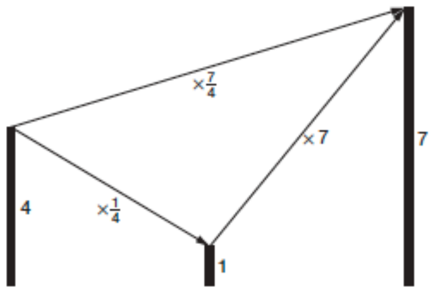
Repeated scaling

In the Year 8 unit, pupils used the image of parallel number lines to explore multiplicative relationships. This image is continued, but simplified to the use of line segments. Note that:

- The diagrams can be rough sketches.
- It helps to think in terms of 'expansion' and 'reduction' of the line segment.
- Depending on the class, you might introduce the mathematical term 'enlargement', for example:
  - expansion by a factor of 4 is equivalent to enlargement by a scale factor of $\times 4$;
  - reduction by a factor of 4 is equivalent to enlargement by a scale factor of $\times \frac{1}{4}$.

This will help pupils to make a link with enlargement in phase 2 of the unit. It will of course be necessary to discuss the apparent contradiction in using the term 'enlargement' for a reduction!

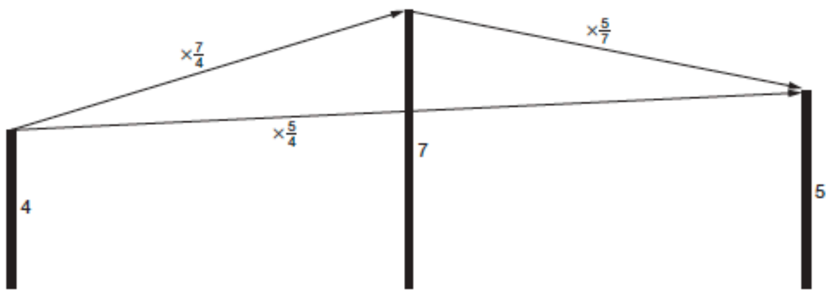


Quickly recap the principle of scaling between any two numbers in either direction using only multiplication. For most pupils the interim steps of scaling down to 1 will be dropped after the initial recap.

For example, ask:

- What single number could you multiply 4 by to scale it to 1?
- What about the inverse… from 1 to 4?
- What single number could you multiply 1 by to scale it to 7?
- What about the inverse… from 7 to 1?
- What number could you multiply 4 by to scale it to 7 in a single step?
- What is the single multiplier coming back from 7 to 4?

Move on to consider successive scalings.



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Use the prompts below for a variety of start, middle and end points including non-integer values. For example, ask:

- What single number could you multiply 4 by to scale it to 7?
- What single number could you multiply 7 by to scale it to 5?
- What number could you multiply 4 by to scale it to 5 in a single step?
- How is this related to the two steps we have described from 4 to 7 and from 7 to 5?
- Can you see that multiplying by the single scale factor ($\frac{5}{4}$) is equivalent to multiplying in two steps by the two separate scale factors ($\frac{7}{4}$ and $\frac{5}{7}$)?

$$4 \longrightarrow \boxed{\times\tfrac{7}{4}} \longrightarrow 7 \longrightarrow \boxed{\times\tfrac{5}{7}} \longrightarrow 5$$

$$4 \longrightarrow \boxed{\times\tfrac{7}{4}\times\tfrac{5}{7}} \longrightarrow 5$$

$$4 \longrightarrow \boxed{\times\tfrac{5}{4}} \longrightarrow 5$$

- Use your calculator to check each step of the scaling and to confirm this equivalence. (Point out the rounding error which can occur here.)

$$4 \longrightarrow \boxed{\times 1.75} \longrightarrow 7 \longrightarrow \boxed{\times 0.714} \longrightarrow 5$$

$$4 \longrightarrow \boxed{\times 1.75 \times 0.714} \longrightarrow 5$$

$$4 \longrightarrow \boxed{\times 1.25} \longrightarrow 5$$

- Can you see that the single multiplier (1.25) is the product of the multipliers used in the two-step scaling (1.75×0.714)?
- Repeat the above for the inverse scalings, from 5 to 7 to 4 noting that beginning with the fractional form of the scale factors is useful here.

Link to multiplication and division of fractions

Although not addressed in this unit, these points are worth noting:

- Successive scalings $\times\frac{b}{a}$ and $\times\frac{c}{b}$ can be thought of as expanding by a factor of *b* and reducing by a factor of *a*, then expanding by a factor of *c* and reducing by a factor of *b*. Expansion and reduction by factors of *b* can be observed to 'undo' one another. This may help to explain the result $\frac{b}{a} \times \frac{c}{b} = \frac{c}{a}$ in terms more meaningful than 'cancelling' the *b*s.
- The inverse of $\times\frac{b}{a}$ is $\div\frac{b}{a}$. However, the inverse can also be expressed as the multiplier $\times\frac{a}{b}$. So $\div\frac{b}{a}$ and $\times\frac{a}{b}$ are equivalent operators, which explains the infamous (familiar but frequently not understood) rule for dividing by a fraction: 'turn the fraction upside down and multiply'.

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