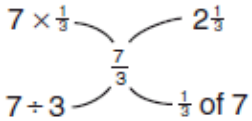
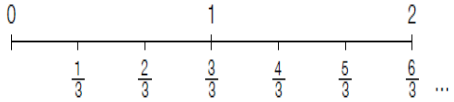
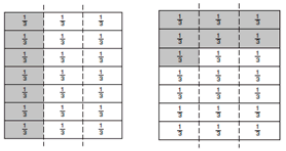
Summary table: support for teaching proportional reasoning

	The teaching notes support these…… **concepts & relationships**	The short mental activities and teaching notes develop this… **language and notation**	The teaching notes and resources for supporting images help to … **make connections**
Fractions as part of proportional reasoning	Fractions as numbers (positions on a number line), multiples of unit fractions: • proper fractions • improper fractions. The importance of identifying what constitutes 'the whole', i.e. what object or quantity is being divided. Fractions as multiplicative operators: • unit fractions, e.g. $\frac{1}{3}$ of • proper fractions, e.g. $\frac{2}{3}$ of • improper fractions, e.g. $\frac{7}{3}$ of	Visual representation of the relationships and equivalences that pupils need to understand and use:  [Note: To be mathematically accurate 3 x 7 is read as '3 multiplied by 7' and means '7 lots of 3'. However in these resources we use the conventional language of 3 x 7 as 'three sevens'.] Language clarifying the object or quantity which is being divided, e.g. half of the chocolate bar; half of the 6 chocolate bars. See: *Short mental activities* section for activities which enable pupils to develop their use of the language of fractions as operators as well as numbers. Such regular opportunities will refine pupils' mental skills in using the commutativity of fraction multiplication as part of proportional reasoning.	Many primary pupils can offer images to explain their understanding of the part/whole aspect of fractions and fractions as numbers, e.g. on number lines or as parts of a chocolate bar.    Add to these images by using a visualiser with *Fraction stacks* or *ITP Fraction stacks* to support understanding of the commutative nature of fraction multiplication. The first image below is shaded to show 'one third of seven' and the second to show 'seven lots of one third'. The aim is to help pupils see that the shaded parts are equal $\frac{1}{3} \times 7 \quad = \quad 7 \times \frac{1}{3}$  You will find further information to establish secure foundations to the understanding of fractions in *What is a fraction?*.

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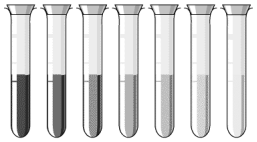
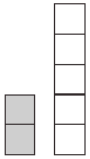
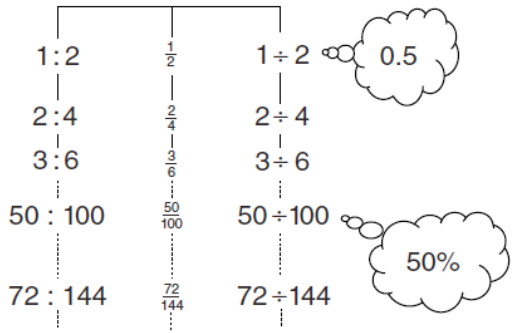
Ratio as part of proportional reasoning	Ratio is a way of comparing quantities: • parts of a single quantity, e.g. mixing paint • one quantity with another quantity, e.g. real life distance and distance on a map. The importance of identifying what quantity is the basis for comparison. Ratios, fractions, decimals and percentages as equivalent ways of comparing.	Ratio (comparisons <u>of parts of a single quantity)</u> When ratio is used to compare quantities that are parts of a whole we are also describing proportions, some connected phrases are: 'in every' and 'out of every' e.g. 1 'in every' 10 children does not eat breakfast, 5 'out of every' 10 dogs dream. Ratio (comparisons of <u>one quantity with another)</u> When ratio is used to compare one quantity with another, some connected phrases are: 'to every' or 'for every' or 'as many as', e.g. the scale drawing shows 1cm to every 50cm, 'for every' 2 bags of crisps you get one sticker, or there are twice 'as many' bags of crisps 'as' stickers. These notations and their associated language describe the relationships and equivalences that pupils need to understand and use: • The ratio of 7 to 3 equals 7:3 • The inverse ratio of 7 to 3 is 3 to 7 which equals 3:7	Ratio (comparisons of <u>parts of a single quantity)</u> Many pupils will have an understanding of ratio as a way of making comparisons between the parts that make up a single quantity, e.g. diluting chemicals, sharing out a bill for a meal.  Ratio (comparisons of <u>one quantity with another)</u>  Develop pupils' thinking about ratio as a comparison of one quantity with another by using the image of ratio strips. • no. of black blocks : no. of white blocks = 2:5 • no. of white blocks : no. of black blocks = 5:2 • no. of black = $\frac{2}{5}$ x no. of white, no. of white = $\frac{5}{2}$ x no. of black • no. of black = 0.4 x no. of white, no. of white = 2.5 x no. of black • no. of black = 40% of no. of white, no. of white = 250% of no. of black

		• 7:3 and $\frac{7}{3}$ are equivalent notations for comparing two quantities. This means that $\frac{7}{3}$ ('seven thirds of') is the relative size of one quantity compared with the other when the ratio of the quantities is 7:3 • $\frac{7}{3}$ is the scaling factor from one quantity to the other when the ratio of the quantities is 7:3 • Equivalence of operators, for example,  See: *Short mental activities* section for activities which focus on the comparison of quantities, giving pupils opportunities to become more fluent in • linking together language and notation of ratio • exploring contexts for equivalent ratios • explaining the connections between ratios and fractions .	See *Ratio strips* resources for notes about using the ratio strips image to help develop pupils' appreciation of ratio as a way of comparing one quantity with another. This underpins many big ideas in secondary mathematics, e.g. scales on maps and drawings, rates, percentages, enlargement, trigonometry.

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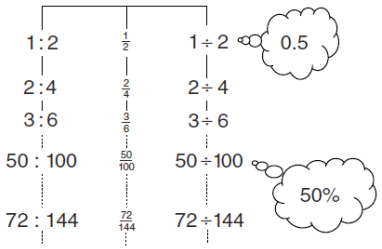
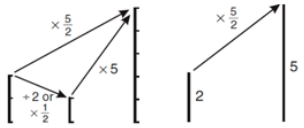
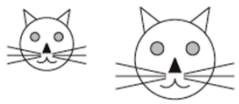
Scaling as part of proportional reasoning	Identifying multipliers/scale factors. Establishing a single multiplier between any two numbers and identifying the inverse multiplier. The importance of identifying what constitutes the 'original' and hence the scaling operation (including the single multiplier) required. Making connections between single operator scaling and unitary methods. Using ratio and scaling to explore similarity comparing: • internal dimensions of a shape • corresponding dimensions between shapes.	Using the fact that dividing by a number is equivalent to multiplying by its reciprocal. Using reciprocals as multiplicative inverses, that is multiplying by $\frac{1}{3}$ is the inverse of multiplying by 3. Multiplying by $\frac{7}{3}$ is equivalent to multiplying by $\frac{1}{3}$ (or dividing by 3) and multiplying by 7. Understanding that $\frac{7}{3}$ is the scaling factor from one quantity to the other when the ratio of the quantities is 7:3. $\frac{7}{3}$ ('seven thirds of') is the relative size of one quantity compared with the other when the ratio of the quantities is 7:3). Equivalence of operators, for example,  See: *Short mental activities* section for activities using the language of scaling to increase or decrease the size of a number. Regular use will refine mental skills in describing and approximating scaling effects and making links to ratio.	The first image below shows three parallel graduated line segments, where the middle one shows an interim step of multiplication or division (as used in unitary method). The second image is without graduation and interim step i.e. shows a single multiplier between any two numbers.  See notes and resources for *Line segments.* Scalings associated with 2D shapes. See *Geometrical images of scaling.*  These can be used to identify 'within' and 'between' aspects of dimensions of similar shapes. Using ratio and scaling to explore similarity comparing: • internal dimensions of a shape • corresponding dimensions between shapes.

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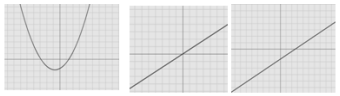
Proportional sets as part of proportional reasoning	Generating proportional sets of numbers by • starting from a pair of numbers, apply the same multiplier to each to create another pair of numbers in the same ratio; repeat the process with any multiplier. E.g. tables of conversions. Recognising the relationships that make two sets of numbers proportional by: • checking for equality of ratios between corresponding pairs of numbers, or • establishing whether the relationship is a linear one, and then whether the line passes through the origin. Application to: • geometry, such as similarity, enlargement, and trigonometry • number and algebra, such as constant rates of change, for example, gradient, speed, and growth.	Using the term 'proportional' to describe relationships, e.g. the circumference of a circle is proportional to its diameter. Comparing quantities that are parts of a whole, e.g. Facts which generate proportional sets: 1 'in every' 10 children does not eat breakfast, 5 'out of every' 10 dogs dream. [Note it is possible to describe a proportion which does not generate sets of proportionally related facts, e.g. 5 'out of the' 8 classes went to the theatre.] Tabular representation of proportional sets, e.g. coordinates of points on a straight line passing through the origin. Algebraic representation of a proportional relationship between two variables, y and x: $y = kx$, where k is a constant. Using the term 'rate' to describe proportional relationships, e.g. currency exchange rates. Connected language is 'for every', 'in every', 'to every' and 'per', e.g. km per hour. See *Short mental activities* for activities enabling pupils to develop their language describing proportional sets and exploring relationships between sets of numbers in and out of contexts. Regular opportunities will refine pupils' mental skills in making connections between proportional sets, rates, and ratios for example.	Graphs of proportional sets of numbers and comparison with other linear and non-linear functions.  Use images of geometric shapes, including right-angled triangles, to identify 'within' and 'between' aspects of dimensions, applying ratio and scaling to explore similarity comparing: • internal dimensions of a shape • corresponding dimensions between shapes and making the link to proportional sets of numbers. See *Photographic enlargements* and *Shadow images* resources which offer images to support exploration of relationships between ratios of dimensions within and between shapes.