Place value and ordering

In primary school pupils will have worked extensively with the image of the number line. As their knowledge and experience of mathematics extends they will welcome the place-value chart as a most flexible and useful image. It will help extend their ability to deal with large and small numbers.

The ability to multiply and divide by any integer power of 10 and to start writing numbers in standard form depends on a secure understanding of place value. This understanding is fundamental in manipulating large and small numbers, both mentally and in written form.

Multiply and divide numbers by powers of 10

Use positive integer powers of 10 and refer to prior knowledge of the way in which a division fact can be derived from a known multiplication fact. Include the vocabulary of multiplication and division as inverse operations.

$5.32 \times 10 = 53.2$ $53.2 \div 10 = 5.32$ $6.95 \times 100 = 695$ $695 \div 100 = 6.95$ $4.78 \times 1000 = 4780$ $4780 \div 1000 = 4.78$	Begin with 10, 100, etc.
$6 \times 0.1 = 0.6$ $0.6 \div 0.1 = 6$ $7 \times 0.01 = 0.07$ $0.07 \div 0.01 = 7$	Extend the same approach and understanding to multiplying and dividing by 0.1, 0.01, etc.

Understand the effect of multiplying and dividing by numbers between 0 and 1

Use powers of 10 as the multiplier to help pupils recognise the logic of the emerging pattern.

$4.2 \times 100 = 420$	Makes bigger ↑	Understand that multiplying by any number between 0 and 1 makes the number smaller
$4.2 \times 10 = 42$		
$4.2 \times 1 = 4.2$	**Stays the same**	
$4.2 \times 0.1 = 0.42$		
$4.2 \times 0.01 = 0.042$	↓ Makes smaller	
$4.2 \div 0.01 = 420$	Makes bigger ↑	Understand that dividing by any number between 0 and 1 makes the number bigger
$4.2 \div 0.1 = 42$		
$4.2 \div 1 = 4.2$	**Stays the same**	
$4.2 \div 10 = 0.42$		
$42 \div 100 = 0.042$	↓ Makes smaller	

Multiply and divide decimals by any number between 0 and 1

Use mental calculations with whole numbers and adjust, using knowledge of the effect of multiplying or dividing by numbers between 0 and 1.

Multiplying	
31×0.4	$31 \times 4 = 124$ 10 times smaller is 12.4
0.25×0.03	$0.25 \times 3 = 0.75$ 100 times smaller is 0.0075

Dividing	
$81 \div 0.3$	$81 \div 3 = 27$ 10 times bigger is 270
$0.24 \div 0.06$	$0.24 \div 6 = 0.04$ 100 times bigger is 4

Alternatively, use the definition of fractions as division and knowledge of equivalent fractions.

Dividing	
$81 \div 0.3$	$\frac{81}{0.3} = \frac{810}{3} = \frac{270}{1} = 270$
$0.24 \div 0.06$	$\frac{0.24}{0.06} = \frac{24}{6} = \frac{4}{1} = 4$

Begin to write numbers in standard form

The *Framework for teaching mathematics: Years 7, 8 and 9*, supplement of examples, pages 38 and 39, provides contexts in which pupils should develop mental processes in place value.

Use movements on a place-value grid. Relate numbers back to the 'baseline' of unit digits and describe movements in terms of multiplication, first by multiples of 10 then by powers of 10.

$235.7 = 2.357 \times 10^2$	Write large numbers in standard form
$0.000\ 92 = 9.2 \times 10^{-4}$	Write small numbers in standard form
6.92×10^{-4}, 2.5×10^2, 3.7×10^2	Order numbers in standard form

Modelling different ways in which pupils can use a place-value chart is fundamental to the follow-up tasks described below.

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

Illustrate the multiplicative relationships within each column of the chart by discussing the multiplication that takes you from one column entry to another. For example:

- $0.08 \times 10 = 0.8$, a step up one row;
- $0.008 \times 100 = 0.8$, a step up two rows.

Link the steps to the powers of ten and establish that multiplying by 10^3 is the same as multiplying by 1000 and is equivalent to a step up three rows.

Confirm that moving down within a column can be considered as division or multiplication, for example:

- $8 \div 100 = 0.08$, a step down two rows;
- $8 \times 0.01 = 0.08$, a step down two rows.
- $8 \times 10^{-2} = 0.08$, a step down two rows.

Hide and reveal tasks engage pupils in the same movements around the chart. Use blank strips to **hide** rows of numbers and ask pupils to explain how they know what numbers are under the strip. Alternatively, **reveal** only one row of the chart and ask pupils to complete the rows above and below it. As a more challenging task ask for the row three steps above or below.

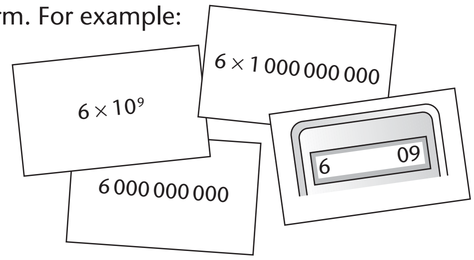
Windows in the chart can be used as the basis of a task in which pupils are asked to create the surrounding entries or even the whole chart, possibly using a spreadsheet.

0.05	0.06
0.005	0.006

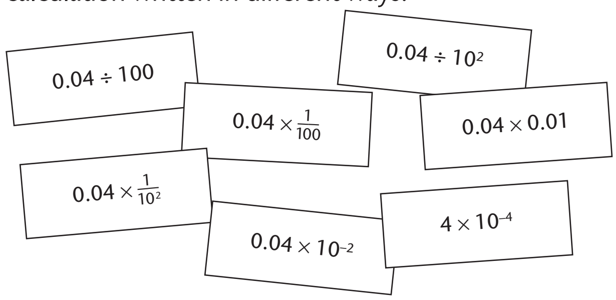
This chart can reinforce pupils' understanding of the effect of multiplying by numbers smaller than 1. It also allows discussion of the fact that the steps can be reversed in two ways:

- by using the same operation (multiplication) with an inverse operator 10^{-4} or 0.0001;
- by using the same number (10^4 or 10 000) and an inverse operation, division.

Matching different forms of representation can provide pupils with the chance to confront misconceptions. For example, pupils could be asked to match cards that show numbers with cards that show calculations and calculator displays in standard form. For example:

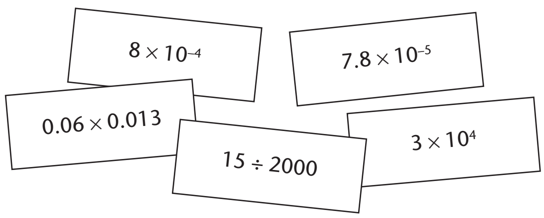


Alternatively, the cards might only show the same calculation written in different ways.



Note that in the task above, and in the one below, numbers are written in standard form without necessarily using the term 'standard form'. The introduction to standard form itself is described in the next modelling activity.

Ordering without using a calculator focuses attention on place value. Write multiplication and division calculations (including numbers in standard form) on cards and ask pupils which cards they can put in sequence, in ascending or descending order, **without calculating the values of the numbers.**



Using cards to match and sequence encourages pupils to discuss their calculations and justify their solutions. Using different sets of cards for different groups of pupils provides a straightforward way of differentiating.

Fill in the missing numbers is an activity in which pupils write in the numbers on cards such as those shown below.

☐ multiplied by 10^3 is ☐

☐ divided by 0.1 is ☐

Pupils work in pairs, using a collection of cards with blank entries. They refer to A4 copies of a place-value chart to support the task. This is about using the structure of the chart, **not** about calculating.

Fill in the missing operations is the reverse of the above task. Pupils continue to work in pairs and to use the structure of the A4 place-value chart. They should describe the operation needed to get from the start number to the second number in as many ways as they can. The task is simplified if only multiplication is used.

3000	0.07
↑	↓
Multiply by 1000 Multiply by 10^3 Divide by 0.001	Divide by 100 Multiply by 10^{-2} Multiply by 0.01
3	0.0007

> The aim is for pupils to become confident in making multiplicative statements linking pairs of numbers from the same column. Their increased confidence will be evidence of a greater understanding of the nature of the base-10 number system.

Modelling how to write numbers in standard form can also begin with the place-value chart. Identify the 'units' digit line as the baseline of the whole chart. All other lines can be generated from this one by using multiplication to step up or down from it. Ask pupils to describe the steps.

$4000 = 4 \times 10^3$ $5000 = 5 \times 10^3$

Generalise to show that all the numbers in the 'thousands' row will be represented by a units digit multiplied by 10^3 ($n \times 10^3$). Give pupils copies of one column from the chart and ask them to annotate each of the entries.

$\times 10^5$	500 000
$\times 10^4$	50 000
$\times 10^3$	5000
$\times 10^2$	500
$\times 10^1$	50
$\times 10^0$	5
$\times 10^{-1}$	0.5
$\times 10^{-2}$	0.05
$\times 10^{-3}$	0.005
$\times 10^{-4}$	0.0005

Encourage pupils to speculate how to write 4500 in this way. Referring to the place-value chart, ask:

- ☐ Where would you start from?
- ☐ Where would you place the resulting number?

Encourage pupils to imagine a number line running across each row. Explain:

- ☐ This helps you to think about 4.5 and 4500 relative to the numbers shown on the chart.

For example, establish that 4.32 is between 4 and 5, closer to 4 than 5. Ask pupils to perform the same operation for 4.32 as above, deciding which multiplication will take it into the row for thousands.

$4.32 \times 10^3 = 4320$

- ☐ What is the equation and where would the number lie on the place-value chart?

Show pupils cards with numbers between 1 and 10 and with × 10 to a suitable index. Ask them to use the place-value chart to identify the resulting number.

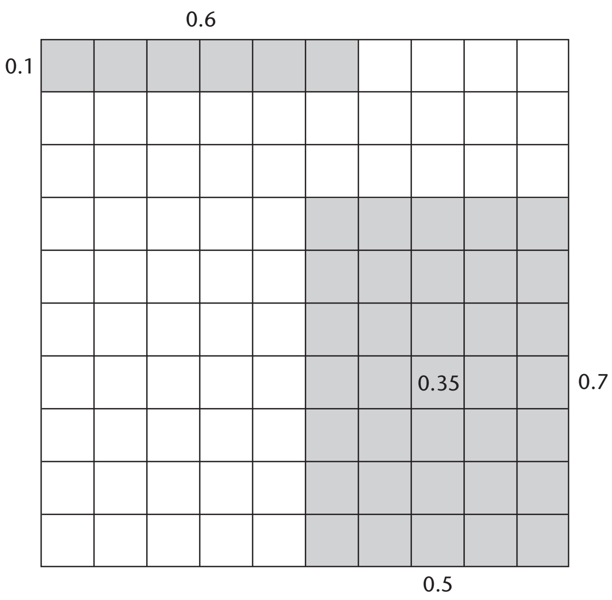
2.36

$\times 10^5$

$\times 10^{-2}$

Use a **Diagrammatic explanation** such as a rectangular array to show multiplication and division. Establish first that the unit lengths are divided into tenths and the unit area is divided into hundredths. For example:

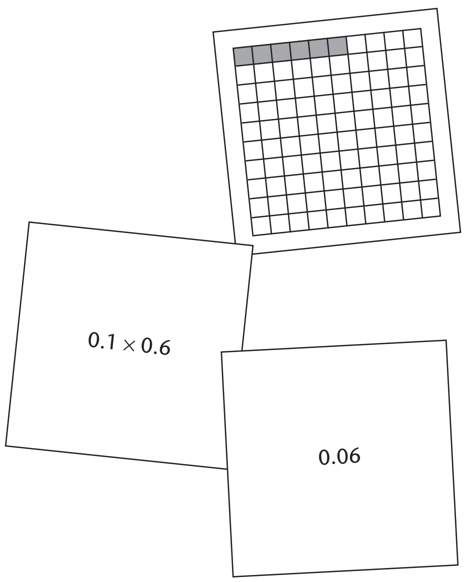
$0.6 \times 0.1 = 0.06$ or $0.35 \div 0.7 = 0.5$.



Ask pupils to match arrays and calculations and then to sketch their own arrays for simple decimal multiplication and division calculations.

Challenge pupils to explain how they could extend this array for larger and smaller numbers.

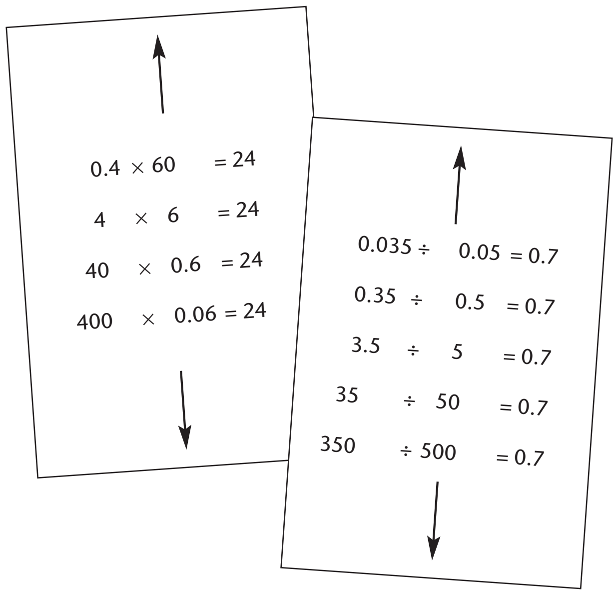
Matching a calculation with an answer and a grid representation will consolidate pupils' understanding of this image. For example, ask pupils to match a set from a mixture of cards including examples such as:



The area model is a useful starting point from which to develop understanding of multiplying and dividing by numbers smaller than 1, especially to show how a number can be partitioned into tenths. It provides a powerful visual image for pupils and can aid mental calculation.

This idea is also illustrated in the *Framework for teaching mathematics: Years 7, 8 and 9*, supplement of examples, page 39.

Asking pupils to **Find as many ways as possible** helps them to appreciate that a calculation can be written in several ways. It also encourages them to demonstrate that one calculation is equivalent to another. Pupils can also discuss which transformation is the most efficient for each calculation.



Give pupils a calculation and ask them to extend and develop it to find equivalent calculations.

Start from a simple known fact, for example, $4 \times 6 = 24$. Ask questions to lead pupils to consider the effect of making the first number 100 times smaller while the answer remains the same.

- ☐ What effect does this have on the magnitude of the second number in the calculation?

Encourage pupils to make generalisations.

Spend some time exploring the different equivalent calculations that are obtained for multiplication and division.

Although pupils may use a calculator to check their answers, they should justify the equivalence through the logic of the calculation.