## Fractions and percentages

## objectives

- Understand percentages as 'the number of parts per 100'.
- Find simple equivalent fractions, decimals and percentages.


## starter

## Vocabulary

fraction
equivalent
simplify

## Resources

OHT N4.4a
mini-whiteboards
Show OHT N4.4a, with a selection of fractions with a denominator of 100.
Q Which is the smallest of these fractions? Which is the largest? How do you know?

Q Which fraction is equivalent to one half? One quarter? Three quarters? Explain why.

Q Could any other fractions on the OHT be written in a simpler form? Which are they? Simplify them.

Ask pupils to answer the next few questions using their whiteboards.
Q Which fraction lies halfway between $25 / 100$ and $75 / 100$ ? Explain your reasoning.

Q Which two fractions on the OHT could add together to make a third fraction on the OHT? Are there any other possibilities?

Gather the complete set of solutions to the last question:

$$
\begin{aligned}
& 20 / 100+25 / 100=45 / 100 \\
& 20 / 100+30 / 100=50 / 100 \\
& 25 / 100+50 / 100=75 / 100 \quad 30 / 100+45 / 100=75 / 100 \\
& 20 / 100+60 / 100=80 / 100 \\
& 30 / 100+50 / 100=80 / 100
\end{aligned}
$$

## main activity

## Vocabulary

percentage
equivalent
fraction
decimal

## Resources

OHT N4.4b
mini-whiteboards

Explain that percentage means per hundred, or in every hundred. 100\% means 100 in every 100, which is the same as one whole. $50 \%$ means 50 in every hundred, and is written as $\frac{50}{100}$ or $1 / 2$.

Show the class how to convert fractions to percentages. Display OHT N4.4b, with four number lines. Say that these represent a tenths fraction line, a 0 to 1 decimal line, a 0 to 1 empty line, and a 0 to $100 \%$ percentage line. Remind the class that $100 \%$ represents one whole, so it is equivalent to 1 .

Locate $1 / 10$ on the fraction line. Draw a straight line from $1 / 10$ on the fraction line, through 0.1 on the decimal line to the equivalent percentage $10 \%$ on the percentage line. Stress the three equivalents: $1 / 10,0.1$ and $10 \%$.

Invite pupils to the projector to do the same with $1 / 2,3 / 10$ and $7 / 10$.
Ask pupils where to mark $1 / 4$ on the fraction line. Establish that this is halfway between 0 and $1 / 2$.

Q What is $1 / 4$ as a percentage?
Confirm that this is $25 \%$. Repeat with $3 / 4$, confirming this as $75 \%$.
Q How can we use the same method to find $3 / 5$ as a percentage?

Ask pupils to start by converting $3 / 5$ to tenths. Draw the vertical line from $\% / 10$, to establish that $3 / 5=6 / 10=60 \%$. Repeat for $2 / 5$ and $4 / 5$.

Explain to the class that it is sometimes easier to convert fractions to percentages rather than to decimals and to use what they know to work out other percentages.

Show the class how to convert percentages to fractions or decimals.
Q What is $25 \%$ in hundredths? $\left({ }^{25} / 100\right)$ Is there a simpler way of writing this fraction? ( $1 / 4$ ) How would we write $1 / 4 / 4$ as a decimal? (0.25) What do you notice?

Establish that $25 \%=25 / 100=1 / 4=0.25$. Write this on the board. Make sure that pupils notice that the two digits after the decimal point are the same as the two digits immediately before the percentage sign.

Q What is $75 \%$ in hundredths? $\left({ }^{75} / 100\right)$ Is there a simpler way of writing this fraction? $(3 / 4)$ How would we write $3 / 4$ as a decimal? (0.75) What do you notice?

Confirm that $75 \%={ }^{75} / 100=3 / 4=0.75$, and that the two digits after the decimal point match the two digits immediately before the percentage sign.

Q Is it also the case for $50 \%$ that the two digits after the decimal point are the same as the two digits immediately before the percentage sign?

Confirm that $50 \%$ is $5 / 100$ and that this could be written as 0.50 . Remind the class that it is not necessary to write the trailing zeros at the end of a decimal, so 0.50 is written as 0.5 .

Q How many hundredths is $10 \%$ ? How would we write this as a fraction? Is there a simpler way to write it? How would we write it as a decimal?

Establish that $10 \%=10 / 100$ and can be written as $1 / 10$ or 0.1.
Ask similar questions to establish that $20 \%={ }^{20} / 100$ and can be written as $1 / 5$ or 0.2 , and that $30 \%=30 / 100$ and can be written as $3 / 10$ or 0.3 .

Q What is $33 \%$ as a fraction?
Write the answer of $3 / 100$ on the board. Ask pupils to find $33 \times 3$. Draw out that $33 \% \times 3=99 \%$, which is very nearly $100 \%$. So $33 \%$ is almost one third.

Q What is two thirds as a percentage?
Establish by doubling that the answer is about $66 \%$ or $67 \%$. Discuss why both answers are reasonable estimates for two thirds.

Ask a series of short questions, asking pupils to answer them by writing on their whiteboards.

Q What fraction is equivalent to $40 \%$ ?
Q What decimal is equivalent to $90 \%$ ?
Q What percentage is equivalent to seven tenths?

## other tasks Unit 5 section 5: Percentages

## Springboard 7

Unit 5

1 Percentages
2 Equivalent fractions and percentages
3 How much petrol?
4 Football fans
Star challenge 7: Favourite sports
Star challenge 8: The TV survey

## Unit 5 section 6: Equivalent fractions

Star challenge 13: Fraction search for a half page 199
Star challenge 14: Fraction search for a quarter page 199

## plenary

## Resources

OHT N4.4c

Show OHT N4.4c, which displays the 27 fractions, decimals or percentages shown below, scattered randomly.

| 0.1 | $1 / 10$ | $10 \%$ |
| :--- | :--- | :--- |
| 0.25 | $1 / 4$ | $25 \%$ |
| 0.75 | $3 / 4$ | $75 \%$ |
| 0.3 | $3 / 10$ | $30 \%$ |
| 0.7 | $7 / 10$ | $70 \%$ |
| 0.2 | $1 / 5$ | $20 \%$ |
| 0.4 | $2 / 5$ | $40 \%$ |
| 0.6 | $3 / 5$ | $60 \%$ |
| 0.8 | $4 / 5$ | $80 \%$ |

Tell the class that there are nine 'families' of equivalent fractions, decimals and percentages on the OHT. Invite individual pupils to identify one of these families and to cross out the trio on the OHT.

Ask pupils to suggest more 'families' (e.g. $0.9,9 / 10,90 \%$ ), including some that are greater than 1 (e.g. $1.25,1 \frac{1}{4}, 125 \%$ ).

## Remember

- Percentage means per hundred, or in every hundred. Percentages like $47 \%$ and $83 \%$ can be written as $47 / 100$ and $83 / 100$.
- One half can be written as $1 / 2,0.5$ or $50 \%$.
- Since one quarter is one half of one half, one quarter is $1 / 4,0.25$ or $25 \%$. One eighth is half of one quarter, so one eighth is $12.5 \%$.
- One tenth can be written as $1 / 10,0.1$ or $10 \%$. From this, you can work out that $7 / 10=70 \%$ or that $3 / 10=30 \%$.
- One third is about $33 \%$ and two thirds is about $66 \%$.


## Finding percentages of whole-number quantities

## objectives

- Understand percentages as 'the number of parts per 100'.
- Calculate simple percentages of whole-number quantities.


## starter

## Vocabulary

equivalent
percentage
Resources
mini-whiteboards
Remind the class that $50 \%, 0.5$ and $1 / 2$ are equivalent. Ask pupils to use their whiteboards to answer some questions.

Q What is $50 \%$ of $24 ?$ Of 70 ? Of 120 ? Of 250 ? Of 9000 ? Of $15 ?$
Q If we know 50\% of something, how do we find 25\%? (halve 50\%)
Q What is $25 \%$ of 40 ? Of 60 ? Of 100 ? Of 1200 ? Of $10 ?$
Remind the class that $10 \%, 0.1$ and $1 / 10$ are equivalent.

Q How do we find 10\% of something? (find one tenth, or divide it by 10)
Q What is $10 \%$ of 560 ? Of 1000 ? Of 53? Of 4.7?
Q If we know $\mathbf{1 0 \%}$ of something, how do we work out $5 \%$ ? (halve 10\%)
Work through with the class finding $15 \%$ of 240 , by first finding $10 \%$, then $5 \%$, then adding $5 \%$ to $10 \%$. Show how to jot down the interim steps.

Ask pupils to use their whiteboards for interim jottings and to answer these questions.

Q What is $15 \%$ of $300 ?$ Of $60 ?$
Stress that $100 \%$ is equivalent to one whole.
Q 45\% of a class are boys. What percentage are girls?
Q $36 \%$ of the shapes in a box are red. What percentage of the shapes in the box are not red?

## main activity

## Vocabulary

method

## Resources

OHP calculator calculators

OHT N4.5a

Remind the class that $1 \%, 0.01$ and $1 / 100$ are equivalent. To find $1 \%$ of a number means finding one hundredth of the number, or dividing it by 100.

Ask pupils:
Q What might weigh about 500 grams? (e.g. a large potato, a small bag of flour, a baby kitten)

Q What is $1 \%$ of 500 g ?
Demonstrate the calculation with the OHP calculator and record the result on the board: $1 \%$ of $500 \mathrm{~g}=5 \mathrm{~g}$. Stress that the units must be included in the answer.

Repeat by finding $1 \%$ of $£ 200,3500$ millilitres, 250 metres. Record on the board:
$1 \%$ of $£ 200=£ 2,1 \%$ of $3500 \mathrm{ml}=35 \mathrm{ml}, 1 \%$ of $250 \mathrm{~m}=2.5 \mathrm{~m}$
Ask pupils to use this information to work out the answers to the following.
$5 \%$ of $500 \mathrm{~g}, 8 \%$ of $£ 200,2 \%$ of $3500 \mathrm{ml}, 4 \%$ of 250 m

Take feedback on each question. Ask the class how they used the information to calculate the percentages. Draw out that the method is to find $1 \%$ by dividing by 100, then to multiply the result by the relevant percentage.

Demonstrate on the OHP calculator how to find $28 \%$ of $£ 540$. Ask pupils to use their own calculators at the same time. Enter 540, and divide by 100. Ask:

Q How do we interpret the 5.4 in the display? (it means $£ 5.40$ )
Multiply by 28.
Q How do we interpret the 151.2 in the display? (it means $£ 151.20$ )
Record on the board: $28 \%$ of $£ 540=£ 151.20$.
Ask pupils to use their calculators to work out $13 \%$ of $£ 550$ ( $£ 71.50$ ) and $4 \%$ of £33.25 (£1.33).

Ask the class:
Q You know that $10 \%$ of a quantity is 8 kg . So $5 \%$ is 4 kg . What other percentages can you work out easily using this information?

Establish that:
$20 \%$ is $10 \% \times 2,30 \%$ is $10 \% \times 3$, and so on;
$15 \%$ is $10 \%+5 \%, 25 \%$ is $20 \%+5 \%$, and so on.
Write randomly on the board a selection of percentages such as:
$50 \%, 25 \%, 75 \%, 10 \%, 1 \%, 20 \%, 60 \%, 90 \%, 33 \%$
Point to one of them and ask:
Q What strategy or method could you use for calculating this percentage of a given amount? Could you work it out in a different way?

Stress any alternative methods. For example:
$90 \%$ is $100 \%-10 \%$ or $10 \% \times 9$;
$60 \%$ is $50 \%+10 \%$ or $10 \% \times 6$;
$75 \%$ is $50 \%+25 \%$ or $25 \% \times 3$.
Work through with the class the questions on OHT N4.5a.

## other tasks Unit 13 section 2: Fractions and percentages

## Springboard 7

Unit 13

1 Using fractions to find percentages
2 10\% and its multiples
Star challenge 3: Reducing prices
Star challenge 4: Percentages and fractions

## plenary

## Vocabulary

chance
interest rate
discount

Ask the class what they think these statements mean and to explain them in their own words.

- This shirt is 70\% cotton and $30 \%$ polyester.
- We spend about $33 \%$ of our lives asleep.
- There is a $10 \%$ chance of rain today.
- The interest rate on my savings account is $3 \%$ per annum.
- I got a 30\% discount on these shoes in a sale.

Supplement with questions such as:

- The shirt weighs 200 grams.

About how much is cotton?

- About how many hours do we sleep in a day? In a week?
- It is twice as likely to rain tomorrow.

What is the chance of rain tomorrow?

- I have $£ 50$ in my savings account.

How much interest will I get in a year?

- The original cost of the shoes was $£ 30$.

What did I pay for them in the sale?
Finish by asking questions such as:
Q Would you prefer to climb $20 \%$ of a 3000 m mountain, or $30 \%$ of a 2000 m mountain?

Q Would you prefer to lose $40 \%$ of $£ 80$, or $80 \%$ of $£ 40$ ?

## Remember

- A quick way to find $20 \%$ of a quantity is to find $10 \%$ by dividing by 10 , then multiply by 2 to find $20 \%$. You can find $30 \%, 40 \%, 50 \%, \ldots$ similarly.
- If there is no quick method for finding a percentage of a quantity, first find $1 \%$, then multiply by the percentage.
- Always include any units in the answer.

| $\frac{50}{100}$ | $\frac{30}{100}$ |
| ---: | ---: |
|  | $\frac{45}{100}$ |

75
100
$\frac{20}{100}$
$\frac{25}{100}$



## OHT N4.5a

The table shows the percentage of people who took part in sports at a sports centre. Label the correct two sections of the pie chart football and squash.

| Badminton | $10 \%$ |
| :--- | ---: |
| Football | $40 \%$ |
| Squash | $5 \%$ |
| Swimming | $30 \%$ |
| Tennis | $15 \%$ |



Altogether 260 people took part. Complete this table.

| Sport | Percentage | Number of people |
| :--- | :---: | :---: |
| Badminton | $10 \%$ |  |
| Football | $40 \%$ |  |
| Squash | $5 \%$ |  |
| Tennis | $15 \%$ |  |
| Swimming | $30 \%$ |  |

