Rounding

Mathematics is sometimes regarded as a subject in which all answers must be either right or wrong but there are many occasions in everyday life when an approximate answer is appropriate. For example, the newspaper headline: '75 000 fans watch Manchester United beat Arsenal' does not mean that exactly 75 000 people attended the match. One of two things may have happened. Either people entering the grounds were counted as they passed through the gate, and the exact number was rounded to the nearest 1000 or 5000, or someone estimated the number. Either case is good enough for the newspaper headline.

Similarly, the value of π is often taken as 3.14 because this is sufficiently accurate for the purpose, even though π, an irrational number, cannot be written precisely as a decimal.

In calculations, it is often difficult for pupils who have been accustomed to giving 'right answers' to understand that it is not always either possible or necessary to be exact. They need time to develop the idea of 'suitable for the purpose'.

Estimating an answer before starting a calculation is important as it can reveal subsequent errors, particularly when a calculator is used. So, for example, before they start to multiply 3.7×0.83, it is useful for pupils to recognise that the answer will be less than 3.7 because they are multiplying by a number less than 1. Rounding the numbers to 4×0.8 gives 3.2, which is a sufficiently close approximation and can be calculated mentally. The accurate calculated answer, 3.071, may be rounded to an appropriate degree of accuracy.

When solving any problem, pupils need to know the stages at which it is appropriate to round the numbers and the effect this will have on the result.

Round to whole numbers and specified numbers of decimal places

Work the problem both ways, considering the values that a number might have taken before rounding, i.e. 'unrounding'. Establish criteria for rounding and when a 'trailing zero' is required.

4.48 = 4 (to nearest whole number) 4.48 = 4.5 (to 1 decimal place) 4.97 = 5 (to nearest whole number) 4.97 = 5.0 (to 1 decimal place)	Round decimals to the nearest whole number or to one decimal place
7.499 = 7 (to nearest whole number) 7.499 = 7.5 (to 1 decimal place) 7.499 = 7.50 (to 2 decimal places)	Round decimals to the nearest whole number or to one or two decimal places

Round to specified numbers of significant figures

This way of rounding is most helpful when dealing with very large and very small numbers. It is a precursor to standard form. For large numbers the result can look the same as 'to the nearest 10 000', for example. For very small numbers it allows rounding without everything shrinking to 0. Draw attention to the roles of zero:

- 'leading zeros' as in 0.000 003 76;
- 'trailing zeros' as in 458 000;
- 'significant zeros' as in 670 006.

In all cases zero is a place-holder maintaining the value of the other digits and therefore the size of the number. In the first two cases the zeros can be established by multiplying or dividing by a power of 10 (as in standard form) but significant zeros cannot be repositioned in this way.

3768 = 4000 (to 1 significant figure) 3768 = 3800 (to 2 significant figures) 3768 = 3770 (to 3 significant figures) 0.002 61 = 0.003 (to 1 significant figure) 0.002 61 = 0.0026 (to 2 significant figures) 0.0296 = 0.030 (to 2 significant figures) 0.005 04 = 0.0050 (to 2 significant figures) 2 083 452 = 2 100 000 (to 2 significant figures)	Round decimals to a given number of significant figures

Understand upper and lower bounds for discrete and continuous data

Draw attention to the fact that rounding effectively maps an interval of numbers onto a single value.

The population of London = 9 million people (to the nearest million) Then the population must be at least 8 500 000 and at most 9 499 999. 8 500 000 ≤ (population of London) ≤ 9 499 999	discrete
The distance from Exeter to Plymouth = 62 km (to the nearest km) Then the distance is 61.5 km or further but not as far as 62.5 km. 61.5 km ≤ (Exeter to Plymouth) < 62.5 km	continuous

The *Framework for teaching mathematics: Years 7, 8 and 9*, supplement of examples, pages 42 to 47, provides contexts in which pupils should develop mental processes in place value.

Four in a row gives pupils practice in rounding to one decimal place. Pupils play the game in groups, using counters in two colours, two identical sets of cards numbered 6, 7, 9, 11, 13 and 14 and a 5 × 5 grid. They write 25 numbers, all in the range 0.1 to 1.9 and with one place of decimals, randomly on one 5 × 5 grid, as a baseboard. Pupils take turns to choose two cards (one from each set) and divide one number by the other, using a calculator as appropriate. They round the answer to one decimal place and then place a counter of their own colour on that number on the baseboard. The aim of the game is to get four counters in a row.

This game provides basic practice in rounding to one decimal place and is a good precursor to further activities.

Before or after? is a task that involves pupils in considering the different effects of rounding numbers before or after a calculation. Pupils work in pairs, using a pile of cards showing four-digit numbers with three decimal places, such as 4.652, 3.894, 2.453, 8.264, 0.675 and 7.329. They each choose a card and put them together to form a multiplication calculation, for example:

4.652 × 3.894

First they predict the effect of rounding **before** the calculation. Ask:

- ☐ Will the result be smaller or larger than the result of rounding **after** the calculation?
- ☐ Is it possible to say?

Then they test their predictions by rounding each number to two decimal places and multiplying them together (with a calculator). They round the result to two decimal places. For example:

4.65 × 3.89 = 18.09

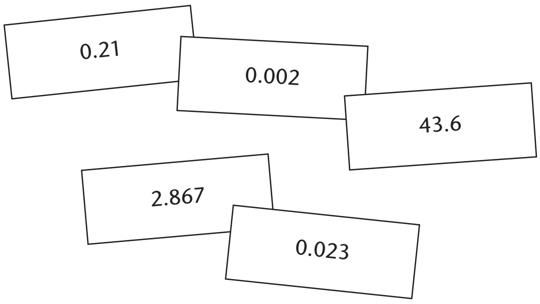
Finally, they multiply the original numbers and round the result to two decimal places.

4.652 × 3.894 = 18.11

Encourage pupils to identify and discuss why rounding before computation gives a different answer. This task can be adapted to use different operations. Extend the activity with questions such as:

- ☐ Which of the numbers you used gave the greatest difference? Why?
- ☐ Which gave the smallest difference? Why?

Sorting sets of numbers enables pupils to identify differences and similarities between them. Give pupils some cards showing numbers that could have been rounded to one, two or three decimal places or written correct to one, two or three significant figures.



Ask the pupils to sort these cards into the correct place on a two-way grid.

		Number of decimal places			
		0	1	2	3
Number of significant figures	0				
	1				
	2				
	3				

Ask questions such as:

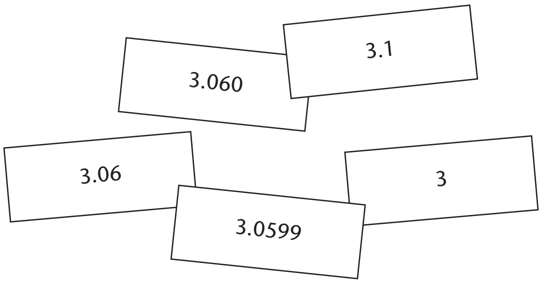
- ☐ What numbers can you find that could be placed in empty cells?
- ☐ Are there any cells in the table that cannot have an entry?

Once cards have been positioned, the pupils should think about the possible values of each number before rounding. They could then speculate about the upper and lower bounds of each number.

Same calculation – different problem is a task in which pupils work out one division calculation that arises from a variety of word problems. The solution for each problem involves the same division calculation but needs a different form of answer or a different degree of accuracy. Some examples are provided on page 25.

Matching numbers to the values they might have taken before rounding forces pupils to recognise that a rounded number could represent a range of values.

From a set of prepared cards, pupils should choose a collection that could represent rounded versions of the same number. In each case they should list the numbers in the order implied by the rounded values and state the degree of accuracy to which the number on the card could have been rounded. Finally, based on the evidence they have, they could add the upper and lower bounds of the collection. For example:



These numbers could be listed as 3 (to 1 s.f.), 3.0599 (to 4 d.p.), 3.06 (to 2 d.p.), 3.060 (to 4 s.f.), 3.1 (to 2 s.f.), with lower bound 3.059 85 and upper bound 3.059 95.

Ensure that pupils are given sets of cards that will provide some conflict, including some cards that do not belong to any collections.

Extend the activity by adding another card to a collection and asking:

- ☐ What changes do you now need to make to the lower and upper bounds?

As a further extension, ask pupils to make up some cards of their own to challenge another group.