, say, 37×53, it is useful for pupils to recognise in advance that the answer will be more than 30×50 and less than 40×60. It may also be helpful to know that a closer approximation is 40×50. This strategy involves the idea of rounding numbers to the nearest ten or hundred, or whatever is appropriate. The fact that these approximations can be done mentally means that they give a way of getting a rough answer even when paper and pencil or a calculator are not available.

Some activities that help pupils to understand the ideas involved with estimation and approximation are given below.

Ask pupils to estimate the number of words on a page of a book or the number of dried beans in a transparent jar. They can see how good their estimate is by actually counting the objects, but help them to recognise that, just because their estimate is different from the actual number, it doesn't mean that their estimate is 'wrong'.

Suggest ways in which they might carry out the estimation. You may also want to discuss what would constitute a good estimate or a poor estimate, and how accurate their estimate might reasonably be (within 10, 20 or 100?).

Estimates like this can usually be made by considering areas or volumes. Pupils can estimate or even count the number of words in a typical line of text, and the numbers of lines on a page, and so arrive at their estimate, if necessary with the aid of a calculator.

Get pupils to estimate some lengths or distances in metres or centimetres. For example, estimate:

- the height of the classroom
- the width of a table
- the length of a corridor
- the distance from the school gate to the bus stop

Stress that, in making estimates like this, it is not possible to count. But it is useful to know a few facts, for example, that the height of an ordinary door is about 2 metres and that the width of a palm of a hand is about 10 centimetres. With these in mind, it is possible to visualise how many doors would fit the height of the classroom, and so on.

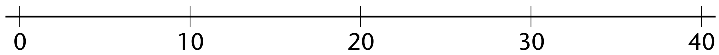
Get a group of pupils to play ‘four in a row’.

They will need two dice or spinners numbered 2, 3, 4, 5, 6, 7 and a 5 by 5 baseboard with 25 randomly placed multiples of 10 from 20 to 80. Pupils take turns to throw the dice and, using the digits in any order, make a two-digit number.

They should round their number to the nearest 10 and then place a counter on that number on the board. The aim of the game is to get four counters in a row.

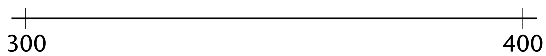
This game provides some simple practice in rounding to the nearest 10.

Use number lines such as:

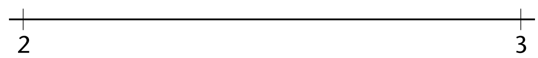


Ask pupils, in turn, where they would place the numbers 18, 25, 7, 32, and so on. Encourage them to explain how they decided where to place their number. What strategies do pupils use?

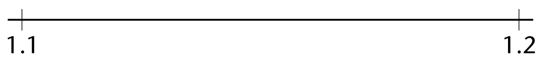
Extend the number lines used to include ones such as:



Where would you place 370, 320, ...?



Where would you place 2.5, 2.8, ...?



Where would you place 1.15, 1.13, ...?

Ask pupils to explain how they decided where to place each number. Discuss their strategies.

Get individual pupils or pairs of pupils to play 'target'.

Give them some digits and ask them to decide how to place them in the boxes so the boxes give an answer as near to the target as they can. For example:

3, 4, 6 □ □ × □ Target 140

6, 7, 8, 9 □ □ × □ □ Target 6000

This game involves an understanding of place value as well as an appreciation of the effect of multiplying by a one- or two-digit number.

Write some calculations on the board, such as 58 + 86 + 31.

Invite pupils to give what they think is an approximate answer. They might find two values between which the answer lies, by saying the answer lies between 50 + 80 + 30 and 60 + 90 + 40. Others might round each number to its nearest multiple of 10, giving, this time, 60 + 90 + 30.

Encourage pupils to make up some examples of their own and work on them in a similar way. You might like to get them to find which of the two methods is closest to the actual answer, and to say why.

Try other examples, such as 953 – 368, 63 × 87 or 953 ÷ 289. These examples pupils could clearly not do mentally, but they can get an approximate answer.

Play the 'rounding game' with a pair of pupils. They need a pile of single-digit cards, with no zeros. Pupils take turns to choose a card and place them to make three two-digit numbers. For example, they might get 73 + 56 + 18:

[7] [3] + [5] [6] + [1] [8]

The first player rounds each two-digit number to the nearest ten and adds them together, getting:

70 + 60 + 20 = 150

The second player adds the original numbers (with a calculator if appropriate):

73 + 56 + 18 = 147

The player with the greater number scores the difference (3 in this case).

On the next round, players swap roles.

The winner is the player with the greatest score at the end of the game.

This game can be adapted. For example, use four or five cards to create multiplication of TU by TU; multiplication or division of HTU by TU, or division of ThHTU by TU. The game can be played by a whole class divided roughly into two teams. Each pupil gives their estimate or the result of the calculation by answering on mini-whiteboards.

Checking

Although in general it is preferable to get pupils to stop and think about the calculation before they embark on it, getting a rough idea of the size of the answer, there is also a strong case for encouraging them to think about the reasonableness of their answer when they have arrived at one. Experience suggests, though, that it is quite difficult to ensure that this actually happens.

There are three types of checking that are helpful: rough checking, partial checking and exact checking. Each type is discussed below and illustrated with examples.

Rough checking

This involves looking at the answer to decide whether it seems reasonable. A check can be made to see if the answer is the right order of magnitude, whether it is far too big or far too small. One might, for example, take a quick glance at a supermarket checkout bill, asking whether the total seems reasonable. Rough checks are usually done mentally and use the same procedures as for the approximations made before the calculation, as described earlier.

Mary bought some goods that cost £3.40, £6.87, £5.97 and £3.56.
The total was given was £81.63.
How do you know it is wrong?
What do you think the error was?

Rounding each item to the nearest £, the goods cost approximately
£3 + £7 + £6 + £4 = £20.
The error was to enter 68.7 rather than 6.87 for the second item.

The answer to 32×27 was given as 288.

How do you know it is wrong?
What is an approximate answer? What is 30×30?
What do you think the mistake was?

In this case, the error was to multiply 32 by 2 and 32 by 7, rather than multiplying 32 by 20 and 32 by 7.

How do you know the answer to 5.6×8.3 cannot be 4648?
What do you think the error was?

The error this time was to omit the decimal points and to multiply 56 by 83.

Partial checking

This is a way of deciding that certain features of the answer suggest that it is wrong. This time the question of whether the answer is reasonable makes use of some basic properties of numbers, such as whether the answer is even or odd, whether it is a multiple of 5 or 10. The final digit of the answer can be checked. In the case of multiplication, for example, a units digit of 6 can come from multiplying 6 and 1 or 2 and 3. Divisibility rules can also be useful: if a number has a factor of 9, for example, the sum of its digits is divisible by 9.

A pupil says that 7×8 is 55.
Why is the pupil wrong?

A pupil says that 13×13 is 167.
Why is that not possible?

A pupil says that 37×3 is 921.
Why is that not reasonable?

In the first example, since 8 is even, the product 7×8 must be even. In the second example, the units digit of the product must be 9, since $3 \times 3 = 9$. In the third example, the product must lie between 30×3 and 40×3 so cannot be 921.

Exact checking

This usually involves carrying out the calculation in a different way. This might involve turning a subtraction into addition or a division into multiplication. Or it might be possible to do the same calculation using a different method.

Check $13 + 27 + 35$ by doing the addition in a different order.

Check $65 - 38$ by adding: does adding 38 to the answer give 65?

Check $144 \div 16$ by multiplying: does multiplying the answer by 16 give 144?

Check 12×15 by doing an equivalent calculation such as 6×30.

It is easy to use inverse operations on a calculator. For example, having multiplied 14.87 by 9 to get 133.83, immediately divide by 9 to see if you get back to 14.87.

Checking calculator work

Mistakes can easily be made when keying numbers into the calculator while carrying out a calculation. This is analogous to the errors that can be made between looking up a number in a telephone directory and then dialling that number. So checking an answer when working with a calculator is every bit as important as checking an answer when working with paper and pencil.

Some of the most common errors that calculator users make are:

- omitting a digit
- entering an incorrect digit
- entering digits in the wrong order
- entering a number from a list twice
- entering the wrong operation
- putting the decimal point in the wrong place.

One way of encouraging pupils to check their calculator answers is to get them to discuss the error that has been made in examples such as:

$14.7 \times 2.3 = 338$
$1001 \times 13 = 77$
$56.71 - 33.47 = 33.24$

Reinforcement

Encouraging pupils to estimate, to make approximations and to check their answers is a long-term process and one that has to be regularly addressed. Many teachers would agree that it is not easy to get pupils to see the importance of these aspects of mathematics. It is probably best to discuss them frequently and to make the most of any opportunity that arises.