Functions and graphs

Unless attention is focused on mental processes involved in work on functions and graphs, there is a real risk that pupils will be expected to move rather too quickly from plotting coordinates to tackling challenging generalisations that link algebraic and graphical forms. Pupils require a higher level of thinking to make connections between real-life contexts and the features of a graph. Development of such skills is best supported by collaborative endeavour, allowing pupils the opportunity to share their emerging understanding and to learn from one another. Pupils are better able to tackle challenging problems independently if they have first experienced some success in those areas through interactive group work.

Activities relating to functions and graphs can take two main forms: interpreting graphs or generating graphs. These should be developed alongside each other.

Interpret graphs of functions

Interpreting pre-drawn graphs provides pupils with opportunities to recognise and generalise the relationship between elements in the function and features of the graph. ICT applications are an ideal medium, both for teacher and pupils, because they provide the means for quickly and accurately testing hypotheses about these links.

$y = mx$ $y = c + x$, $y = x + c$ $y = mx + c$	Recognise these graphs for integer values of m and c Note the relationship between families of graphs as values of m and/or c increase or decrease Note which functions represent proportional relationships
$y = ax^2$ $y = x^2 + c$ $y = ax^2 + c$ $y = (x + b)^2$ $y = x^2 + c$ $y = (x + b)^2 + c$ $y = (x + b)(x + a)$	Recognise these graphs for integer values of a, b and c Note the relationship between families of graphs as values of a and/or b and/or c increase or decrease

Generate graphs of functions

An important skill is the ability to summarise the key features of a graph through a sketch. This can be developed alongside skills involving graphical calculators or graph-plotting software. In all cases it is crucial to explore problems, discuss results and explain the relationship between the features of a function and the consequent features of the graph.

$y = mx$ $y = c + x$, $y = x + c$ $y = mx + c$	Sketch these graphs for integer values of m and c Explain the relationship between families of graphs as values of m and/or c increase or decrease Explain which functions represent proportional relationships
$y = ax^2$ $y = x^2 + c$ $y = ax^2 + c$ $y = (x + b)^2$ $y = (x + b)^2 + c$ $y = (x + b)(x + a)$	Sketch these graphs for integer values of a, b and c Explain the relationship between families of graphs as values of a and/or b and/or c increase or decrease

Interpret graphs arising from real-life problems

Consider using graphs from other subject areas, such as science or geography, or those that appear in newspapers, other published material or on the internet. Ask pupils to explain what they think the graph might be about. Discussion about the shape of a graph and how it is related to the variables and the context represented supports pupils' understanding.

linear conversion	a single straight line, interpreting the meaning of points and sections
distance–time	linear sections, interpreting the meaning of points and sections
temperature change	curved sections, interpreting the meaning of points and sections

Generate graphs arising from real-life problems

Use ICT to generate graphs of real data, including application data from other subject areas. Focus on the degree to which the graph is an accurate interpretation of a real situation (recording temperature change) or part of a mathematical model (distance–time for a cycle journey). Hypothesising about graphs without scales and headings can draw attention to the way in which different scales and starting points can lead to different interpretations.

The *Framework for teaching mathematics: Years 7, 8 and 9*, supplement of examples, pages 164 to 177, provides contexts in which pupils could develop mental processes in sequences.

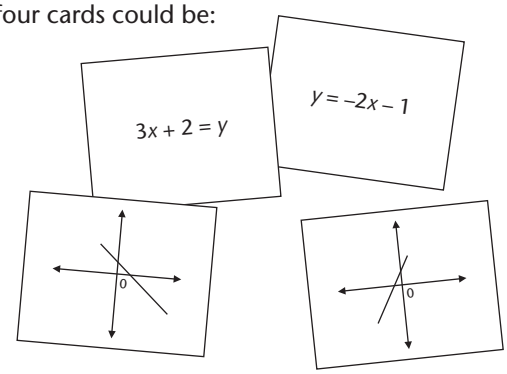
linear conversion	a single straight line, interpreting the meaning of points and sections
distance–time	linear sections, interpreting the meaning of points and sections
temperature change	curved sections, interpreting the meaning of points and sections

Matching activities can develop and consolidate pupils' understanding. Provide cards showing tables of values, sets of points, equations and graphs of lines or curves. Model this task, thinking aloud to provide a running commentary. Invite a few pupils to do the same. Agree on helpful prompts or questions such as:

- Do both values increase?
- Which point is important to note?
- What would be a good checking value?

Resources to support an example of a *Functions and graphs* matching activity are given on pages 27 to 31. Encourage pupils, in pairs, to challenge each other's choices and decisions. Circulate but do not intervene unless pupils are really stuck. Take notes of interesting tactics and discussion points to feed into the plenary. Put pairs together into fours where you can see that a discussion about different choices would be useful.

Wise words is a versatile task suitable for developing understanding of most visual forms. Pupils work in pairs with a set of up to eight cards. For example, four cards could be:



Pairs compose one or two statements to describe a card, which another pair must try to identify. Each statement may use only one key word and may not say what form the representation takes (for example, whether it is a table, a set of points, a function or a graph). Key words might be 'gradient' and 'intercept' and a statement describing the graph on the bottom left card (above) could be: 'The gradient is negative.'

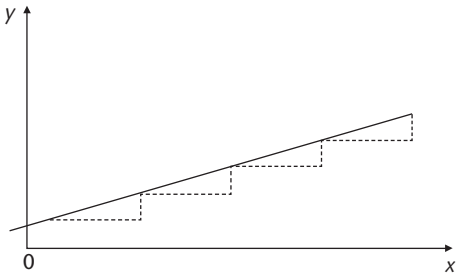
Each pair passes their statements to another pair, for them to work out which card is being described. The second pair can write one 'yes/no' question and, after getting a response to this, they must identify the card. A suitable question could be: 'Is the intercept positive?'

The design of the cards, the number of cards and the key words makes this a rich and adaptable activity, which engages pupils in discussion and forces them to consider the precision of the language they are using.

Say what you see helps pupils to understand the process of drawing graphs.

Use ICT to plot the graph of a linear function, building up the line step by step, giving a running commentary. Use dotted lines to show changes in the *x*-values (horizontal) and corresponding changes in the *y*-values (vertical). As you plot points and trace the graph, ask pupils to say what they see. For example:

- The steps all look the same.
- The line is made up of smaller lines all the same.
- It could go on for ever … both ways.



Ask if other lines could grow in the same sort of steps. Draw some more parallel lines in the same way. Give groups copies of a worksheet, prepared by pasting an A4 sheet (showing a family of parallel straight lines on unmarked axes) in the centre of a blank A3 sheet. Ask pupils to use graphing calculators to explore a possible interpretation of these lines and to annotate this image.

Note: This task is open to many levels of response. The absence of labels and scales allows groups to generate different functions, making for a rich plenary. Some pupils or groups may use coordinates or tables to justify the functions illustrated. Others may take the opportunity to generalise and offer more than one solution.

Living graphs is an activity in which prepared images prompt pupils to think around a 'real-life' context. It leads to pupils composing their own interpretations and ultimately constructing the graph for themselves. An example, *The speed of a racing car*, is given on page 32. Choose a suitable subset of the statement cards. Use the statement cards to model the process, showing that they can be placed in various positions on the graph, and to give pupils confidence. Ensure that the justification for positioning each statement is clear. Pupils have to decide where the statement would be best positioned from the information they have been given and justify their reasoning to the group.