## Problems involving direct proportion

## objectives

- Find doubles and halves of numbers.
- Solve simple problems involving direct proportion.
- Solve word problems.


## starter

## Vocabulary

halves quarters

## Resources

counting stick

## main activity

## Vocabulary

multiply
divide
estimate
problem

## Resources

OHP calculator
calculators
OHTs N4.7a, N4.7b

Use a counting stick and count along and back in halves from zero.
Q What are $\mathbf{7}$ halves? $\mathbf{3 6}$ halves? How did you work it out?
Q How many halves are there in 9 ? In 54 ? How did you work it out?
Q What fractions do you know that are equivalent to one half?
Encourage pupils to use doubling and halving strategies.
Now count on and back along the stick in quarters.
Q What are 8 quarters? 32 quarters? How did you work it out?
Q How many quarters are there in 11? In 23? How did you work it out?
Q What fractions do you know that are equivalent to one quarter? To three quarters?

Write on the board: 1 bowl $\rightarrow 6$ tomatoes. Explain that to make one bowl of soup you need 6 tomatoes.

Q How many tomatoes do you need to make 2 bowls of soup?
Q What operation do you need to do to find out? (multiply 6 by 2)
Write on the board: 2 bowls $\boldsymbol{\rightarrow} 12$ tomatoes.
Q How many tomatoes do you need to make 3 bowls of soup?
Q What operation do you need to do to find out?
Quickly build up a table to 6 bowls $\rightarrow 36$ tomatoes.
Ask questions such as:
Q How many bowls of soup can you make with 48 tomatoes? How do you know?

Encourage pupils to consider different strategies for working out the answer to the question, making sure that the strategy of dividing 48 by 6 is included.

Record on the board: 48 tomatoes $\rightarrow 6$ bowls.
Q Can you predict how many tomatoes you will need for 10 bowls of soup? How did you work it out? What about 100 bowls of soup? 1000 bowls of soup?

Record on the board:

```
    1 bowl \(\rightarrow 6\) tomatoes
100 bowls \(\rightarrow 6 \times 100\) tomatoes
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Q Can you predict how many bowls of soup you can make with 66 tomatoes? With 120 tomatoes? With 6 million tomatoes?

Record on the board:
6 tomatoes $\rightarrow 1$ bowl
66 tomatoes $\rightarrow 66 \div 6$ bowls
Show the first problem on OHT N4.7a. Show pupils how to solve the problem and how to show their working.

There are 9 pencils in a box.
A school buys 24 boxes.
How many pencils does the school buy?
Start by recording a statement about one thing, based on the information given in the question:

1 box $\rightarrow 9$ pencils
Underneath, record what the question is asking, making sure that the item to be found is on the right.

24 boxes $\rightarrow$ ? pencils
Q What operation do we need to do to find the answer? (multiply 9 by 24)
Q What is an estimate of the answer? $(10 \times 20=200)$
Record $24 \times 9$ and ask pupils to work it out using their preferred method (using a calculator only if time in the lesson is short). Compare the answer of 216 pencils with the estimate and check that it makes sense in the context of the question.

Read through the second part of the first problem:
Another school has ordered 126 pencils.
How many boxes of pencils has the school ordered?
Record again the statement about one thing, and write below it what the question is asking, making sure that the unknown is on the right.

$$
\begin{aligned}
& 9 \text { pencils } \rightarrow 1 \text { box } \\
& 126 \text { pencils } \rightarrow \text { ? boxes }
\end{aligned}
$$

Q What operation do we need to do to find the answer? (divide 126 by 9)
Q What is an estimate of the answer? $(130 \div 10=13)$
Q Does the answer of 14 boxes make sense in the context of the question?
Work though the second and third problems on OHT N4.7a in a similar way, stressing how pupils should show their working.

Show OHT N4.7b and ask pupils to work through the problems in pairs. Assist the pairs, making sure that they identify the operation needed and an estimate for the calculation. Make sure that they 'show their working' by recording the complete calculation.

## other tasks Unit 15 section 1: Mental calculations - multiplication

Star challenge 2: Two-star mental challenge
page 477

## Springboard 7

Unit 15
$\begin{array}{ll}\text { Unit } 15 \text { section 3: Multiplication - written methods } & \\ \text { Star challenge 5: Multiplication problems } & \text { page } 483\end{array}$
Unit 15 section 4: Division - written methods
3 Problems page 485
Star challenge 6: Mental challenges page 485

## plenary

## Resources

OHT N4.7c
OHP calculator

Show OHT N4.7c. Work through each problem to identify the calculation needed.
Q Which of these calculations could be done without a calculator, and which would need a calculator?

Ask an individual pupil to use an OHP calculator to demonstrate to the class the steps they would take to do one of the calculator calculations. Ask the same pupil to write on the board what they would do to 'show your working'. Stress that this needs to show the complete calculation. Stress also the use of approximation to check the answer.

## Remember

- Read the question carefully. Look for key words in the question to help decide what operation to use and what calculations to do.
- Decide what information you need for your calculations. It is often useful to write down the information given in the problem, for example a statement about one thing.
- Write down the calculation that you did to show your working.


## Ratio and proportion

## objectives

- Divide a quantity into two parts in a given ratio.
- Solve simple problems involving direct proportion.


## starter

## Vocabulary

proportion
ratio
fraction
percentage

## Resources

interlocking cubes

Count together in multiples of 5 , to 50 .
Use 6 blue and 4 yellow interlocking cubes to make a stick like this.


Hold up the stick and tell the class that you have made a pattern where, for every 3 blue cubes, there are 2 yellow cubes. Ask:

Q What colour would the next cube in the pattern be? (blue) What colour would the 15th cube be? (yellow) How do you know?

Q What colour would the 31st cube be? (blue) How do you know?
Establish that the cubes are grouped in fives, and that the first cube in every five will be blue. Draw this table on the board.

| Blue cubes | Yellow cubes | Total cubes |
| :---: | :---: | :---: |
| 3 | 2 | 5 |
|  |  |  |
|  |  |  |
|  |  |  |

Rearrange the stick of cubes to look like this.


Q What proportion of the stick is blue?
Explain that proportion means the same as 'fraction' or 'percentage'. Establish that $6 / 10$ or $3 / 5$ of the stick is blue.

Q In a stick of 10 cubes, how many are blue? (6) How many are yellow? (4)
Say that the pattern could also be described as 'for every 6 blue cubes, there are 4 yellow cubes' or 'for every 4 yellow cubes, there are 6 blue cubes'. Add 6, 4 and 10 to the table.

Q If the stick had 9 blue cubes, how many yellow cubes would there be?
Establish that 3 more blue and 2 more yellow cubes would be needed, and add 9, 6 and 15 to the table.

Q If the stick had 18 blue cubes, how many yellow cubes would there be? How do you know?

Discuss pupils' responses and add 18, 12 and 30 to the table.
Q If the stick had 60 blue cubes, how many yellow cubes would there be? How do you know?

Discuss pupils' responses and add 60, 40 and 100 to the table.
Point to all the numbers in the first column.
Q What are all these numbers? (multiples of 3 )
Establish that the numbers in the second column are all multiples of 2 . Explain that the ratio of blue cubes to yellow cubes is 3 to 2 , and is written as $3: 2$. A ratio can be simplified in the same way as a fraction, by dividing each side by the same number. For example, a ratio of $5: 10$ is equivalent to a ratio of $1: 2$.

## main activity

## Vocabulary

problem
recipe

## Resources

OHTs N4.8a, N4.8b
Show OHT N4.8a, a recipe for fish pie for two people.
Q What is the ratio of butter to fish in the recipe?
Establish that for every 25 g of butter 250 g of fish are needed. The ratio of butter to fish is $25: 250$, or $1: 10$.

Q What is the problem asking us to do?
Q How shall we begin to tackle it?
Ask pupils to discuss these questions in pairs, then take their suggestions. Establish that a good way to start would be to halve the recipe to make enough for one person. Ask the pairs to do this, then write it up on the board.

Q How much fish would be needed for 3 people?
Establish that $125 \mathrm{~g} \times 3$ would be needed for 3 people.
Q What fraction of $\mathbf{1} \mathbf{k g}$ is $\mathbf{3 7 5} \mathbf{~ g}$ ? (three eighths of a kilogram) How can we write this as a decimal? ( 0.375 kg )

Q How much potato would be needed for 3 people? $(600 \mathrm{~g})$ What is 600 grams in kilograms? ( 0.6 kg )

Q How much butter would be needed for 3 people? ( 37.5 g )
Read though the second part of the problem.
Q How many grams is $\mathbf{2} \mathbf{k g}$ ? (2000 grams)
Q How many grams of potato are needed for 1 person? (200 g)
Q How many people would need 2000 grams of potato?
Record on the board:

$$
\begin{aligned}
200 \mathrm{~g} & \rightarrow 1 \text { person } \\
2000 \mathrm{~g} & \rightarrow \text { ? people }
\end{aligned}
$$

Q What operation do we need to do to find the answer? $(2000 \div 200)$
Q Does the answer of 10 people seem appropriate?
Q How much butter would be needed for 10 people? What operation do we need to do? (multiply 25 g by 5 )

Q How much fish would be needed for 10 people? What operation do we need to do? (multiply 250 g by 5 )

Q What is 1250 g in kilograms? ( 1.25 kg )
Show OHT N4.8b, a recipe for raspberry ice cream. Ask:
Q What is the ratio of sugar to raspberries?
Establish that both amounts must be in the same units, and that the ratio is 250 : 1000 or 1: 4.

Ask pupils to work in pairs to tackle the problem. Establish that one way to start might be to halve the recipe to make enough for four people.

Collect and discuss solutions, inviting pupils to the board to explain their methods.
Stress what they need to do to show their working.

## other tasks Unit 13 section 5: Ratio and proportion

## Springboard 7

Unit 13

1 In every ... and for every ...
page 436
2 Ratio and the words that go with it page 437
3 Proportion
Star challenge 9: Ratio and proportion problems

## plenary

## Resources

OHTs N4.8c, N4.8d

Show OHT N4.8c. Discuss the ratios and proportions illustrated.
Show OHT N4.8d and complete the table.

## Remember

- Ratio is a way of comparing two quantities. For example, the ratio of a 2 cm rod to a 3 cm rod is $2: 3$.
- A proportion is similar to a fraction or percentage. If 1 in every 4 beads in a necklace is red, then the proportion of red beads is $1 / 4$ or $25 \%$.
- '2 for every 3' means that 5 units are being shared out, and the fractional parts are $2 / 5$ and $3 / 5$. The ratio of these two parts is $2: 3$.
- '2 in every 3' means that 3 units are being divided up, and the fractional parts are $2 / 3$ and $1 / 3$. The ratio of these two parts is $2: 1$.
- When changing a recipe, work out the quantities for one person, then increase the quantities by multiplying by the number of people involved.

1 There are 9 pencils in a box.
A school buys 24 boxes.
How many pencils does the school buy?
Another school has ordered 126 pencils.
How many boxes of pencils has the school ordered?

Show your working.

2 Plants are sold in trays of 20.
David wants 240 plants.
How many trays of plants does he need to buy?
Ivana buys 7 trays of plants.
How many plants is this?
Show your working.

3 One length of a swimming pool is 25 metres. How many lengths are there in a 150 metre race?

Laura swims 14 lengths.
How many metres does she swim?
Show your working.

## OHT N4.7b

1 Apples are sold in packets of 4. How many apples are in 72 packets?

Alex buys 96 apples. How many packets does she buy?

2 There is 60 g of rice in one portion. How many portions are there in a 3 kg bag of rice?

Harry cooked 8 portions of rice.
How many grams of rice did he cook?

3 A spoonful of medicine is 5 ml . How many spoonfuls of medicine can you get from a bottle holding 375 ml ?

Tim had 32 spoonfuls of medicine when he was ill. How many millilitres of medicine did he have?


## OHT N4.7c

1 Sue went camping for 6 nights.
It cost $£ 2.20$ to camp each night.
How much did Sue pay to camp?
Ram paid £26.40 to camp.
For how many nights did Ram stay at the camp?

2 Emma saves $£ 3.50$ each week. How much has she saved after 16 weeks?

Paul has saved $£ 4.50$ each week.
Altogether he has saved $£ 40.50$.
For how many weeks has he saved?


Here are the ingredients for fish pie for 2 people.


Omar makes fish pie for 3 people. How many grams of fish should he use?
grams

Mary used 2 kg of potato to make a fish pie. How many people did her fish pie feed? $\qquad$ How much butter was in her fish pie? grams How much fish was in her fish pie? $\qquad$ grams

## OHT N4.8b

Here is a recipe for raspberry ice cream for 8 people.

## Raspberry ice cream for 8 people <br> $\frac{1}{2}$ litre of cream <br> 1 kg raspberries <br> 250 g sugar

Josie makes enough raspberry ice cream for 12 people. How much cream does she use? $\qquad$ litre

Fred makes raspberry ice cream in the same way. He uses $2 \frac{1}{2} \mathrm{~kg}$ of raspberries.

How much sugar does he use?
grams

For each set of circles, complete the statements below.


Write this proportion as:
a fraction $\square$
a decimal $\square$
a percentage $\square$

The ratio of black circles to white circles is $\square$ to $\square$
Complete this table.

| in every | fraction | decimal | percentage |
| :---: | :---: | :---: | :---: |
| 1 in every 5 |  |  |  |
|  |  |  | $75 \%$ |
|  | $\frac{2}{3}$ |  |  |
| 3 in every 8 |  |  |  |

