Solving linear equations

All pupils should aim to achieve mental agility with the linear form. Their ability to solve algebraic equations at the higher National Curriculum levels can be greatly improved if their responses to these stages of a problem become automatic.

The expectations listed below refer exclusively to the mental facility that will support written solution at level 5 and above.

The progression in the table below is described in two ways.

- The four sections show how the form of the equation is perceived to become more difficult to deal with because of the change in position of the unknown term.
- Within each section the solution involves values that are harder to deal with, such as non-integer or negative integer values.

One-step linear equations

The position of the unknown value is not a source of difficulty if pupils are practised in thinking flexibly about the form of an equation. Encourage them to generalise their pre-knowledge of arithmetical commutativity and inverse in 'families of facts' such as $3 \times 5 = 15$, $5 \times 3 = 15$, $3 = 15 \div 5$, $5 = 15 \div 3$.

One-step linear equations with the unknown in a 'standard' position:	
$x + 4 = 7$	positive integer solutions
$\frac{x}{4} = 6$	
$x - 9 = 34$	
$8x = 56$	
$3x = 5$	non-integer solutions
$x + 14 = 9$	negative integer solutions

One-step linear equations with the unknown perceived to be in a 'harder' position:	
$13 = 8 + x$	positive integer solutions
$\frac{20}{x} = 10$	
$\frac{20}{x} = 3$	non-integer solutions
$13 = 8 - x$	negative integer solutions

Equations involving brackets

One way of solving such equations would involve multiplying out brackets. Another, often simpler, way is to encourage pupils to see the expression in brackets as an object or term. For example, $3(x + 4) = 27$ has the same structure as $3p = 27$, so pupils can mentally move through this step and see that $x + 4 = 9$.

$3(x + 4) = 27$	positive integer solutions
$\frac{(x - 5)}{3} = 7$	

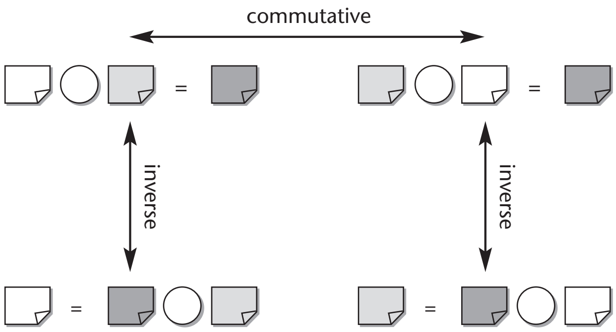
Inequalities

Pupils can better understand an inequality (or 'inequation') if they have have the ability to think clearly about the meaning of the 'equals' sign in an equation. Reading 'equals' as 'makes' is a limited interpretation. Use the language of 'equals' meaning 'is the same as' to develop understanding of equations. The consequent step to 'is greater than' can develop form this much more clearly. Consider building understanding by using visual and mental images such as number lines and coordinate axes. Ask pupils to express inequalities both ways round, to develop flexible and thoughtful use of language and symbols.

The *Framework for teaching mathematics: Years 7, 8 and 9*, supplement of examples, pages 122 to 125, provides contexts in which pupils could develop mental processes in algebraic solution.

$5x < 10$	one boundary to solution set, one-step solution
$-4 < 2x < 10$	two boundaries to solution set
$5x + 3 < 10$	two-step solution

Show a **Family of equations** by using a template for the elements and operations in an equation, as shown below. This is illustrated in the example on page 23.



Use colour-coded sticky notes to hide two numbers, one unknown term and the operations (use circles) needed to make a 'family' of equations. Show one of the equations and use pupils' knowledge of arithmetical relationships (families of facts) to generate the three related equations. Encourage pupils to judge which of the four equivalent forms enables them most easily to evaluate the unknown term, using a mental process. Consider changing one element and map through the effect on other equations. *Does the change affect pupils' judgements?*

It is important that pupils remember the structure of these relationships and can recall this as a mental image.

In the above activity, terms can be replaced by expressions, for example, considering $(x + 4)$ as a single term in equations such as:

$$3 \times (x + 4) = 27 \text{ or } \frac{(x + 4)}{2} = 19$$

Pupils' understanding is strengthened by the regular layout, use of colour, 'hide and reveal' tasks, pupils using their own cards (moving and annotating) and as much talk and shared explanation as possible. These tactics support pupils as they move from the concrete (arithmetic equations) to the abstract (algebraic equations). In particular, pupils have to recognise those operations that are commutative and can be positioned at the top of the template.

The task of **Clouding the picture** is described on page 10 of the booklet, *Constructing and solving linear equations, Year 8*. It is also shown in an example on page 24 of this document. Pupils are asked to express an equation in as many ways as they can. Use this principle to ask: 'What else can you see?' For example, two equations in **Clouding the picture** are:

$$5x + 8 = 11$$

$$5x + 8 = 3 + 8$$

Reading the equals sign as 'is the same as', ask pupils: 'What else can you see?' Read out the second equation and ask if they can see that $5x$ must be the same as 3. Extend to more complex examples such as:

$$5x + 8 = x + 11$$

$$4x + x + 8 = x + 8 + 3$$

Pupils see that $4x$ must be the same as 3.

Allow time for pupils to consider this point as it is not obvious to all. Those who can follow the reasoning should be given the chance to explain it.

Extend this activity to inequalities such as:

$$3x + 7 < 13$$

$$3x + 7 < 7 + 6$$

Point out that this is not an equals sign. Ask pupils:

- 'What else can you see?'

Clarify that since 7 'is the same as' 7 the inequality (or imbalance) must come about by the value of the $3x$ term being less than 6.

Be careful with the use of a balance analogy. The idea of matching items that are the same on both sides is more easily extended than the concept of 'taking off' from each side.

Ask pupils to **Annotate written solutions**. Provide a selection of very detailed written solutions to the same algebraic equation and ask them, first, to choose a method that supports their thinking and then to highlight those stages that they would confidently omit when writing out the solution. Pair pupils and ask them to compare the mental stages and to explain how they process the steps in their heads. Give each pair another equation to solve and ask them to give each other a commentary on what they write and what they think.

Discussing strategies in this way stimulates pupils to think about their thinking (metacognition). Many pupils will refine their own mental processes following collaborative work. This also gives pupils opportunities to discuss the purpose of presenting working and to establish what is required for different audiences. For example, compare introductory notes against revision notes, coursework with a timed examination, a mental with a written test or a calculator and non-calculator examination paper.