## More sequences

## objectives

- Count on and back in steps of constant size.
- Recognise the first few triangular numbers.
- Generate terms of a simple sequence, including from practical contexts.
- Solve problems and investigate in number and algebra.


## starter

## Vocabulary

steps
halfway
interval

## Resources

OHT A3.3a
counting stick
Use a counting stick. Tell the class that one end is 4, and you want them to count in steps of 3 . Count together along the stick to 34 . Point to the midpoint of the stick.

Q What is this number? (19) How do you know?
Establish that it is halfway between 4 and 34 . Each interval or step is worth 3, so that five intervals or steps are worth 15.

Q What is the next number? (22) How do you know? (it is 3 more)
Q What is the number before the middle number? (16) How do you know?

Tell the class that one end is still 4, but that this time you want them to count in steps of 9 .
Q What is the quick way to add on $\mathbf{9}$ ? (add 10 and subtract 1)
Q What will be the last number on the stick? How do you know?
Establish that it will be 94, since the starting number is 4 and there are ten intervals or steps, each worth 9 . Write on the board $4+(9 \times 10)$.

Q What number is halfway between 4 and 94 ?
Invite a pupil to explain why 49 is halfway between 4 and 94.
Show OHT A3.3a, with five number lines. Invite pupils to the projector to fill in the missing numbers on the lines. As they do so, ask them to explain their reasoning. As each line is completed, ask the class to complete the sentence: 'The numbers on this line go up in steps of ...'

## main activity

## Vocabulary

sequence
term

## Resources

OHT A3.3b
Resource A3.3c
mini-whiteboards

Write the sequence 11, 21, 31, 41 on the board. Ask:

## Q How does this sequence continue?

Confirm that the next few terms are $51,61,71$, and the rule is 'add 10 '.
Q Will there ever be a multiple of 10 in this sequence? Explain why.
Establish that each term is a multiple of 10, plus 1.
Repeat with the sequence 2, 7, 12, 17. Ask for the next few terms then ask:
Q Will there ever be a multiple of 5 in this sequence? Explain why.
Establish that each term is 2 more than a multiple of 5 , and that the rule is 'add 5'.

Show the first table on OHT A3.3b. Tell the class that the rule is 'add 4'. Invite a pupil to enter the next four terms $(9,13,17,21)$ along the top row of the table, with the help of the class.

Q How will the pattern continue? $(25,29,33,37)$
Q What do you notice about all the numbers in the sequence? (they are all odd numbers)

Q Will 66 be in the sequence? Explain why or why not. (no - it's even)
Draw attention to the pattern of dots. Ask:
Q What do you notice about this pattern of dots? Can you describe the pattern?

Establish that each pattern has one dot, plus rows of four dots. Complete the bottom row of the table: $1+(4 \times 2), 1+(4 \times 3), 1+(4 \times 4), 1+(4 \times 5)$.

Q How would this row continue?
$1+(4 \times 6), 1+(4 \times 7), 1+(4 \times 8), 1+(4 \times 9)$
Work out these expressions and establish that they match the extended sequence of 25, 29, 33, 37.

Q Will 101 be in the sequence? How do you know?
Ask pupils to discuss this question in pairs, then take feedback on their decisions and reasons.

Q How many rows of four dots will there be in the pattern that represents 101?

Use the pattern to explain that the number of rows of four dots will be $(101-1) \div 4=25$. Check by calculating $1+(25 \times 4)=101$. As 101 can be represented by a pattern with one dot, plus 25 rows of four dots, it is a term in the sequence.

Q Will 51 be in the sequence? How do you know?
Use a similar method to establish that $51-1=50$, and that 50 is not a multiple of 4 , so 51 is not in the sequence.

On the lower table on OHT A3.3b, write 3 in the first box of the top row. Enter a rule of 'add 5'. Point to the sixth box in the top row and ask:

Q How can we work out this term without completing all the boxes in between?

Ask pupils to discuss this question in pairs for a minute or two. Take feedback on their strategies and establish that the sixth term will be 3 plus 5 rows of 5 dots. This can be written as $3+(5 \times 5)=28$.

Give out copies of Resource A3.3c. Refer to the first question. Ask pupils to use their whiteboards and to draw the next pattern in the sequence. Ask:

Q What is the same as the previous pattern? What is different?
Repeat with the second question.

Ask pupils to complete the two questions in pairs. Take feedback on their answers, inviting pupils to explain to the rest of the class the strategies that they used.

## other tasks There are no relevant exercises on sequences involving spatial patterns in the

 Springboard 7 folder. Choose suitable tasks or activities from textbooks or other resource materials, or devise your own. Other exercises on sequences are:
## Unit 9 section 2: Multiples

3 Counting on and back in $6 \mathrm{~s}, 7 \mathrm{~s}, 8 \mathrm{~s}$ and 9 s

## plenary Draw on the board this pattern of dots.

|  |  | $\because: \because$ | $\because \because: 8$ | $\because \because: 8:$ |
| :---: | :---: | :---: | :---: | :---: |
|  | $\because$ | $\because \because:$ | $\because \because 8:$ | $\because 8: 8:$ |
| $1 \times 1$ | $2 \times 2$ | $3 \times 3$ | $4 \times 4$ | $5 \times 5$ |

Q How would you describe this sequence? (square numbers)
Q What would be the tenth term in the sequence? $(10 \times 10=100)$ The hundredth term in the sequence? $(100 \times 100=10000)$

Write on the board and complete with the class:

| 1 | $=1$ |
| :--- | :--- |
| $1+3$ | $=4$ |
| $1+3+5$ | $=\ldots$ |
| $1+3+5+7$ | $=\ldots$ |
| $1+3+5+7+\ldots$ | $=\ldots$ |

Q Can you describe the numbers we are adding? (odd numbers)
Q What special numbers are the answers? (square numbers)
Establish that adding the first three odd numbers produces the third square number, and so on.

Q What is the answer to $1+3+5+7+9+11$ ?
Q What is the sum of the first ten odd numbers?

## Remember

- Decide if the rule means that you add or subtract a number each time.
- If the sequences can be represented by a pattern, use the pattern to help work out terms.
- To decide whether a given number is a term in a sequence, work out the gap between the first number and the given number. Then check whether the gap is a multiple of the number in the rule.


## OHT A3.3a

Fill in the missing numbers on these number lines.


For each line, finish this sentence:
The numbers on this line go up in steps of
The rule is add 4.

| 1 | 5 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\bullet$ | $\bullet \bullet \bullet \bullet$ |  | $\begin{array}{ll}\bullet & 0 \\ \because \because 0 \\ \bullet \bullet\end{array}$ |  |  |
| 1 | $1+(4 \times 1)$ |  |  |  |  |

The rule is

|  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |

## Resource A3.3c

This series of patterns grows in a regular way.

|  | $\bullet \bullet$ |  | $\times$ | $\times$ |
| :---: | :---: | :---: | :---: | :---: |
| $\bullet$ | $\times$ | $\times$ | $\bullet$ | $\bullet$ |
| $\times \quad \times$ | $\bullet$ | $\bullet$ | $\times$ | $\times$ |

How many dots would be in pattern 5 ? $\qquad$
How many crosses would be in pattern 5 ? $\qquad$

This is a series of patterns with white and grey tiles.


How many white tiles and grey tiles will there be: in pattern 8 ?
$\qquad$ white tiles and $\qquad$ grey tiles in pattern $16 ?$
$\qquad$ white tiles and $\qquad$ grey tiles

