Fractions, decimals, percentages, ratio and proportion

After calculation, the application of proportional reasoning is the most important aspect of elementary number work. Proportionality underlies key aspects of number, algebra, shape, space and measures, and handling data.

In order to make progress through levels 6, 7 and 8 in the number section of the National Curriculum, pupils must recognise which number to consider as 100% or a whole. This enables them to reverse proportional change and to calculate repeated proportional change. In many problems, making the appropriate choice between fractions, decimals, percentages or ratio will be crucial. Pupils can only make such a choice if they have a sound understanding of equivalence.

Developing mental mathematics from level 5, in this context, means securing a flexible approach to a problem, underpinned by confidence with equivalence and multiplicative strategies. This rationale was the basis of the suite of minipacks in the series:

Interacting with mathematics in Key Stage 3:

- *Enhancing proportional reasoning in Year 7*
  Year 7 Fractions and ratio; (minipack DfES 0093/2002)
- *Interacting with mathematics in Year 8*
  Year 8 Multiplicative relationships; (minipack DfES 0220/2002)
- *Interacting with mathematics in Year 9*
  Year 9 Proportional reasoning. (minipack DfES 0588/2002)

Each minipack offers a range of strategies for developing mental mathematics in this area. There are supporting resources and some frames for lesson plans.

The following pages in this section offer a small, additional contribution to the wealth of ideas in the minipacks. They focus on:

- understanding and using equivalencies between fractions, decimals and percentages;
- use proportional reasoning to solve a problem.

Understanding and using the equivalencies between fractions, decimals and percentages

Pupils should already have some knowledge of the equivalences between different representations of fractions, decimals and percentages. As they progress beyond level 5 it is important they recognise that the choice of form can affect the ease and efficiency with which a calculation can be performed, particularly mentally. They need to become confident in spotting appropriate forms and converting between them. This is usually a mental process, even when the subsequent calculation is performed on paper or by calculator.

Build this confidence and fluency by creating chains of equivalences, using striking visual arrangements to draw attention to the structures and patterns.

The *Framework for teaching mathematics: Years 7, 8 and 9*, supplement of examples, pages 3 to 5, 60 to 81 and 98 to 101, provides contexts in which pupils should develop mental processes in fractions, decimals, percentages, ratio and proportion.

Know and use the equivalence between fractions, decimals, percentage and ratio:	
Know that $\frac{3}{5}$, 3/5 and $3 \div 5$ all mean the same	Relate fractions to division
Know that $7 \times \frac{1}{3}$, $7 \div 3$, $\frac{1}{3}$ of 7, $\frac{7}{3}$ and $2\frac{1}{3}$ are equivalent	Interpret different meanings of fractions
$0.365 = \frac{365}{1000} = \frac{73}{200}$ $0.365 = 36.5\%$	Convert terminating decimals to fractions or percentages
$\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{11}$	Convert common recurring decimals to fractions
$\frac{1}{3} = 0.3333\ldots = 0.\dot{3}$ $\frac{2}{9} = 0.2222\ldots = 0.\dot{2}$ $\frac{7}{6} = 1.1666\ldots = 1.1\dot{6}$	For simple fractions with recurring decimal equivalents, convert between the fraction and decimal forms
$\frac{1}{3}, \frac{a}{3a}, \frac{2(a+b)}{6(a+b)}$	Simplify or find equivalent algebraic fractions

Within and between is a task in which pupils use arrays to simplify or find equivalent fractions or ratios. For example:

	Equivalent fractions		
Numerator	0.7	7	35
Denominator	?	?	150

$\times \frac{30}{7}$

$\times \frac{1}{5}$

Pupils can simplify the single multipliers as shown; for example, to get from 7 to 35, use $\frac{35}{7}$ or 5, to get back to 7 from 35 use $\frac{7}{35}$ or $\frac{1}{5}$. They can write the calculation, $150 \times \frac{1}{5}$, to find the number, 30, to write in the cell. They should notice that the link between the numerator and denominator is the same for every pair. The link between a pair of numerators (and their equivalent denominators) is different for different pairs.

When working with ratio the array would look similar.

$\times \frac{1}{7}$

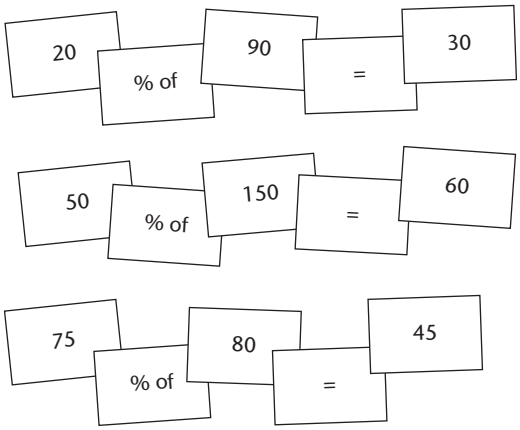
2	:	?
14	:	35
?	:	85

$\times \frac{2}{5}$ $\times \frac{5}{2}$

Observation and explanation of this 'within and between' relationship for a proportional set is very important. The emphasis is on noting down the calculations, for example, $150 \times \frac{30}{7}$, rather than the actual computation.

Sorting activities in the form of a puzzle can help pupils to find different equivalences.

Pupils work in pairs to sort a set of number cards and arrange them to make three correct calculations.

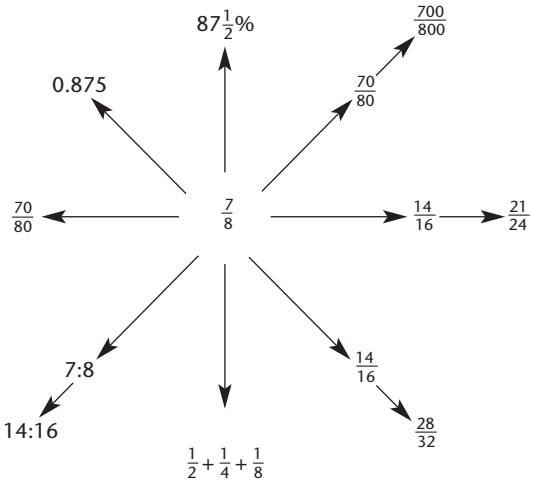


This activity requires pupils to calculate mentally while sorting and resorting the cards until they make all the calculations correct. The activity may be varied by using fractions or decimals rather than percentages.

Percentages: a chain of reasoning is a task that encourages the pupils to think flexibly about a calculation. It also encourages pupils to give reasons and justify steps they are taking. In the example on page 26 the chain ultimately shows that 60% of 25 is the same as 25% of 60.

Clouding the picture is a technique to enable pupils to identify other related facts by using equivalence. For example, ask pupils to complicate the central fraction $\frac{7}{8}$ in *as many ways as they can*. They should:

- start by giving another couple of examples along each branch;
- stop after a few examples and try to explain what is happening along the branch (*generalising the process*);
- start a new branch that does something different to complicate the fraction.



This technique could be modelled by the teacher and then repeated by pupils using different fractions, decimals, percentages or ratios as the centre numbers. The task can easily be adjusted to match the challenge to the group of pupils. Another example is shown on page 27.