Geometric reasoning: lines, angles and shapes

While the whole of the contents of this booklet could be described as geometrical reasoning, the National Curriculum and the *Framework for teaching mathematics: Years 7, 8 and 9* both identify work on lines, angles, triangles and other rectilinear shapes and properties of circles and 3-D shapes under this heading. Much of this work in written mathematics relates to traditional areas of study. In terms of ideas and understanding, however, links to other aspects of shape, space and measures are very evident. Work on similarity (in the geometric reasoning section), for example, arises out of enlargements (transformations) and contributes significantly towards pupils' understanding of trigonometry (measures and mensuration). Teachers often build upon pupils' experiences of using *repeat* in Logo to construct regular polygons (constructions and loci) to establish a means of calculating exterior angles (geometric reasoning).

Mental imagery and visualisation are key elements in developing pupils' capacity to solve problems, especially those in geometry. Pupils need to become practised in constructing chains of reasoning for their hypotheses about properties of shapes or sizes of angles. They should be taught to distinguish between the conventions of geometry, the definitions or givens and the properties that are derived from them. Developing understanding and using chains of reasoning, as part of collaborative activities in the classroom, increase the likelihood that this reasoning will become a routine part of geometric thinking and thus allow greater access to more challenging problems. It is common to deal with conventions rather too quickly but, as geometry is about linking ideas and communication, these are the necessary building blocks of written proof and justification.

This section deals with the processes required to solve geometric problems mentally. The intention is to develop a mental facility to recognise the geometric features in a shape. These might be inherent in the shape or introduced to the shape by construction. Recognising these features is helpful in solving problems. Time spent developing these skills at an early stage can help to increase pupils' confidence and competence when later problems become quite complex.

Recognise and explain properties of given shapes to calculate angles and/ or derive other properties:
knowing the angle sum at a point, on a straight line and in a triangle
using angle properties of equilateral, isosceles and right-angled triangles
using angle properties of intersecting and parallel and perpendicular lines
knowing and using geometric properties of triangles and quadrilaterals in classifying different types of quadrilateral
knowing angle and symmetry properties of polygons and using them to solve problems
explaining and justifying geometric inferences and deductions, using mathematical reasoning
knowing and understanding conditions for congruent triangles to find angles and in geometric proofs
identifying and using properties of circles to solve problems

The Framework for teaching mathematics: Years 7, 8 and 9, supplement of examples, pages 178 to 191, provides contexts in which pupils should develop mental processes in geometric reasoning.

Further support is available in the Key Stage 3 National Strategy publication:

- *Key Stage 3, Mathematics study modules*, modules 5 and 6: resource 10g, Transformations. (DfES 0156/2004 G)

Classifying cards could include:

- diagrams of shapes;
- names of shapes;
- properties of shapes, for example, *angles*, *sides*, *symmetries*, *diagonals*.

This task will involve pupils, working in pairs or small groups, in negotiating the meaning of geometric terms. Although written output may be minimal, important diagnostic feedback can found by listening and observing as pupils, quite literally, lay their thinking out on the tables.

Sorting properties could be a follow-up to the classifying task. Examples of such tasks can be found in the photocopiable resource from ATM, *Geometry games* (ISBN 1 898611 38 6). A simple activity from this collection is called *Sort the properties*. In this game, pupils work in pairs or groups of four to sort two types of card showing names and properties of quadrilaterals (see resource 1, *Sort the properties*, pages 33 and 34). They arrange the cards in pairs, so that each pair makes a true statement. Once this has been done you can explore whether the solution is unique or not. Further discussion may involve a notion of 'minimum' collections that, by themselves, are sufficient to define a shape. This can lead to some unlikely definitions, for example, 'a quadrilateral in which diagonals bisect each other'. Pairs could attempt to compile different minimum collections of cards and present them to another pair asking, 'Is this enough? Is it too much?' The checking pair should look at two criteria.

- Is it possible to use these properties to draw any shape other than the target shape?
- Is it possible to remove a property card and still only be able to draw the target shape?

For ideas for card sets for these activities, see the Year 8 expectations in *The Framework for teaching mathematics: Years 7, 8 and 9*, supplement of examples, page 187.

For a similar activity to use with an interactive whiteboard or computer screen projection system, see *Embedding ICT @ Secondary: Mathematics* (DfES 0812/2004), page 19.

Geometry games (ISBN 1 898611 38 6) from ATM includes a wealth of games aimed at developing geometrical reasoning (**www.atm.org.uk**).

Sharing the image involves one pupil interrogating another in order to identify a mystery shape. The mystery shape is visible only to the answering pupil, as a sketch or object, on a geoboard or on the screen of a computer running dynamic geometry software. The interrogating pupil must identify the shape by asking questions that require only 'yes/no' answers, using precise language relating to the geometrical properties of the shape. The aim is to identify shapes by asking as few questions as possible. This task helps pupils to communicate accurately and concisely, to develop their ability to transmit and receive information, to enhance their powers of mental imagery and to develop mathematical vocabulary.

Always, sometimes or never true? generates fruitful discussion. Refer to the geometric properties of quadrilaterals outlined on page 187 in the Year 8 outcomes of the *Framework for teaching mathematics: Years 7, 8 and 9*. From these properties, compose a set of statements that pupils can classify as 'always true', 'sometimes true' or 'never true'. For example:

- The diagonals of a parallelogram bisect each other.
- A kite has rotational symmetry of order 2.
- An isosceles trapezium has unequal diagonals.

Pupils, working in pairs on this activity, can usefully share their solutions with another pair, who may have reached different conclusions to their own. They must use geometrical argument to justify their conclusions. Plenary prompts might be:

- How do you know that the diagonals of a parallelogram *always* bisect each other?
- Is it possible for a kite *sometimes* to have rotational symmetry of order 2? In this case, how would you rename the shape?
- What property of an isosceles trapezium means it will *never* have unequal diagonals?

An alternative to this approach is to ask pupils to devise statements that fit the criteria *always*, *sometimes* or *never true* in the context of a particular type of quadrilateral.

Rather than using the terms *always*, *sometimes* or *never true*, Year 9 pupils working on such activities could be introduced to the language of proof: *necessarily true*, *undetermined* and *necessarily not true*.

Visualising a square dissection sets pupils the task of visualising a square and creating 'in their mind's eye' a right-angled triangle and a trapezium, formed by a single cut from a vertex to the mid-point of an opposite side. After agreeing this image, pupils could attempt further tasks that involve:

- creating a mental picture of the different shapes made by joining equal sides of the right-angled triangle and trapezium;
- naming and describing (not drawing) the shapes to check that all possibilities have been found;
- producing a chain of reasoning to explain how they know they have found all the shapes possible.

A variety of short visualisation activities is available in the *Framework for teaching mathematics: Years 7, 8 and 9*, supplement of examples, pages 184 to 185.

Designing quadrilaterals is an activity in which pupils are asked to choose any quadrilateral with a specified property. They consider changes they could make to the shape while maintaining the stated property. They state what new shape is produced and why. For example, 'diagonals at right angles' could link to a square as a start point. The square could be modified by making the two intersecting diagonals longer or shorter rather than the same length. So pupils would nominate *kite*, *rhombus* and *square*. They could then pick one image and describe it to their neighbour. They should focus on particular combinations of properties and how the collections of these produce new shapes, for example, diagonals equal in length with both/one/neither bisected; diagonals unequal in length with both/one/neither bisected. In all cases it is important that pupils try to generate mental images first but they might usefully support these in subsequent discussions by means of dynamic geometry software.

Further opportunities for pupils to explore the links between properties of diagonals of a quadrilateral and the quadrilateral itself can be found in the QCA booklet, *Developing reasoning through algebra and geometry*, page 13: Rich activity, local deduction, and page 17: Is it still true? (ISBN 1 85838 550 4) See **www.qca.org.uk**.

Wise words requires pupils to describe succinctly what they see. They will need a set of eight cards, each showing a different quadrilateral. Pupils, working in pairs, choose one image and make two statements about it. One statement should be about *either* sides *or* angles and the other statement should be about the diagonals. The information given should be sufficient to define the generic quadrilateral; for example, *opposite sides are parallel* and *diagonals bisect each other* would not be sufficient to define a square. Each statement must use only one key word (*parallel, equal, bisect, perpendicular, supplementary)* and must not reveal the name of the quadrilateral shown on the card. The other pair tries, by asking as few 'yes/no' questions as possible, to identify the shape.

Using a **Two-way table**, such as the one below, can engage pupils in discussing geometrical properties of quadrilaterals and other shapes. Pupils are asked to write the names of appropriate quadrilaterals in the spaces and to consider whether any spaces will remain empty and, if so, why.

		Number of pairs of parallel sides		
		0	1	2
Number of pairs of equal sides	0			
	1			
	2			

Other properties for sorting include:

- number of right angles;
- number of lines of symmetry;
- number of sides;
- number of sides of different length;
- order of rotational symmetry.