Algebraic conventions

Pupils need to be as familiar with the conventions of algebra as they are with those of arithmetic. Algebraic conventions should become a routine part of algebraic thinking, allowing greater access to more challenging problems. It is a common error to deal with these conventions rather too quickly. How pupils understand and manipulate algebraic forms is determined by their mental processing of the meaning of the symbols and the extent to which they can distinguish one algebraic form from another.

We are seeking to develop a mental facility to recognise which type of algebraic form is presented or needs to be constructed as part of a problem. Some time spent on this stage of the process can reduce misconceptions when later problems become quite complex.

Recognise and explain the use of symbols

Explicitly model and explain the correct vocabulary. For example, in the equation $p + 7 = 20$ the letter p represents a particular unknown number, whereas in $p + q = 20$, p and q can each take on any one of a set of different values and can therefore be called variables. Equations, formulae and functions can describe relationships between variables. In a function such as $q = 3p + 5$ we would say that $3p$ was a variable term, whereas 5 is a constant term. Be precise and explicit in using this vocabulary and expect similar usage by pupils.

$3x + 5 = 11$	Represent an unknown value in equations with a unique solution
$p + q = 20$	Represent unknown values in equations with a set of solutions
$2l + 2b = p$	Represent variables in formulae
$y = \frac{x}{2} - 7$	Represent variables in functions

Identify equivalent terms and expressions

It is often the case that pupils do not realise when an equation or expression has been changed, or when it looks different but is in fact still the same. The ability to recognise and preserve equivalent forms is a very important skill in algebraic manipulation and one in which pupils need practice. One way of approaching this is to start with simple cases and generate more complex, but equivalent, forms. This can then be supported by tasks involving matching and classifying.

$2x + x + 5$	simple chains of operations
$ax + 5$	some with unknown coefficients
$7(x + 2)$	brackets (linear)
$(x + 2)(x + 5)$	brackets (quadratic)
$x^3 \times x$	positive indices

Identify types and forms of formulae

This will build on the understanding of equivalence and will rely on knowledge of commutativity and inverse. Encourage pupils to see general structure in formulae by identifying small collections of terms as 'objects'. These objects can then be considered as replacing the numbers in 'families of facts', such as 3 + 5 = 8, 5 + 3 = 8, 3 = 8 – 5, 5 = 8 – 3. The equations are then more easily manipulated mentally.

The *Framework for teaching mathematics: Years 7, 8 and 9*, supplement of examples, pages 112 to 117, provides contexts in which pupils should develop mental processes in algebraic conventions.

To develop pupils' understanding of the dimensions of a formula, make explicit connections between the structure of the formula and its meaning. Consider the units associated with each variable and how these build up, term by term. Involve pupils in generating and explaining non-standard formulae, for example, for composite shapes.

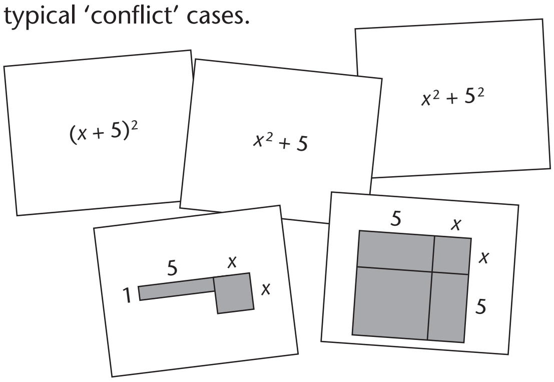
$\frac{a}{b} = l$, $a = l \times b$	equivalence of formulae
$a = l \times b$, $2l + 2b = p$	dimensions of a formula

Classifying cards will help to build pupils' understanding of the connections within different algebraic forms. A set of cards could include:

- expressions, some equivalent, some not;
- equations with unique solutions, some with sets of solutions, some formulae and some functions;
- formulae of different dimensions (linear, area, volume, compound).

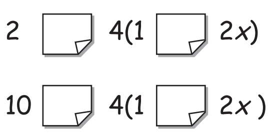
> For an illustration of the types of cards see the *Framework for teaching mathematics: Years 7, 8 and 9*, supplement of examples, page 116, or the booklet, *Constructing and solving linear equations, Year 9*, page 22.

Matching different forms of representation is also helpful. For example, pupils could be asked to match a set of cards with algebraic expressions to another set showing area diagrams. The cards could include typical 'conflict' cases.



> The *Framework for teaching mathematics: Years 7, 8 and 9*, supplement of examples, page 116, uses this same area image to develop pupils' understanding of the expansion of brackets. It builds on the grid method of long multiplication which, in algebra, can provide a strong mental image of the source of each element within an expansion of brackets.

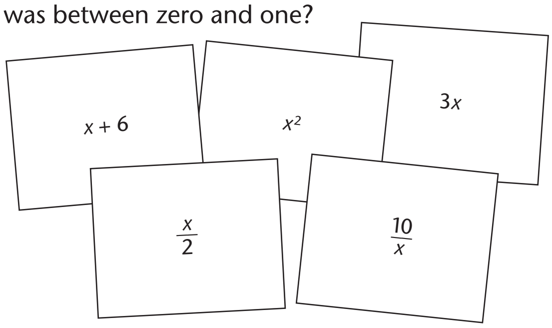
Matching equivalent expressions is illustrated as part of a lesson plan on pages 17 to 19 of the booklet, *Constructing and solving linear equations, Year 9*. For example, could these expressions be equivalent to $6 - 8x$?



> The operations in these expressions are hidden by sticky notes. Pupils are asked to apply their mental reasoning, not laboriously expand expressions.

Sequencing expression cards in order of size, by taking x to be a given positive integer, could be used as a preliminary task for pupils, before setting more challenging questions such as:

- Which cards would change position if x was a given negative integer? (For example, –2 instead of 2.)
- Which cards would change position if the value of x was between zero and one?



> Considering this point can encourage pupils to check their written strategies, especially where it is important to have an idea of the relative size and nature of the solution.

Pyramid activities give pupils opportunities to construct expressions of various degrees of complexity, as illustrated in the *Framework for teaching mathematics: Years 7, 8 and 9*, supplement of examples, page 116.

> Keep the task centred on talk and jottings rather than written manipulation and simplification. Focus on mental strategies by asking questions about preceding layers of the pyramid, for example:
>
> - What possible entries in the early stages of this pyramid could result in this expression?
> - Could this be one of the entries? Why not?

Clouding the picture, as illustrated on pages 24 and 25 of this document, could easily be adapted from equations to expressions. Pupils are asked to write an expression 'in as many ways as you can', using a systematic way of 'complicating' the expression. For example, one system for 'complicating' $2x + 8$ might appear as $3x + 8 - x$ and $4x + 8 - 2x$. Ask: 'Are they equivalent? How do you know?'

> Look out for pupils explaining the system that they devised to ensure that the expressions were equivalent. Extend to more complex examples such as $5x + 6 + y$ and look for challenging 'complications' such as $\frac{1}{2}(10x + 12 + 2y)$.