Fraction operators

Rationale

The first of the following sequences (which are much easier to demonstrate than to explain on paper) establishes connections between multiples of a fraction and fractions of a quantity, leading to a recognition of the equivalence of different expressions, such as $\frac{7}{3}$, $7 \times \frac{1}{3}$, $\frac{1}{3}$ of 7 and $7 \div 3$. In the second sequence, this is extended to finding non-unitary fractions of a quantity, for example $\frac{5}{3}$ of 7.

Note that:

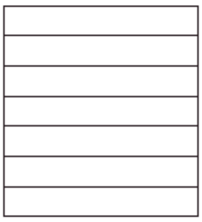
- Pupils will have met the representation of a fraction as part of a single shape. What may be new in the image here is stacking a set of rectangular strips, each representing 1, to represent a bigger number. Descriptions assume use of an OHP, alternatives being the interactive teaching program (ITP) *Images of fractions* or hand drawings on a flipchart or board. The inaccuracy of hand drawn lines should be discussed but should not be a problem.
- Drawing on their understanding of symbols and language, pupils will usually read '×' as 'times', meaning 'lots of'. When dealing with fractions it makes sense to interpret 'of' as meaning 'times' and to write '×' in place of the word. (Eventually pupils realise the order of numbers in multiplication is unimportant.)

The development of the main part of the first two lessons in phase 1 is now described in two stages. The description uses thirds. Quarters and fifths can be approached in a similar way. Refer to the unit plan for a suggested teaching order.

Stage 1

Images of thirds: making the connection between seven thirds and one third of seven

When working through this sequence it is important to emphasise that the stack of strips represents the number seven.



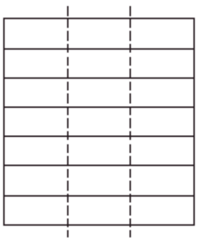
Show OHT 'Seven stack' (from resource FR1), covering all but the top strip. Say:

Here is a strip representing the number 1.

Now reveal the strips one by one. Say:

Together, count the strips as I show them: 1, 2, 3, ..., 7.

The whole stack represents the number 7.



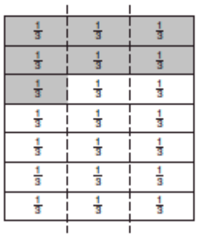
Q How could I draw vertical lines to divide the stack of seven into three equal parts?

The lines do not have to be exactly right – we can imagine that they are!

Q How could we check that each part is the same?

Q If I look at any one strip how could we label each small section?

Q Can I mark every small section as $\frac{1}{3}$? Why?



Show OHT 'Seven stack in thirds' and say:

Here is a marked-up diagram.

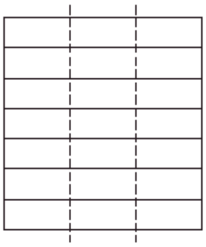
Q How many thirds are there altogether? **(record $\frac{21}{3} = 7$)**

We will now count the first few multiples of one third together and record them.

Beginning at the top left, count from left to right along the strips, shading each $\frac{1}{3}$ as you go.

Say together:	Record:
one times a third is one third	$1 \times \frac{1}{3} = \frac{1}{3}$
two times a third is two thirds	$2 \times \frac{1}{3} = \frac{2}{3}$
three times a third is three thirds which is one	$3 \times \frac{1}{3} = \frac{3}{3} = 1$
four times a third is four thirds which is one and a third	$4 \times \frac{1}{3} = \frac{4}{3} = 1\frac{1}{3}$
...	...
seven times a third is seven thirds which is two and a third	$\mathbf{7 \times \frac{1}{3} = \frac{7}{3} = 2\frac{1}{3}}$

Say that you will come back to this diagram.



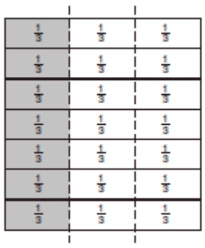
Now start with a fresh copy of OHT 'Seven stack'. As before, cover all but the top strip, then reveal them one by one.

Together, count the strips as I show them: 1, 2, 3, ..., 7.

As before, the whole stack represents the number 7.

Q In the last example, how did we divide the stack into three equal parts?

Q Looking at any one strip, how did we label each small section?



Show OHT 'Seven stack in thirds' (from resource FR1) and say:

Here is a marked-up diagram.

Q When we divide any shape into three equal parts how could we describe what we have done to the shape?

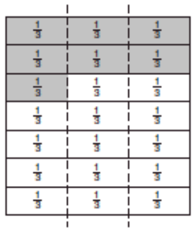
Q Could someone shade one third of this whole stack?

Q If I cover some of the strips is it still correct to say that one third of the visible stack is shaded?

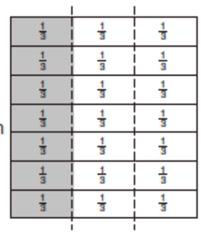
Cover the lower part of the stack, revealing a strip at a time as you count from top to bottom.

Say together:	Record:
one divided by three is one third of one which is one third	$1 \div 3 = \frac{1}{3}$ of $1 = \frac{1}{3}$
two divided by three is one third of two which is two thirds	$2 \div 3 = \frac{1}{3}$ of $2 = \frac{2}{3}$
...	...
seven divided by three is one third of seven which is seven thirds	$\mathbf{7 \div 3 = \frac{1}{3}}$ **of** $\mathbf{7 = \frac{7}{3}}$

Q Imagine the stack is larger. Could we continue counting?



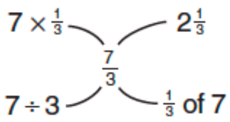
Now display the two diagrams side by side.



Q Do you remember how we counted seven thirds in the first diagram?

Q Do you remember how we counted to find one third of seven in the second diagram?

Q Are the answers the same? Why? Can we convince everyone?



Point out that you have linked lots of equivalent expressions. Illustrate these links in a web diagram, as on the left.

Point to different expressions in the web and ask pupils to read and relate each expression to one of the diagrams. *(See the notes for the key lesson on page 12.)*

Q Could we draw a similar web diagram for five thirds? For two thirds? ...

Repeating stage 1 for quarters and fifths

A decision needs to be made depending on the extent to which pupils appreciate the generality of the image, i.e. they can apply it to other fractions and see a definition of a non-unit fraction which is different to collecting together a number of unit fractions. For many classes, a repetition of the sequence for quarters and fifths will be appropriate. As a test of their understanding you might ask:

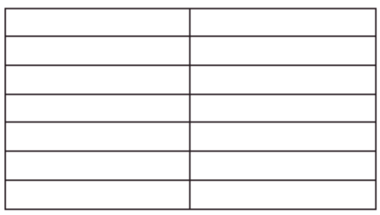
Q Can you draw a web of expressions equivalent to $\frac{11}{8}$ and explain all the connections in your diagram?

Stage 2

Images of thirds: extending beyond unit fractions to thirds of seven

In this second sequence, as well as extending beyond unit fractions of quantities, part of the aim is to create a link between 'lots of' and 'fractions of', hence linking the term 'of' with the operation multiplication.

Show OHT 'Two seven stacks' (from resource FR3) and say:



Q Can you see how this diagram shows two lots of seven?

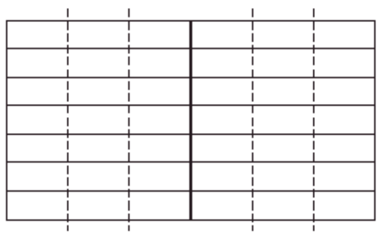
Now cover the right-hand stack.

Q How could I divide this lot of 7 into three equal parts?

Now cover the left-hand stack.

Q How could I divide this lot of 7 into three equal parts?

Show OHT 'Each seven stack in thirds' (resource FR3), saying: 'Here is a marked-up diagram.'



Cover the right-hand part of the stack, revealing a single column at a time as you count from left to right.

Say together:	Record:
one third of seven is seven thirds	$\frac{1}{3}$ of 7 = $\frac{7}{3}$
two thirds of seven is fourteen thirds	$\frac{2}{3}$ of 7 = $\frac{14}{3}$
three thirds of seven is twenty-one thirds...	**$\frac{3}{3}$ of 7 = $\frac{21}{3}$ = 7**
(point out this is the same as the initial stack, one lot of 7)	
four thirds of seven is twenty-eight thirds	$\frac{4}{3}$ of 7 = $\frac{28}{3}$
five thirds of seven is thirty-five thirds	$\frac{5}{3}$ of 7 = $\frac{35}{3}$
six thirds of seven is forty-two thirds	**$\frac{6}{3}$ of 7 = $\frac{42}{3}$ = 14**

Reflect on the final picture which shows 2 'lots of' 7.

Say together:	Record:
two **lots of** seven is two **times** seven	2 'lots of' 7 = 2 × 7
six thirds **of** seven is six thirds **lots of** seven	$\frac{6}{3}$ of 7 = $\frac{6}{3}$ 'lots of' 7
which is six thirds **times** seven	= $\frac{6}{3}$ × 7

Go back through the other 'fractions of' 7:

- Say and record in terms of multiplication.
- Ask pupils whether the answer will be less than, equal to or greater than 7.

Record these facts, asking pupils to provide the explanations:

- $\frac{1}{3}$ of 7 is the same as $\frac{1}{3}$ × 7 – 'of' means 'multiply'.
- These are the effects of fraction multipliers on the number 7:
  - $\frac{1}{3}$ × 7 and $\frac{2}{3}$ × 7 have answers less than 7 (a decrease).
  - $\frac{3}{3}$ × 7 is equal to 7.
  - $\frac{4}{3}$ × 7, $\frac{5}{3}$ × 7 and $\frac{6}{3}$ × 7 have answers more than 7 (an increase).
- The answer to $\frac{5}{3}$ × 7 can be found by doing the calculation (7 ÷ 3) × 5.

Q If I want to find '$\frac{5}{3}$ of' any number, will it be an **increase** in the number or a **decrease**? Why?

Q Using a calculator how would you find $\frac{2}{3}$ of 7? Or $\frac{4}{3}$ of 7? ...

Q Could we do something similar for other fractions such as quarters or fifths?

